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RESEARCH ON AEROSOL SCATTERING IN THE INFRARED

Final Report

Prepared by

Rudolf B. Penndorf

RESEARCH AND ADVANCED DEVELOPMENT DIVISION
AVCO CORPORATION
Wilmington, Massachusetts

Technical Report

RAD-TR-63-26

Contract AF19(604)-5743

Project No. 7670

Task No. 76704

June 1963

Prepared for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
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Space Science Department

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ABSTRACT

Theoretical studies have been carried out to investigate the scattering of light by spherical aerosols with the objective to obtain basic information useful for practical applications. Numerical data for Mie scattering have been analyzed to find general trends, to simplify interpolation problems, and to establish simple relationships between important parameters. Results of the research are given in the form of abstracts of 10 scientific reports and 6 related papers issued during the course of this contract. Revisions and additions to each report are given. An extensive bibliography of tables concerning Mie scattering and an atlas of scattering diagrams for six refractive indices from n = 1.1 to 1.5 and α = 0.5 (0.5) 10 are given as appendixes.

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ILLUSTRATIONS

ACKNOWLEDGMENTS

Appreciation is expressed for the long-term cooperation, assistance, and interest given by Dr. J. Howard, Dr. J. Garing, and Mr. L. Elterman of AFCRL. The studies and analyses reported in this document would not have been possible without the aid of a number of supporting personnel, including Misses Estelle Guilbault, Susan Tyler, and B. Willey; Mrs. Carol Kotce and Mrs. T. Schaler; and Mr. Frank Walsh; all of whom performed many hours of computation, tabulation, and plotting.

I. INTRODUCTION

The scattering of light by aerosols poses interesting theoretical as well as experimental problems. The atmosphere contains large quantities of particular matter, which herein are called "dust," that affect direct (solar) and diffuse radiation. Scattering and absorption of light occur not only in the ultraviolet (UV) and visible spectral range, but also in the atmospheric windows of the infrared (IR). Dust consists of particles which differ in size and chemical composition. The natural aerosol population in the troposphere is composed of particles originating from the ground. In the stratosphere, Junge (1961) has suggested that the natural population consists of three families: (a) one of tropospheric origin in the size range of less than 0.1 micron, (b) one of stratospheric origin in the size range between 0.1 and 1.0 micron, and (c) one of extraterrestrial origin in the size range greater than 1.0 micron. As to the chemical composition of stratospheric aerosols, particles composed of ammonium persulfate and ammonium sulfate make up almost all of the aerosol in the radius range from 0.1 to 1.5 microns,which particles, in turn, make up more than 90 percent of the total mass of the aerosol.

Light is scattered in the atmosphere by the molecules and the aerosol, and attempts have been made to infer the size distribution and altitude distribution of aerosols by using this effect; one of the more recent is that of Newkirk and Eddy (1963).

To interpret experimental results based on scattering of light, the theoretical data have to be available in a useful form. Nowadays, scattering functions can be computed for any size and any refractive index by electronic computers. Hence, no difficulty arises in getting the basic numerical data; but this is only the beginning because, in designing the experiments, it is helpful to have a feeling for how large the effect of aerosol scattering will be. In some cases, one desires to maximize the scattering effect; in other cases, one wants to minimize it. Furthermore, it is helpful to know the angular distribution of the scattering intensity and polarization. A point one has to watch out for is the uniqueness of the solution. This important point has to be checked very carefully. Identical scattering coefficients exist for different size parameters (within particular size ranges) because of the oscillating nature of the functions; or,to express it in physical terms, interference.

The purpose of this contract was to investigate theoretical scattering data computed by electronic computers for various refractive indices and size ranges to find general trends, simple but for experimental work accurate enough interpolation methods, and approximations useful for rough calculations. The work accomplished has been reported in scientific reports, which are summarized in this final report. Revisions and additions to each report are given if new information was obtained after the report was completed, or if later studies necessitated revisions of the original findings.

The applications of our results are manifold, and since scattering by spherical particles has been proven as an extremely helpful tool in many disciplines, no listing of possible applications of the results will be given.

References

Junge, C. E., C. W. Chagnon, and J. E. Manson, J. Meteorol. 18, 81 (1961).

Newkirk, G. and J. A. Eddy, J. Atmos. Sci.(to be published in 1963).

II. SCIENTIFIC RESULTS

In this section, a short description of the highlights of the results will be attempted. Three areas have been investigated: (a) the total scattering coefficients, (b) the forward scattering, and (c) the angular scattering coefficients.

2.1 TOTAL SCATTERING COEFFICIENT

The total scattering coefficient K has been investigated to find an approximation method which allows to construct a curve of K versus α, the size parameter, for any refractive index $n < 2$. Maxima and minima of K occur at specific size parameters. Choosing a normalized size parameter $\rho = 2\alpha(n - 1)$, it was found that

$$\rho_y = 2\pi(y \pm 1/4) \tag{1}$$

is a good approximation; the subscript y stands for the order of the extreme values, $y = 1, 2, \ldots$; the + sign applies to the maxima, the - sign to the minima. For $n = 1$, equation (1) is correct; but for $n > 1$, a correction term should be added, so that

$$\rho_y \quad n = 2\pi(y \pm 1/4) + 0.3\,(n - 1) \tag{2}$$

represents the best approximation.

The absolute value of K_y at the extreme values has also been obtained. It was found that a resonably good approximation for a smoothed extreme value $\overline{K}_y$ can be described by

$$\overline{K}'_y = 2 + \frac{4}{\rho_y} + \frac{4}{\rho_y{}^2} + \frac{29M}{\rho_y} - \frac{51M}{\rho_y{}^2} \tag{3}$$

for the maxima, and

$$\overline{K}''_y = 2 - \frac{4}{\rho_y} + \frac{4}{\rho_y{}^2} + \frac{8.01M}{\rho_y} - \frac{27.3M}{\rho_y{}^2} \tag{4}$$

for the minima. Here $M = (n^2 - 1) / (n^2 + 2)$. A graphical method to determine smoothed $\overline{K}$ for any α and any n has been described, based on computations of

ρ_y, $\overline{K}'_y$, and $\overline{K}''_y$ only.

Another problem, which has been tackled, concerns the absorption and scattering coefficients for small aerosols. Exact derivations of the equations for $K^{(e)}$, $K^{(a)}$, and $K^{(s)}$ have been obtained. The asymptotic expansion of the exact solution is given, where the leading term is identical with the Rayleigh approximation.

For nonabsorbing spheres, the total scattering coefficient for small size parameters is

$$K_{RRR} = \frac{8\,\alpha^4}{3}\left(\frac{n^2-1}{n^2+2}\right)^2 \left[1 + \frac{6}{5}\,\alpha^2\left(\frac{n^2-2}{n^2+2}\right) + \alpha^4\left\{\frac{3}{175}\left(\frac{n^6+41\,n^4-284\,n^2+284}{(n^2+2)^2}\right) + \frac{1}{900}\left(\frac{n^2+2}{2n^2+3}\right)^2\left[15+(2\,n^2+3)^2\right]\right\}\right] \quad . \tag{5}$$

For all practical purposes, the error remains below $\pm$ 1 percent to n = 1.4 and $\alpha < 1$. For $n \sim 2$, the error reached 15 percent at $\alpha = 1$. Figure 3 in Scientific Report 3 shows the error as function of α and n and can be used as a reliable guide in determining the upper limits of α and n for which equation (3) is useful.

For absorbing spheres, the complex index of refraction

$$\tilde{n} = n - i\varkappa \tag{6}$$

has to be used, and the total extinction coefficient for small size parameters is given by

$$K^{(e)}_{RR} = \frac{24 n \varkappa}{Z_1}\, \alpha + \left[\frac{4}{15}\, n \varkappa + \frac{20\, n \varkappa}{3\, Z_2} + 4.8\, n\varkappa \left(\frac{7\left(n^2 + \varkappa^2\right)^2 + 4\,(n^2 - \varkappa^2 - 5)}{Z_1^2} \right) \right] \alpha^3 + \frac{8}{3} \left\{ \frac{\left[\left(n^2 + \varkappa^2\right)^2 + (n^2 - \varkappa^2 - 2)\right]^2 - 36\, n^2 \varkappa^2}{Z_1^2} \right\} \alpha^4 , \qquad (7)$$

with

$$Z_1 = \left(n^2 + \varkappa^2\right)^2 + 4\,(n^2 - \varkappa^2) + 4, \text{ and}$$

$$Z_2 = 4\left(n^2 + \varkappa^2\right)^2 + 12\,(n^2 - \varkappa^2) + 9.$$

The total scattering coefficient is given by

$$K^{(s)}_{RR} = \frac{8}{3\, Z_1^2} \left\{ \left[\left(n^2 + \varkappa^2 \right)^2 + n^2 - \varkappa^2 - 2 \right]^2 + 36\, n^2 \varkappa^2 \right\} \alpha^4 \times \left\{ 1 + \frac{6}{5\, Z_1} \left[\left(n^2 + \varkappa^2\right)^2 - 4 \right] \alpha^2 - \frac{24}{3\, Z_1}\, n \varkappa\, \alpha^3 \right\} , \qquad (8)$$

and the total absorption coefficient as

$$K_R^{(a)} = K_R^{(e)} - K_R^{(s)} , \qquad (9)$$

or

$$K^{(a)}_{RR} = \frac{24\,n\,\varkappa}{Z_1}\,\alpha + \left[\frac{4}{15}\,n\varkappa + \frac{20\,n\,\varkappa}{3\,Z_2}\right.$$

$$+\ 4.8 n\varkappa \left(\frac{7\,(n^2+\varkappa^2)^2 + 4\,(n^2-\varkappa^2-5)}{Z_1^2}\right]\alpha^3$$

$$-\ 192\,(n\varkappa/\,Z_1\,)^2\ \alpha^4\ , \qquad (10)$$

leaving out higher terms in equation (8) because terms with α^5 have already been left out in equation (5).

Again the error against the exact solution has been determined (figures 7 and 11 in Scientific Report 3), showing that these formulas are useful for $\alpha < 1$ if $n < 2$ and $\varkappa < 1$.

These formulas, equations (5) to (9), are very useful because this small size range is difficult to compute using the exact Mie equations since the two recursion relations which provide the input for the first term are losing all eight significant figures for $K^{(a)}$ for large n, and $\varkappa$ in the normal programming procedure. It is therefore helpful to use these equations for $\tilde{n}$ and very small values of α, where special methods have to be employed to obtain reliable results using the exact Mie equations.

2.2 FORWARD SCATTERING

The forward scattering has been investigated because of its fundamental importance, and results are given for the range of n = 1.1 to n = 2. Again, the major oscillations appear as in K, and also the ripples. The advantages of various scattering parameters are outlined, showing the importance of the phase function p_M. The interpolation for small and large size parameters is possible because approximation formulas have been developed for the occurrence of the first "plateau," its width, the position of succeeding plateaus, as well as asymptotic values for large α.

The practical limits of the Rayleigh, Mie, and diffractive regions have been determined as function of α and n.

2.3 ANGULAR SCATTERING COEFFICIENTS

A large part of the effort has been devoted to an interpretation of the angular scattering coefficients i_1 and i_2 for the refractive indices n = 1.33 to 1.5. The objective has been to compose an atlas of angular scattering coefficients, which is now completed and given as appendix 6.2. Since the basic data are computed in steps of 5 degrees (for n = 1.33) and 10 degrees (for all other refractive indices), interpolative methods had to be devised so that continuous curves for the scattering coefficients could be drawn. It was found that the "altitude chart" offers the best method for graphically interpolating scattering data. It is important to find systems for the maxima (bright rings) and minima (dark rings) because the interpolation of the computed data is very much simplified if the angular position and the absolute value of the extreme values of i_1 and i_2 are know.

Many different and laborious attempts have been made to find a systematic behavior of the maxima and minima and to understand it. All the data have been investigated again, and the present position, modifying earlier results reported in Scientific Reports 4, 5, and 10, follows. It is believed now that the first interpretation of the systematic behavior of the bright and dark rings is not always correct.

For graphical interpolation, the "altitude chart" method has been developed for i_1, i_2, and i_θ and described in Scientific Reports 5 and 10. Based on the recent experience in finding a system for the diffractive as well as the reflective minima, it would seem that this altitude chart can be improved. The new altitude chart is based on the two parameters $u = \alpha \sin \theta$ and α, instead of u and ρ. Hence, one can use the symmetry at θ = 90 degrees and have two separate areas; i.e., one for θ = 0 to 90 degress, and the other for θ = 90 to 180 degrees; at θ = 90 degrees the two systems become identical. Ordinate α and abscissa $u = \alpha \sin \theta$ are used, covering all angles between θ = 0 and 90 degrees. At θ = 90 degrees, the system is changed so that, for the realm θ = 90 to 180 degrees, the abscissa is α and the ordinate $u' = \alpha \sin \theta'$, where $\theta' = \pi - \theta$. In figure 1, a schema of the new altitude chart is shown. It will be seen, in the examples which follow, that the diffractive minima and maxima are starting now at the top and move downward, whereas in the old system they start at the right-hand side, move horizontally to the left-hand side, a knee appears at θ = 90 degrees, and only in the forward scattering area do they move downward. The reflective minima will now move more or less horizontally from left to right hand, whereas in the old system each one possesses a different inclination to the basic parameters ρ and u. It is believed that this new altitude chart is better than the old chart in recognizing the two basic systems, especially around θ = 90 degrees, in aiding the interpolation and in determining the position and depth of the minima at points of interference of the diffractive and reflective systems.

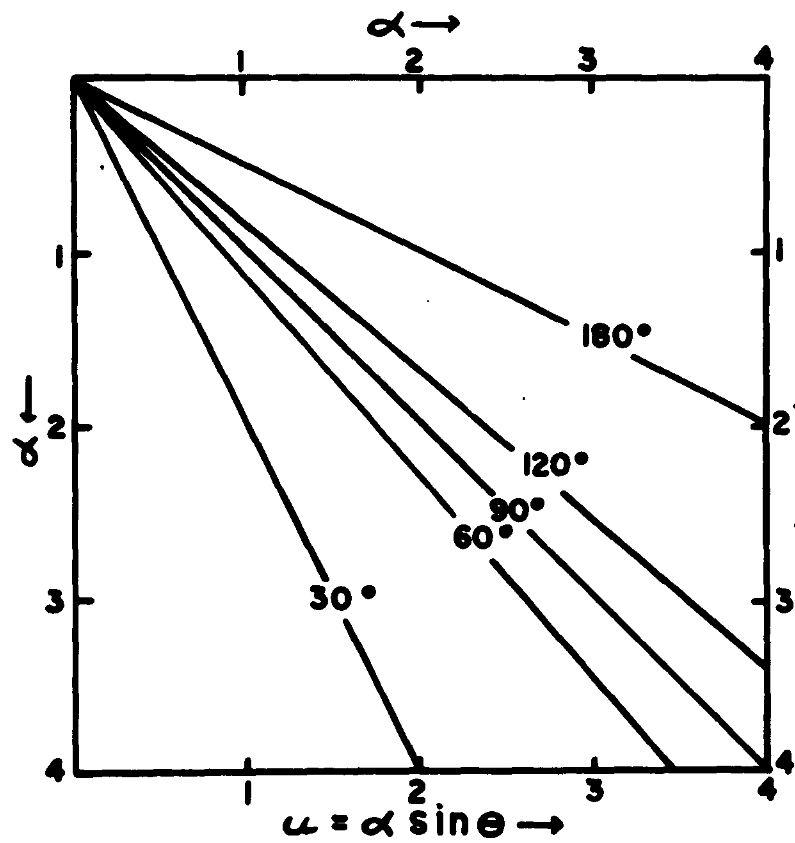


Figure 1 SCHEMA OF MODIFIED ALTITUDE CHART

Scattering angles (θ = 0 to 180 degrees) are represented by straight lines. Data are plotted at intersection of α with θ.

New diagrams of the angular position of maxima and minima have been constructed in the range n = 1.05 to 1.5, where the data for n = 1.05 to 1.3 are taken from Pangonis and Heller (1960), those for n = 1.486 from Rowell (1963), and the rest from the Penndorf and Goldberg tabulations (1953). Rowell's data are extremely valuable because they are based on computations in steps of $\Delta\theta$ = 1 degree. However, they are available only in diagrams and had to be transformed into their system. For i_1, the position of the trenches could also be determined from his diagram. Several samples are selected from the present collection of diagrams of angular position of maxima and minima (figures 2 and 4) and modified altitude charts (figures 3 and 5). For n $<$ 1.33, both the maxima (solid line) and minima (dashed line) are shown; for larger refractive indices only the minima, otherwise the diagrams would appear overloaded. The results for the complete collection are discussed, but it is not believed necessary to show them all.

For i_1, $\alpha < 7$, and low refractive indices (n$<$1.15), only diffraction plays a role; and the system of maxima and minima presents itself in a very simple fashion not only in figure 3 but also in figure 2. For n = 1.2, the first disturbance in the diffractive system is noted; i.e., the effect of internal reflection and refraction (reflective system for short). This disturbance begins in the backscattering area and becomes much clearer for n = 1.25 and 1.3 than for n = 1.2, and finally for n$>$1.3, only the first diffractive minimum remains undistrubed.

The reflective system starts for such refractive indices in the backward area and moves toward larger scattering angles as figure 2 indicates and more or less horizontal in the altitude charts (figure 3). The case for n = 1.3 is a good example of this behavior. The first diffractive minimum (solid line) is undisturbed moving from top to bottom. A maximum (dashed line) and a minimum start at about α=2.7 and u' = 1.4 (all extrema start as pairs); they follow the diffractive system rather well. The next pair of extrema starts at about α= 3.9 and u' = 1.6. This pair is primarily caused by reflection, and it moves more or less horizontally from left to right hand, some ripples being superimposed. The pair starting at about α= 4.7 and u' = 4.1 is part of the diffractive system, whereas the next pair at α = 5.7 and u' = 4.7 belongs to a reflective system. The position of the trenches (on the minima) and the peaks (on the maxima) are shown by large dots. The dots can be connected to distinguish the two systems. Dotted lines represent the minima of the diffractive system, thin solid lines the minima of the reflective system, and thin dashed lines the maxima of the reflective system.

It is quite obvious from this and other diagrams in figure 3 that trenches appear always at the intersection of the minima of the two basic systems and peaks at the intersection of the maxima of the two basic systems. If a minimum of one system intersects with a maximum of the other system, the minima or the

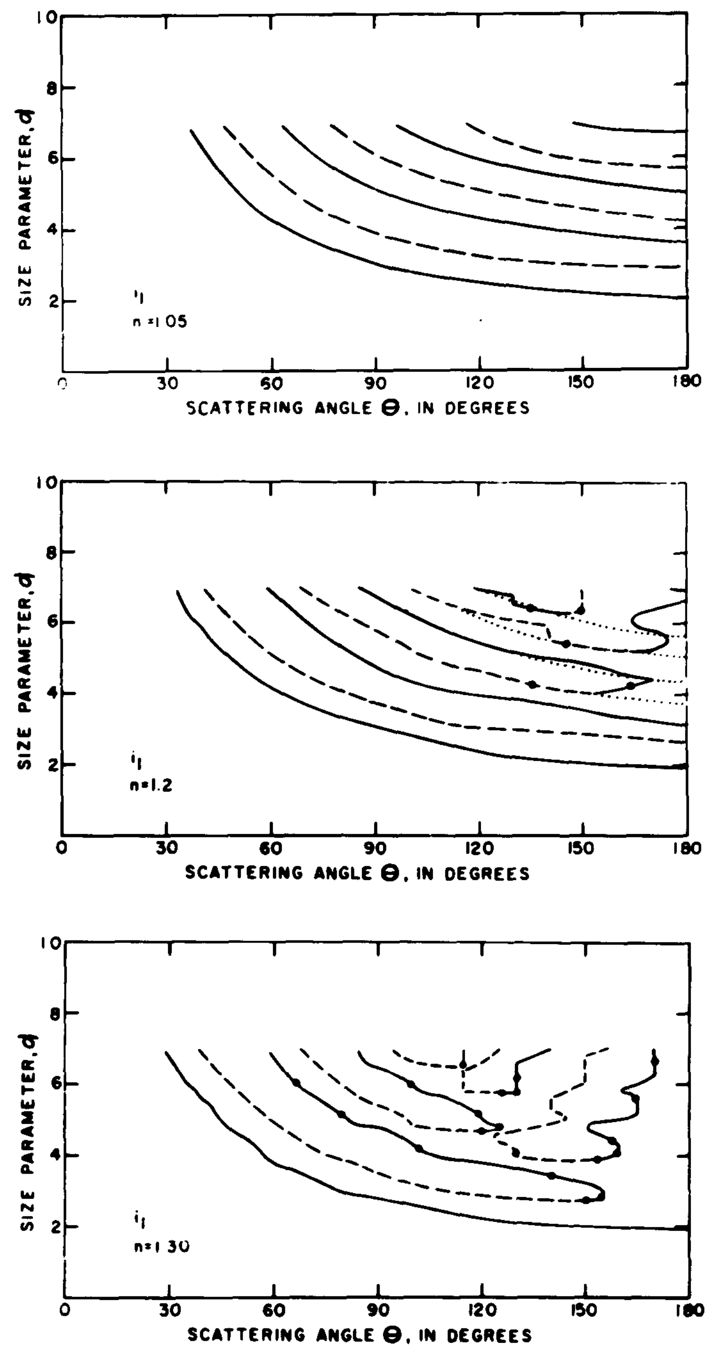


Figure 2 DIAGRAMS OF ANGULAR POSITION OF MAXIMA (BRIGHT RINGS) AND MINIMA (DARK RINGS) FOR INTENSITY FUNCTION i_1

Position of minima is indicated by solid lines, of maxima by dashed lines, and center position of trenches and peaks by full circles. Dotted lines indicate interpolation of diffractive minima, thin solid lines of reflective minima, and thin dashed lines of reflective maxima

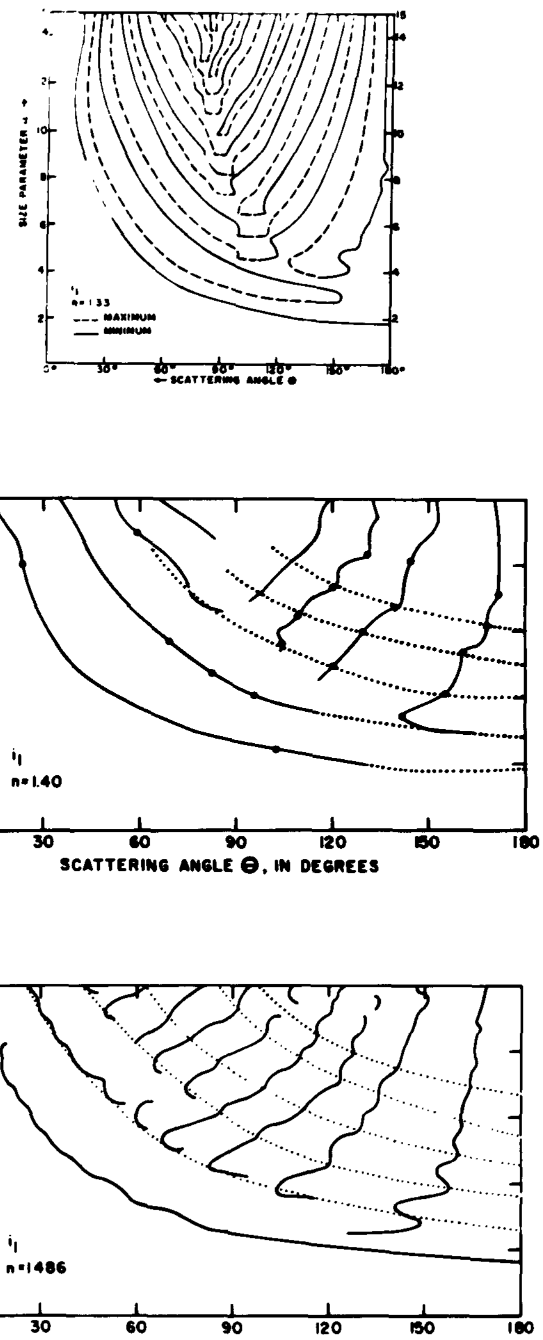


Figure 2 (Concl'd)

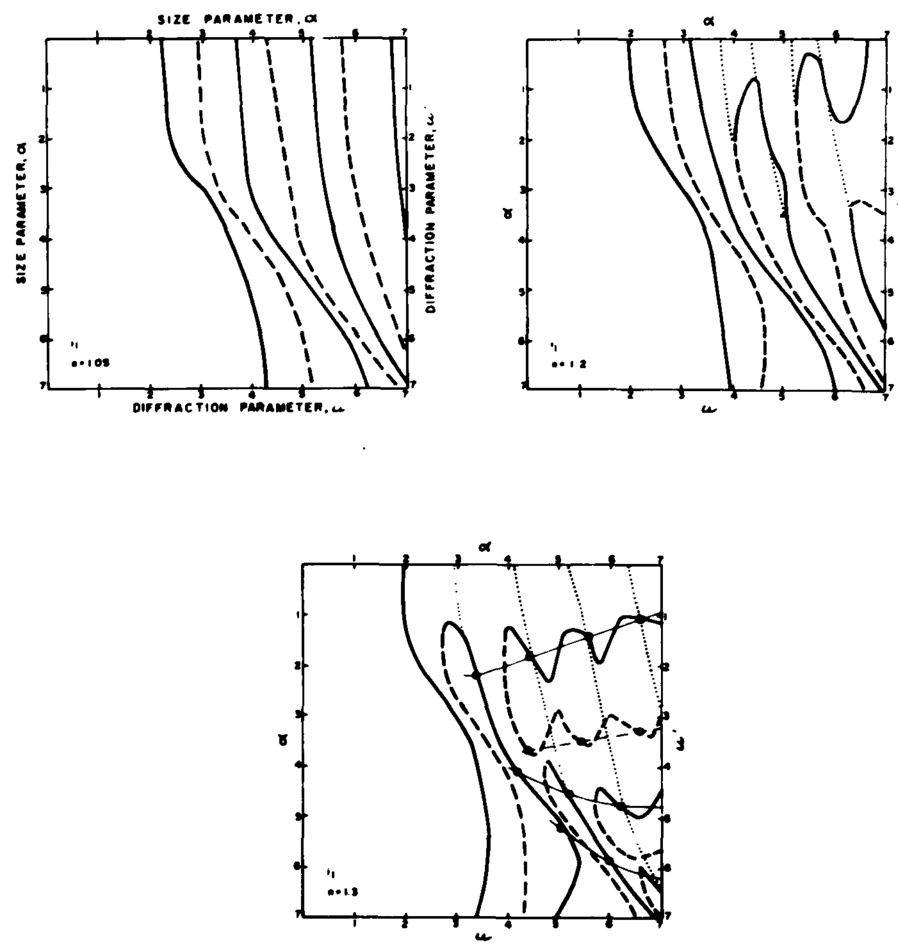
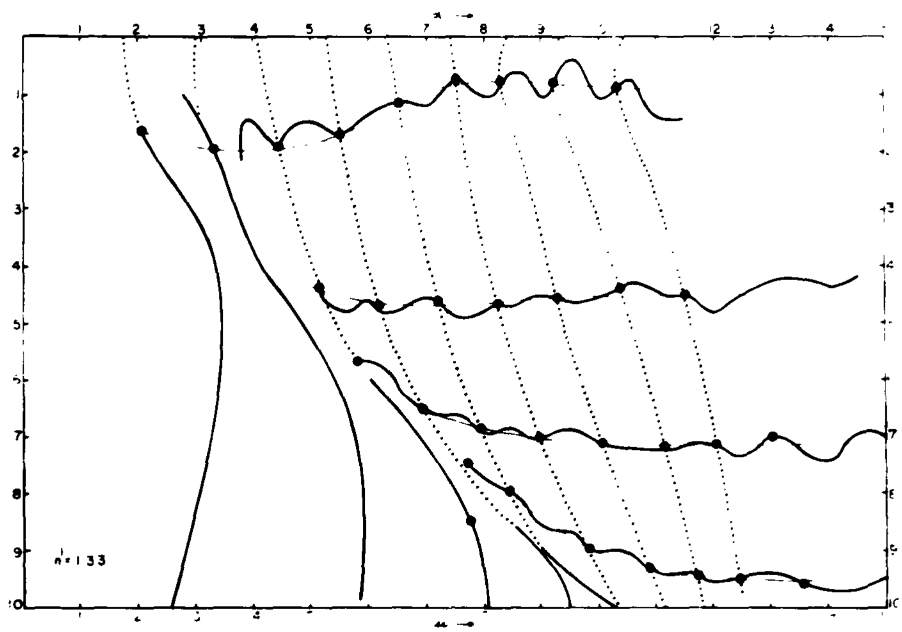


Figure 3 ALTITUDE CHARTS FOR INTENSITY FUNCTION i_1

Position of minima is indicated by solid lines, of maxima by dashed lines, and center position of trenches and peaks by full circles. Dotted lines indicate interpolation of diffractive minima, thin solid lines of reflective minima, and thin dashed lines of reflective maxima.



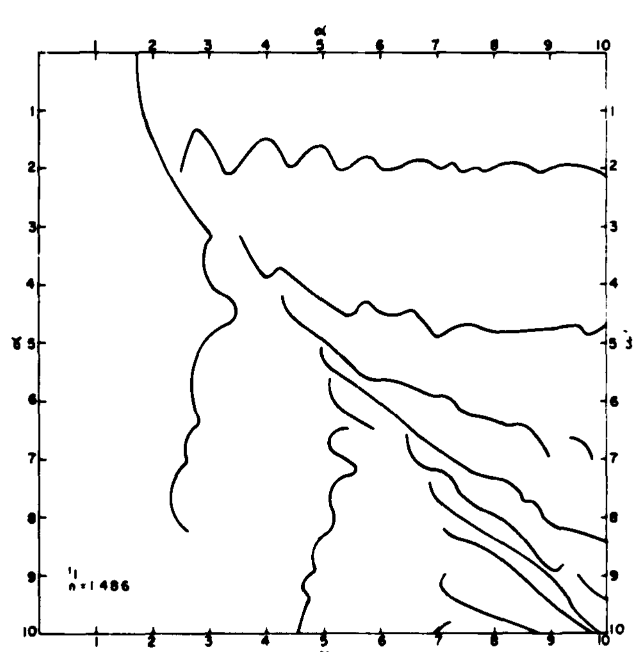


Figure 3 (Concl'd)

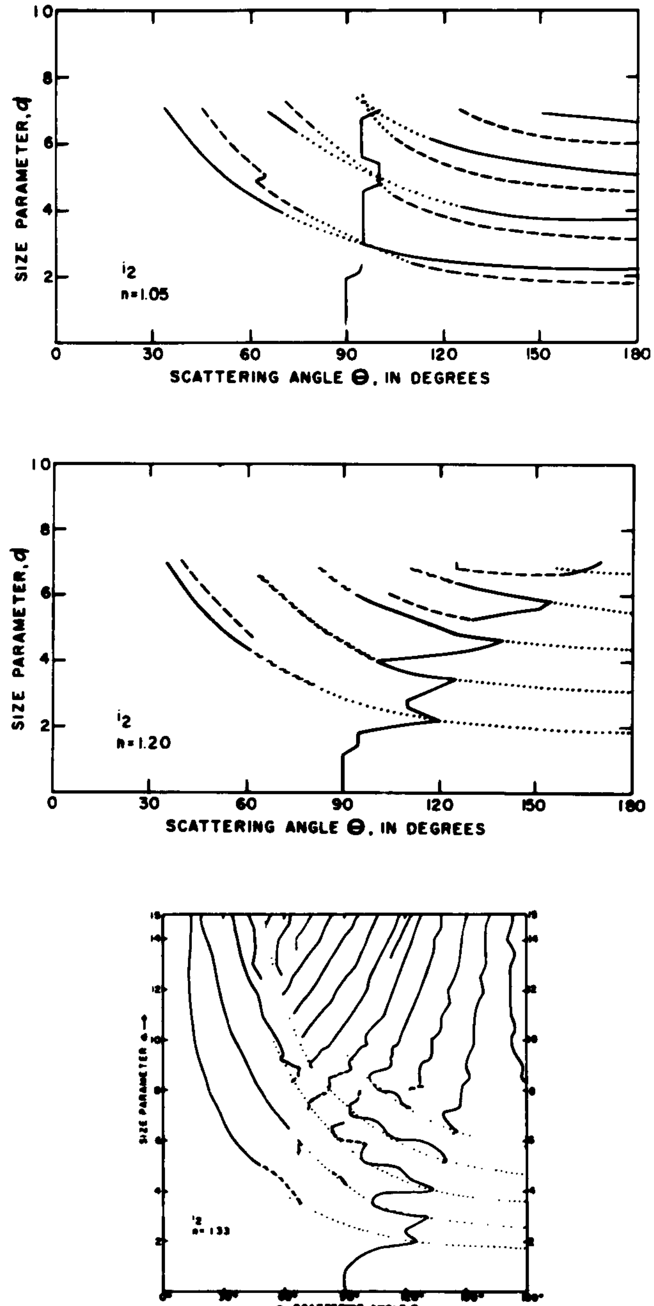


Figure 4 DIAGRAMS OF ANGULAR POSITION OF MAXIMA (BRIGHT RINGS) AND MINIMA (DARK RINGS) FOR INTENSITY FUNCTION i_2

Position of minima is indicated by solid lines, of maxima by dashed lines, and center position of trenches and peaks by full circles. Dotted lines indicate interpolation of diffractive minima, thin solid lines of reflective minima, and thin dashed lines of reflective maxima.

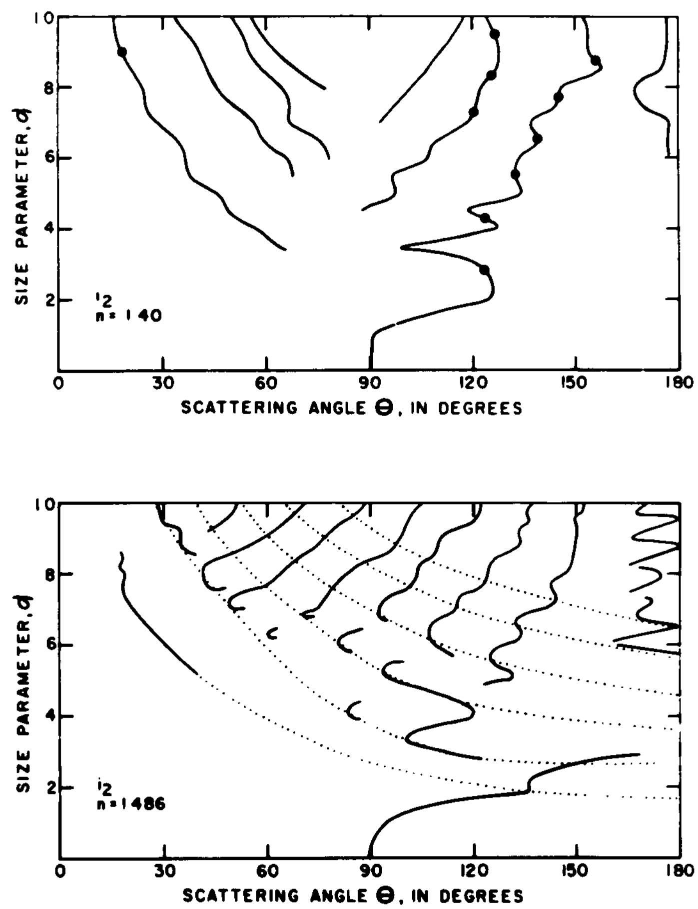


Figure 4 (Concl'd)

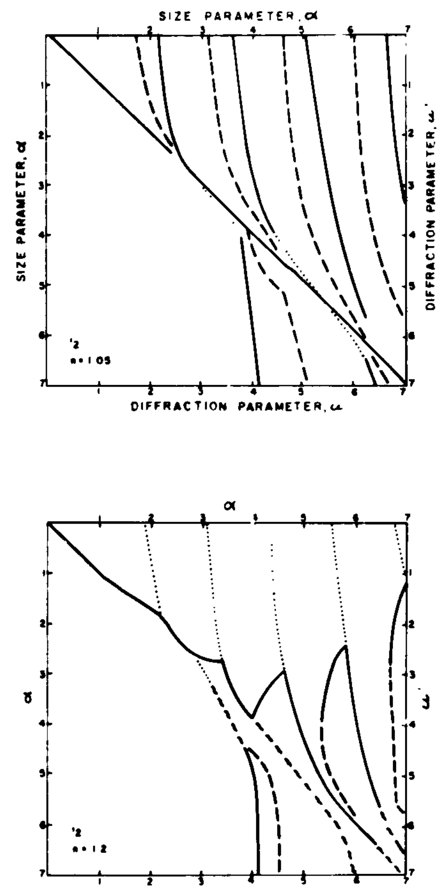


Figure 5 ALTITUDE CHARTS FOR INTENSITY FUNCTION i_2

Position of minima is indicated by solid lines, of maxima by dashed lines, and center position of trenches and peaks by full circles. Dotted lines indicate interpolation of diffractive minima, thin solid lines of reflective minima, and thin dashed lines of reflective maxima.

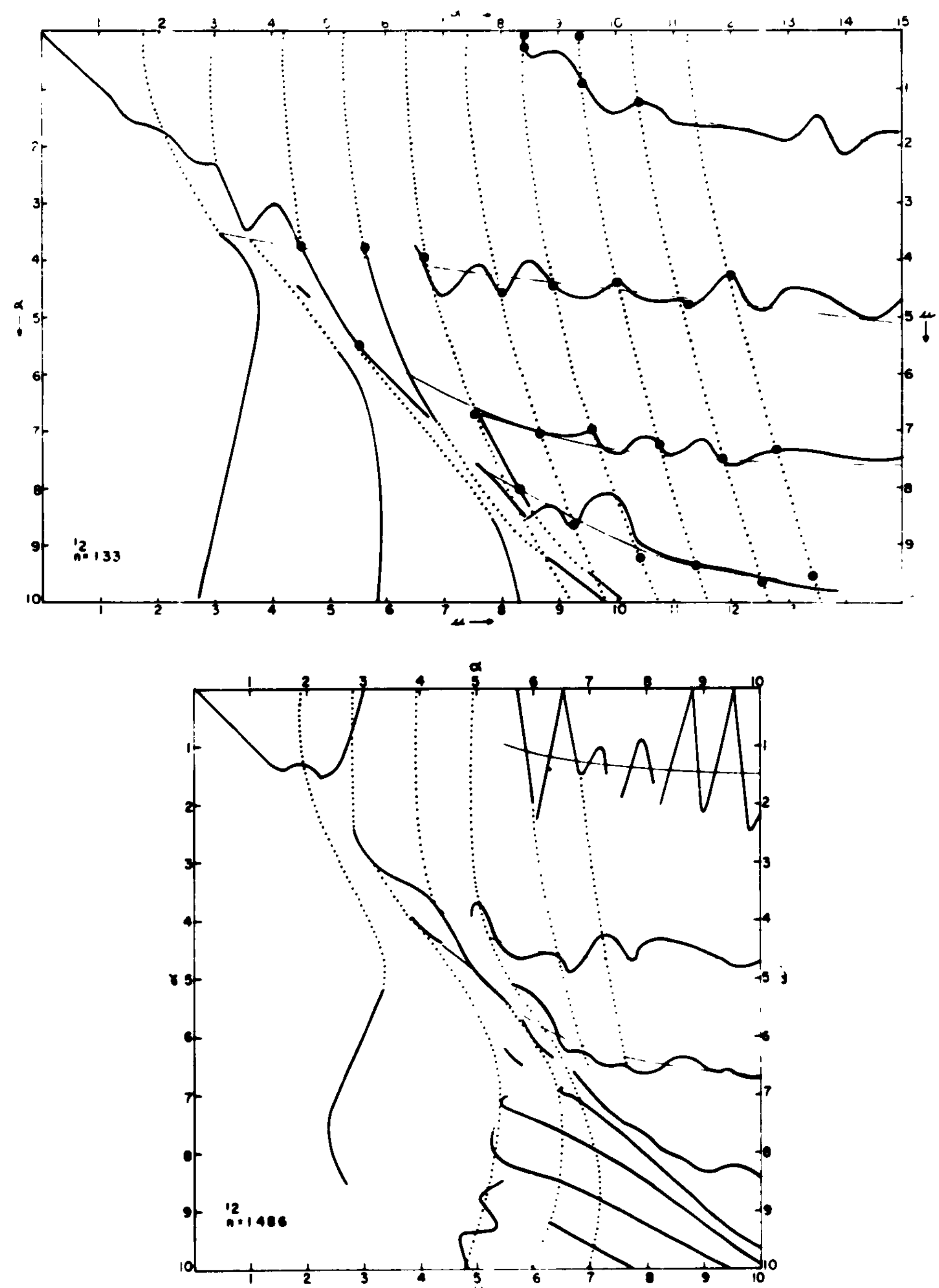


Figure 5 (Concl'd)

maxima are shallow. This is also the position where a system may suddenly "vanish," or where an analysis of the minima suddenly jumps from one system into the other system. Theoretically, this can be explained on the basis of ray optics. At the intersection of the two systems, both rays are of opposite phase, so that the intensity becomes extremely low. For the maxima, the two systems must be in phase so that the intensity reaches peaks. It seems possible to investigate this phase effect for some of the points and corroborate intuition. The cases for $n = 1.33$ and $n = 1.486$ are the best examples for this interpretation although only the minima are shown.

For the intensity function i_2, the conditions are somewhat different because the minimum around $\theta = 90$ degrees exercises a dominant influence. This situation presents itself already for $n = 1.05$ (figures 4 and 5). The diffractive minima and maxima can be interpolated around $\theta = 90$ degrees (dotted lines), and they cross each other where they intersect the 90-degree minimum. The 90-degree minimum moves always into the backward area where it intersects the diffractive minima. Some of this behavior is seen better in figure 4 than in figure 5. For $n = 1.15$, the matter starts to look more complicated because the second and third diffractive minimum does not start at $\theta=180$ degrees, but a pair of maxima and minima starts around 135 and 150 degrees. The case for $n = 1.20$ (figures 4 and 5) shows the condition very well. The minimum starting at $\theta = 90$ degrees for $\alpha = 0.1$ undergoes large angular fluctuations between $\theta = 100$ and 140 degrees; its behavior is best seen in the altitude chart, where it follows a diffractive minimum for a while and then moves suddenly to large scattering angles until it meets the next diffractive minimum. The first reflective system is indicated by these sharp tips. Inflective points, instead of well developed minima, are shown by the sign $\sim$. The altitude chart for $n = 1.3$ allows to find the first and second reflective system very clearly, and figure 5 shows the solution for $n = 1.33$ and $n = 1.486$ as the best examples for the existence of these two systems. The minima for i_2 and small α, say, α below 6, are very often interrupted, they just "disappear" on figure 4; but the altitude charts reveal clearly that two systems exist, and the position of the minima can be interpolated without difficulty.

On the various diagrams, the diffractive minima have been interpolated (dotted lines) through the backscattering area to $\theta = 180$ degrees. The value for $\theta = 180$ degrees has been computed according to the following scheme. It was investigated whether a system existed for the beginning of the diffractive minima at $\theta = 180$ degrees. Finally, such a system emerged, so that for i_1 and i_2

$$\alpha_j = \frac{\pi}{4}\left[\frac{1}{n} + j\left(\frac{3}{n} - 2n + n^2\right)\right], \tag{11}$$

where α_j denotes the α value at which a new diffractive minimum begins at $\theta = 180$ degrees, and $j = 1, 2, 3$ stands for the first, second, third, ..., diffractive minimum. Thus, one is able to draw the "undisturbed" curve for each diffractive minimum (dotted line). A system must also exist for the reflective minima, but so far an approximation formula has not been found.

The difference between succeeding minima is then given by

$$\Delta\alpha = \frac{1}{n}\left(\pi - \frac{\pi}{4}\right) - \frac{\pi}{4} + \frac{\pi}{4}(n-1)^2$$

$$= \frac{1}{n}\left(\pi - \frac{\pi}{4}\right) - \frac{\pi}{4}\left[1 - (n-1)^2\right]. \qquad (12)$$

These approximation formulas are valid only for $n < 1.5$; they have not been checked for $n > 1.5$.

In summary, graphical methods for interpolation of computed data have been developed which promise to lead to reliable interpolation for practical application of the Mie theory.

References

Pangonis, W. T. and W. Heller, Angular Scattering Functions for Spherical Particles, Wayne State Univ. Press, Detroit (1960), 222p.

Penndorf, R. and B. Goldberg, Unpublished Tables (1953).

Rowell, R. L., Private Communication (1963).

III. CONTENT OF SCIENTIFIC REPORTS

Scientific Report 1, Results of an Approximation Method to the Mie Theory for Colloidal Spheres, by R. Penndorf, AFCRC-TN-59-608, RAD-TR-59-38 (September 1959), 43p.

"An approximation method is outlined for computing the total Mie scattering coefficient K in aerosols using infrared light sources. The method is valid for any size parameter α, from small to very large spheres, and any arbitrary real refractive index $n \leq 2$. Analytical expressions are developed for the phase and the amplitude of K at the maxima and minima. The constants in these analytical expressions are determined from existing data computed by the exact Mie theory. Finally, a graphical interpolation process allows for determination of K for any arbitrary size. This method leads to an accuracy of ±2 or 3 percent for K as compared with results obtained by the Mie theory.

"This approximation method is than applied to 14 refractive indices n between 1.05 and 2.0. The results for smoothed $\overline{K}$ values are listed for the range α = 0.2 (0.2) 30 (0.5) 40, which is considered sufficient for most applications. For very large spheres, the laws of geometrical optics can be used successfully. Graphical representations of all cases are also included. From these figures, the appropriate $\overline{K}$ value can be read off directly for any given size parameter, and also for any given radius of the aerosol particle, if the wavelength of the incoming infrared radiation is selected."

Scientific Report 2, Mie Scattering in the Forward Area, by R. Penndorf, AFCRC-TN-60-285, RAD-TR-60-10 (February 1960), 87p.

"The scattering of light is investigated from a theoretical point of view. After explaining the basic Mie formulas, several useful scattering functions are defined, such as the angular Mie scattering coefficient i_θ and the Mie phase functions p_M. Next, the scattering pattern for the forward, sideward, and backward area is briefly described, followed by a very detailed investigation for the forward direction, which is based on all available data computed according to the Mie formulas. A wide range of refractive indices n from n = 1.05 to n = 2.0 has been selected, and all values of i_θ and p_M are extensively listed in the appendix. The behavior of these scattering coefficients is investigated and described in detail. It is found that, for example, while i_θ shows major oscillations, only plateaus are found for p_M, and their width is related to the refractive index. The amplitude of the major oscillation decreases with increasing size parameter. Superimposed on these oscillations are ripples, the phase of which agrees exactly with the phase of the ripples for the total scattering coefficient K. Several ripple systems are noted, and some regularity in their appearance is clearly indicated. The multivalueness of the scattering function and its consequences are also pointed out.

"This is followed by a careful investigation of practical limits for the Mie region, which can be defined only arbitrarily. The lower limit against the Rayleigh region is defined by a "crossover" between the Rayleigh scattering coefficient and the Mie scattering coefficient. The upper limit against the geometrical optics region is defined by the lowest α value for which the ratio $p_M - p_R/p_M$ reaches a value of less than 25 percent. Consequently in the geometrical optics region, this ratio always remains below 25 percent. For $n = 1.33$ for example, the true Mie region extends from $\alpha = 4.7$ to $\alpha = 25$, and for this region, there is no other choice except that of using Mie's formulas whereas simple formulas can be applied successfully below and above these limits.

"Furthermore, methods are developed for computing the scattering functions by simple means. Those given for the extended Rayleigh region, and the geometrical optics region, lead to acceptable values for practical problems, and errors can easily be reduced to less than 10 percent. A new approximation method, derived from the cross section theorem, is developed. Since total scattering coefficients K are known to a very large extent, this new method, which uses only K to determine the scattering functions in the forward direction, can become of great value for $\alpha \geq 5$. Its accuracy is carefully estimated, and it is found that the errors will stay below 10 percent with a few exceptions for small particle sizes and will decrease rapidly with increasing size of the aerosol.

"Finally, applications of these theoretical data are given to compute the angular Mie cross section and the volume scattering coefficient by a graphical method. These scattering values can be derived also for given size distributions of the aerosol."

Corrections

page 2, paragraph 3, line 2, $\leq 3 \times 10^{-6}$cm instead of $\leq 3 \times 10^{6}$cm.

page 12, Ordinate, angular scattering coefficient values for $n = 1.75$ revised; see figure... below.

page 21, Table I, extended in Table I of Scientific Report 7.

page 31, Table II, extended and revised in Table II of Scientific Report 7.

page 34, Table III, extended and revised in Table III of Scientific Report 7.

page 38, Table IV, extended and revised in Table IV of Scientific Report 7.

page 41, Figure 16, the area $1.6 \leq n \leq 2.0$ can be revised based on new data.

page 42, Figure 17, the curve for $n = 1.5$ can be revised for $n > 30$ using the publications by Giese, Bullrich, and de Barry (1962).

page 53, Figure 19, arrow on right-hand side goes in wrong direction.

Scientific Report 3, Scattering Coefficients for Absorbing and Nonabsorbing Aerosols, by R. Penndorf, AFCRL-TN-60-667, RAD-TR-60-27 (October 1960), 92p.

"Applied problems involving light scattering by aerosols demand reliable numerical data of the scattering and absorption coefficients. When the size of the aerosol is smaller than the wavelength of the incident light, approximation formulas can be derived from the Mie theory. The asymptotic expansion of the exact solution is given, where the leading term is identical with the Rayleigh approximation. For the first time, the first three terms as function of the size parameter α are correctly derived for nonabsorbing, absorbing, and metallic spherical particles. Next, the error of our approximation is determined using one, two, and three terms. This approximation extends the useful size range by about a factor 2 for nonabsorbing aerosols.

"The new approximation allows for computing the scattering and absorption coefficients for small aerosols simpler and faster than the Mie formula. Numerical results are given for nonabsorbing aerosols having refractive indices between $n = 1$ and $n = 2$. For absorbing spheres, the numerical values are given for the coefficients in the appropriate formulas. A wide range of indices of refraction $\tilde{n} = n - i\kappa$ are selected, values between 1 and 9 are chosen for n and between 1 and 4 for κ.

"A comparison is made between results obtained by the approximation formulas and the Mie formulas for $\tilde{n} = 1.25 - i\kappa$, $\tilde{n} = 1.29 - i\kappa$, $\tilde{n} = 1.50 - i\kappa$, and $\tilde{n} = 1.75 - i\kappa$. The new approximation formulas lead to errors in the total extinction coefficient $K^{(e)}$ of less than 20 percent if $\alpha < 0.8$ and $\kappa < 1$. The ratio $K^{(s)}/K^{(e)}$ shows that our formulas can be used up to $\alpha = 1.0$ because the error resulting from the approximations remains small and within tolerable limits.

"Finally, numerical data are computed for $\tilde{n} = 1.29 - i\kappa$, $\tilde{n} = 1.33 - i\kappa$, and $\tilde{n} = 1.40 - i\kappa$. For the case $\tilde{n} = 1.29 - i\kappa$, a complete new set of $K^{(e)}$ values has been derived to $\alpha = 7$ by carefully interpolating those new data as well as other existing data."

Corrections

page 15, paragraph 3, the statement made is wrong. van de Hulst's formula for $K_{RR}^{(e)}$ is correct. It leads to the correct formula in agreement with equation (17a), provided that n = n - iϰ is inserted for m in van de Hulst's formula.

page 44, paragraph 4, last sentence, change signs to read: "For the chosen values, the error ranges for $\alpha = 1$ from -11 to -30 percent for n = 1.25, from 3 to -6 percent for n = 1.5, and from 10 to 16 percent for n = 1.75."

page 46, Figure 11, the three refractive indices are n = 1.25, 1.50, and 1.75.

Scientific Report 4, Bright and Dark Rings, by R. Penndorf, AFCRL-425, RAD-TR-61-16 (March 1961), 24p.

"The scattering diagrams of spherical aerosols possess maxima and minima, which can be interpreted as sequences of bright and dark rings. These rings are investigated for n = 1.10, 1.20, 1.30, 1.33, 1.40, 1.44, and 1.50. The results show a systematic behavior exists for the appearance of these rings, such as the number of rings and their regular shift in angular position as function of the size parameter α. For the intensity function i_1, the rings due to diffraction move with increasing size parameter into the forward scattering area, whereas those caused by refraction and reflection within the sphere stay in the backward scattering area. The rings appear first in the backward area as long as the size parameter is small, but from $\rho > 4$ (ρ = normalized size parameter) on they begin to appear at 80 to 90 degrees; i.e., in the right-angle scattering area. For the intensity function i_2 however, the systematic behavior is poor and ill defined. Nevertheless, some of the basic features are similar to those for i_1. Finally, a system similar to i_1 exists for the angular Mie scattering coefficient i_3. In the last case, data are given for n = 1.33 only."

Corrections

page 5, Figure 5, the minima possess no cusp as indicated. The minima should show a smooth transition occurring somewhat higher than indicated. The same is true for figures 3 and 4.

page 10, Figure 5-7 should be revised, see the discussion in Scientific Report 10, page 7. We believe now that our first interpretation, as given in this report, is not always correct. There are minima which "vanish" as α increases. The better solution is indicated by figures 3-6 and 3-7 in Scientific Report 10, and in section II of this final report.

Scientific Report 5, Atlas of Scattering Diagrams for n = 1.33, by R. Penndorf, AFCRL-1044, RAD-TR-61-32 (October 1961), 69p.

"Angular scattering diagrams are constructed for spherical aerosols of refractive index n = 1.33. They show the intensity function i_1 and i_2 as function of the scattering angle for size parameters $\alpha = 0.5\ (0.5)\ 15$. The basic data have been computed using Mie's theory and graphical interpolation techniques have been used to fill the gaps.

"The diagrams presented in this atlas allow to determine the intensity functions i_1 and i_2 for any desired scattering angle with a high degree of reliability."

Corrections

Figure 1, the insert is taken from Giese for n = 1.33 and $\alpha = 10$.

Figure 2, a new schema of an improved altitude chart is discussed in section II of this final report.

Figure 9, a new interpretation of this figure can be given based on our discussion in section II of this final report.

page 24, line 3, polynomials.

page 25, Table I, the "number of rings" given under "Penndorf" is now considered too large. Since some of the minima can "vanish," it is believed that the number of rings for $\alpha > 12$ is overestimated. The "suggested steps of $\Delta\theta$ in degrees," however, will not change very much because the interval between minima can still be on the low side as indicated.

page 29, it should be added that, in the atlas, i_1 is shown by a solid line and i_2 by a dashed line.

page 56, Dr. Diermendjian (personal communication) informed us that his values for n = 1.33, $\theta = 45$ degrees, and $\alpha = 13$, 14, and 15 differ somewhat from those given in the report. The differences have not been settled as yet.

Scientific Report 6, Angular Mie Scattering, by R. Penndorf, AFCRL-62-1025, RAD-TR-62-54 (September 1962), 7p., reprinted from J. Opt. Soc. Am. 52, 402-408 (1962).

"The basic Mie formulas are explained, and several useful scattering functions are defined, such as the angular Mie scattering coefficients i_θ and the Mie phase function p_M. Finally, the scattering pattern for the forward, sideward, and backward area is surveyed, based on data for the refractive index n = 1.33."

Scientific Report 7, Mie Scattering in the Forward Area, by R. Penndorf, AFCRL-62-1026, RAD-TR-62-55 (September 1962), 18p., reprinted from Infrared Phys. 2, 85-102 (1962).

"The scattering of light in the forward area is investigated from a theoretical point of view, and it is based on all available data computed according to the Mie formulas. A wide range of refractive indices n from $n = 1.05$ to $n = 2.0$ has been selected. The behavior of these scattering coefficients is investigated and described in detail. It is found that, for example, while i_θ shows major oscillations, only plateaus are found for p_M, and their width is related to the refractive index. The amplitude of the major oscillation decreases with increasing size parameter. Superimposed on these oscillations are ripples, the phase of which agrees exactly with the phase of the ripples for the total scattering coefficient K. Several ripple systems are noted and some regularity in their appearance is clearly indicated. The multivalueness of the scattering function and its consequences are also pointed out. This is followed by a careful investigation of practical limits for the Rayleigh and the Mie region."

Scientific Report 8, Approximation Formula for Forward Scattering, by R. Penndorf, AFCRL-62-1027, RAD-TR-62-56 (September 1962), 4p., reprinted from J. Opt. Soc. Am. 52, 797-800 (1962).

"By using the cross section theorem, an approximation formula is derived for the scattering of light in the forward direction. It is valid for spherical aerosols if the radius of the particles is larger than the wavelength of the incident light."

Scientific Report 9, Scattering and Extinction Coefficients for Small Absorbing and Nonabsorbing Aerosols, by R. Penndorf, AFCRL-62-1131, RAD-TR-63-4 (November 1962), 9p., reprinted from J. Opt. Soc. Am. 52, 896-904 (1962).

"Approximation formulas are derived for small spherical aerosols ($r < \lambda$) by using the series expansion of the exact solution (Mie theory). The first three terms of the series are given for real, complex, and infinite indices of refraction. The new formulas allow for computing the scattering and absorption coefficients for small aerosols more simply and faster than the Mie formulas. The error of our approximation formulas is determined, and it is found that the useful size range is extended by about a factor 2."

Scientific Report 10, Atlas of Scattering Diagrams for n = 1.5, by R. Penndorf, AFCRL-62-1131, RAD-TR-63-9 (January 1963), 46p.

"Angular scattering diagrams are constructed for spherical aerosols of refractive index $n = 1.5$. They show the intensity functions i_1, i_2, and $i_1 + i_2$ as functions of the scattering angles for size parameters $\alpha = 0.5\,(0.5)\,10$.

"The basic data have been computed using Mie's theory, and a graphical interpolation technique (altitude chart technique) has been used to determine accurately the position and numerical value of the maxima and minima.

"The scattering diagrams presented in this atlas allow to determine the intensity functions i_1, i_2, and $i_1 + i_2$ for any desired scattering angle with a high degree of reliability. The basic data limit the construction of reliable scattering diagrams to $\alpha < 10$."

Corrections

Figures 3-1 and 3-4 have been inadvertently interchanged.

page 9, Figure 3-4, instead of Figure 3.1.

page 13, Figure 3-1, instead of Figure 3-4.

IV. ABSTRACTS OF JOURNAL ARTICLES

Penndorf, R., Angular Mie scattering, J. Opt. Soc. Am. 52, 402-408 (1962).

See under Scientific Report 6 for abstract.

Penndorf, R., Scattering and extinction coefficients for small spherical aerosols, J. Atmos. Sci. 17, 193 (1962).

"Formulas for total scattering and extinction coefficients are given for small spherical aerosols; the refractive index can be either real or complex. The formulas have been derived from Mie's formula using the series expansion method. Their limits have been described."

Penndorf, R., Approximation formula for forward scattering, J. Opt. Soc. Am. 52, 797-800 (1962).

See under Scientific Report 8 for abstract.

Penndorf, R., Mie scattering in the forward area, Infrared Phys. 2, 85-102 (1962).

See under Scientific Report 2 for abstract.

Penndorf, R., Scattering and extinction coefficients for small absorbing and nonabsorbing aerosols, J. Opt. Soc. Am. 52, 896-904 (1962).

See under Scientific Report 9 for abstract.

Penndorf, R., Scattering Diagrams in the Mie Region, Conf. Electromag. Scat. (ICES), p. 73-86 (1963).

"Scattering diagrams showing the intensity function i_1 as function of the scattering angle have been constructed for 4 refractive indices between $n = 1.33$ and 1.50. They are given in an atlas for size parameters $\alpha = 0.5$ (0.5) 10 and in the case of $n = 1.33$ to $\alpha = 15$. The basic data have been computed using Mie's theory, and graphical interpolation techniques have been used to fill the gaps. The various interpolation techniques are discussed in detail, especially the altitude chart techniques. They are very important to fill the gaps in the basic data.

"The scattering diagrams allow determination of the intensity functions i_1 and i_2 for any desired scattering angle with a high degree of reliability. Examples of the results are shown."

V. LIST OF SYMPOSIUM PAPERS

Penndorf, R., Approximation Methods to the Mie Scattering Theory for Colloidal Spheres, 2nd Conf. Anal. Chem. Nucl. Reac. Tech., Gatlinburg, Tenn. (1959).

Penndorf, R., Total and Angular Mie Scattering Functions for Spherical Particles, Am. Opt. Soc. Mtg., Columbus, Ohio (October 1957).

Penndorf, R., Scattering Diagrams in the Mie Region, Conf. Electromag. Scat. (ICES), Potsdam, New York (August 1962).

VI. APPENDIXES

6.1 BIBLIOGRAPHY OF NUMERICAL COMPUTATIONS ON SCATTERING AND ABSORPTION OF ELECTROMAGNETIC RADIATION FOR SPHERICAL PARTICLES BASED ON THE MIE THEORY

6.2 ATLAS OF SCATTERING DIAGRAMS FOR n = 1.1, 1.2, 1.33, 1.4, 1.44, AND 1.5

6.1 BIBLIOGRAPHY OF NUMERICAL COMPUTATIONS ON SCATTERING AND ABSORPTION OF ELECTROMAGNETIC RADIATION FOR SPHERICAL PARTICLES BASED ON THE MIE THEORY

CONTENTS

TABLES

I. INTRODUCTION

This bibliography is based on notes collected over the past 10 years, and it summarizes what is known about existing numerical computations using the Mie theory. The computations for single scattering are only considered, and those for multiple scattering have been excluded.

Lists already exist; for example, some are given by van de Hulst in his book on pages 165-171 and 273-275, and another by Penndorf (reference 42 in section 3. 3 of this appendix). However, these and similar lists and bibliographies are superseded by the vast amount of new computations, which are a consequence of the electronic computers. Hence, there seems to be a need for an updated list.

The user of this bibliography should be familiar with the field. That means he should have looked in van de Hulst's book, and he should be familiar with the basic equations; otherwise, the tables will be useless to him. Unfortunately, no general agreement exists on symbols, and this makes it difficult for the user of tables scattered over a large number of journals. He should in all cases read very carefully the definitions of each author, and he should compare them with those used in van de Hulst's book to derive the proper conversion factors. It seems hopeless to indicate all the differences and conversion factors in the following tables.

For reasons of convenience, reference always is made to van de Hulst's book. It is an excellent book, and it is available in all libraries; whereas, less known references may be harder to obtain.

Basic References

van de Hulst, H. C., Light Scattering by Small Particles, Wiley, New York (1957), 483p.

Hawksley, P. G. W., [The] Physics of particle size measurements, Monthly Bull. Brit. Coal Utiliz. Res. Assoc. 15, 105 (1951); 16, 117 (1952); and 16, 181 (1952).

Fishman, M. M., Light Scattering by Colloid System, An Annotated Bibliography, Tech. Service Laboratories, River Edge, N. J. (1957), 84p. (Supplement 1958), 106p.

Logan, N. A. and M. E. Sherry, Bibliography on the Theory of Diffraction and Propagation of Radio Waves, AF Cambridge Res. Ctr., AFCRC-TR-57-102, ASTIA AD-117044 (1957).

Weil, H., M. L. Barach, and T. A. Kaplan, Scattering of Electromagnetic Waves by Spheres (Studies in Radar Cross Sections X), Eng. Res. Inst., Univ. Michigan, Contract AF30(602)-1070, 2255-20-T (1956).

Morse, P. M. and H. Feshbach, Methods of Theoretical Physics, McGraw-Hill, New York (1953).

II. DESCRIPTION OF SYMBOLS AND TABLES

To understand the tables listed in this report, the following quantities and formulas are needed. We have excluded a discussion and listing of all auxiliary functions, such as Bessel functions and Legendre polynomials; some of these can be found on p. 165 in v.d. Hulst's book, although his list is incomplete.

The definitions, symbols, and formula are arranged in tabular form, similar to Hawksley's. This seems to be the most concise form. Table 1 contains the definitions and symbols, Table II the formula for total extinction and scattering, Table III the formula for angular scattering, and Table IV the abbreviations used in sections III to V.

Some notes and explanations are deemed advisable.

1. The refractive index is

$$\tilde{n} = n - i\kappa . \qquad (1)$$

This is the complex index of refraction; for a nonabsorbing particle $\kappa = 0$. Some authors prefer the symbol m instead of $\tilde{n}$. So far no agreement exists about the use of m or n. The SUN Commission (Physics to-day, 1962) suggest the symbol n. We should also note that some authors prefer

$$\tilde{n} = n\ (1 - i\kappa). \qquad (2)$$

In addition, the dielectric constant

$$\varepsilon = \varepsilon' - i\varepsilon'' \qquad (3)$$

is given, especially in radar work. This can be converted into $\tilde{n}$ by standard procedure ($\varepsilon = \tilde{n}^2$).

2. The total Mie coefficients K, K^s, K^e, K^a pose no problem, except K/2 is given in the older literature.

3. The angular scattering coefficients are i_1, i_2. The scatter-

ing angle in the forward direction is defined as $\theta = 0^\circ$, in agreement with v.d. Hulst and the present-day use. In the older literature it will be found that $\theta' = 180^\circ$ is defined as the forward direction, which goes back to Mie's definition. However, the use of $\theta = 0^\circ$ for forward scattering is advisable, and our tables are listing the scattering angle in our definition irrespective of what definition the author used.

Since this mix-up introduces difficulties in using the literature properly, a special symbol (@) is added in our lists to indicate to the reader that he has to look out for the definitions used by the author.

Some authors do not tabulate i_1, i_2 directly, but related quantities such as i_1/α^3, $i_\theta = (i_1+i_2)/2\pi\alpha^2$, phase functions, or other quantities. In those cases we have used the notation i_1, i_2 and qualified it by the symbol (#) to indicate to the reader that a related quantity is tabulated.

4. The <u>Mie functions</u> are a_m, b_m. They are useful if additional computations are required. Problems arise for two reasons. First the definition of forward scattering $\theta' = 180^\circ$ instead of $\theta = 0^\circ$ introduces a constant factor. If a_m is the Mie function using $\theta = 0^\circ$ for forward scattering and a_m^T that for $\theta' = 180^\circ$ for forward scattering, both are related by

$$\begin{aligned} a_m &= (-1)^m \, i \, a_m^T \\ b_m &= (-1)^{m-1} \, i \, b_m^T \end{aligned} \tag{4}$$

This change is important. Secondly, the term $\frac{2m + 1}{m(m + 1)}$ has been included in the definition of a_m, for example by Lowan and by Gumprecht and Sliepcevich. Hence, their a_m and b_m must deviate from those given in Table II (Eq. 2.5).

5. The <u>radar cross-section</u> σ. The tables indicate by the letter R that this quantity has been computed. It is an unfortunate definition but

so entrenched in the radar literature that it is hopeless to change it. Equation 3.3 (Table III) and van de Hulst (p.284) show the relationship between σ and i_1 and i_2. One has to be careful if the author uses polarized or unpolarized radiation for the source or receiver, or both. Hence, a factor 2 can appear. Again the definitions of the author have to be watched before using tabulated data.

6. Letters and Symbols. No standard notation exists. Hence, the reader will find all kinds of letters for the same quantity. It seems hopeless to include such a list, although it can be prepared.

For example, for the amplitude functions we find Σ_I, Σ_{II}; Σ_1, Σ_2; i_1^*, i_2^*; E_ϕ, E_θ; and A_1, A_2. A complete list of conversion factors, although extremely helpful, requires a lot of work but may be needed only by a few.

Again we can only warn the reader to pay attention to the definitions used by each author and compare them with Table I, II, and III or van de Hulst's book. From there the reader will be able to arrive at a conversion factor, if it is really necessary.

7. Odd parameters. Problems arose in compiling our tables when the author did not use our basic system of n and α in giving his results. For example, we will find that an author specified only λ and r and refers to a specific chemical substance for the refractive index at those λ's. Frequently the source for n is listed, but not the adapted value of n. We have not checked further since we are not sure what values have been used for the computations. In such cases we give the information as it appears in the listed source and have refrained from a conversion into our system. This applies especially to section IV.

8. Footnotes. At times it seemed necessary to add a footnote to a specific listing. This is done only rarely and to save space in the last column. The footnotes are collected at the end of each section, (3.1, 4.1, 5.1 and 6.1).

Footnotes are indicated by (F1, F2...) in the last column.

TABLE I

DEFINITIONS AND SYMBOLS

	This Bibliography	v. d. Hulst
r	radius of particle	a
λ	wavelengths of light	λ
$\tilde{n}$	complex index of refraction	$\tilde{m}$
	$\tilde{n} = n - i\kappa$	$\tilde{m} = n - in'$
n	real part of refractive index	n
n	refractive index for non-absorbing particles	m
κ	complex part of refractive index	n'
α	size parameter, $\alpha = 2\pi r/\lambda$	x; $x = 2\pi a/\lambda$
ρ	normalized size parameter $\rho = 2\alpha(n-1)$	ρ; $\rho = 2x(m-1)$
K	Total Mie scattering coefficient	Q Efficiency factor for scattering
K^s	Total Mie scattering coefficient (for complex index of refraction)	Q_{sca} Efficiency factor for scattering
K^e	Total Mie extinction coefficient	Q_{ext} Efficiency factor for extinction
K^a	Total Mie absorption coefficient	Q_{abs} Efficiency factor for absorption
θ	Scattering angle	θ
i_1, i_2	Angular scattering coefficients	i_1, i_2
S_1, S_2	Amplitude functions	S_1, S_2
a_m, b_m	Mie functions	a_n, b_n Mie coefficients
π_m, τ_m	Angular functions	π_n, τ_n

TABLE II

FORMULA FOR EXTINCTION, SCATTERING, AND ABSORPTION

2.1 Extinction $K^e = \frac{2}{\alpha^2} \sum_{m=1}^{\infty} (2m+1) \operatorname{Re}(a_m + b_m) = \frac{2}{\alpha^2} \operatorname{Re} \left\{ S_1(0) \right\}$

2.2 Scattering $K^s = \frac{2}{\alpha^2} \sum_{m=1}^{\infty} (2m+1) \left(|a_m|^2 + |b_m|^2 \right)$

$$= \frac{2}{\alpha^2} \sum (2m+1) \left[\operatorname{Re} |a_m|^2 + I_m |a_m|^2 + \operatorname{Re} |b_m|^2 + I_m |b_m|^2 \right]$$

2.3 For the particular case of nonabsorbing spheres (n real, including $n = \infty$)

$$K^s = K^e$$

2.4 Absorption $K^a = K^e - K^s = \frac{2}{\alpha^2} \sum_{m=1}^{\infty} (2m+1) \left[\operatorname{Re}(a_m) - |a_m|^2 + \operatorname{Re}(b_m) - |b_m|^2 \right]$

2.5 Mie coefficients

$$a_m = \frac{S_m'\{\beta\} S_m\{\alpha\} - n\, S_m'\{\alpha\} S_m\{\beta\}}{S_m'\{\beta\} \phi_m\{\alpha\} - n\, \phi_m'\{\alpha\} S_m\{\beta\}}$$

$$b_m = \frac{n\, S_m'\{\beta\} S_m\{\alpha\} - S_m'\{\alpha\} S_m\{\beta\}}{n\, S_m'\{\beta\} \phi_m\{\alpha\} - \phi_m'\{\alpha\} S_m\{\beta\}}$$

where $\beta = n\alpha$,

and S_m, ϕ_m are related to spherical Bessel functions of half integral order and of first, second and third kind (Riccati-Bessel functions) see Morse-Feshbach, p. 1573

TABLE III

FORMULA FOR ANGULAR SCATTERING

3.1 Angular Mie scattering coefficients

$$i_1 = \left| S_1 (\theta) \right|^2$$

$$i_2 = \left| S_2 (\theta) \right|^2$$

3.2 Amplitude functions

$$S_1 = \sum_{m=1}^{\infty} \frac{2m + 1}{m(m + 1)} \left[a_m \pi_m - b_m \tau_m \right]$$

$$S_2 = \sum_{m=1}^{\infty} \frac{2m + 1}{m(m + 1)} \left| a_m \tau_m - b_m \pi_m \right|$$

a_m, b_m are defined by Eq. (2.5)

π_m, τ_m are Angular functions based on Legendre polynomials

$$\pi_m = \frac{d}{d \cos \theta} P_n \left\{ \cos \theta \right\} = \frac{1}{\sin \theta} P_n^1 \left\{ \cos \theta \right\}$$

$$\tau_m = \pi_m \cos \theta - \pi_m' \sin^2 \theta$$

3.3 Radar cross section

$$\sigma_b = \frac{\lambda^2}{\pi} \left| S_1 \left\{ \tilde{n}, \alpha, 180 \right\} \right|^2$$

TABLE IV

ABBREVIATIONS

The following abbreviations are used in all tables listing the existing computations (sections 3.1, 4.1, 5.1 and 6.1). The abbreviations are selected on the basis of avoiding subscripts, to facilitate typing and to save space.

a,b	Mie coefficient: $Re(a_m)$, $Im(a_m)$, $Re(b_m)$, $Im(b_m)$
S	Amplitude function: S_1, S_2 (or i_1^*, i_2^*)
K	Total Mie scattering coefficient: K
Ks	" " " " : K^s
Ke	Total Mie extinction coefficient: K^e
Ka	Total Mie absorption coefficient: K^a
i	Angular Mie scattering coefficient: i_1, i_2
I	$1/2\ (i_1 + i_2)$
P	Degree of polarization: $(i_1 - i_2)/(i_1 + i_2)$
R	Radar cross section: Eq. 3.3 (see van de Hulst p. 284)
α	Size parameter
θ	Scattering angle ($\theta = 0^o$ in the forward direction)
U	Unpublished, or microfilm deposited and available from Library of Congress
L.C.	Library of Congress
T	Results given in tabular form
G	Results given in graphical form
D	Desk calculator
C	Electronic computer
N	Results not checked
●	Definition $\theta' = 180^o$ for forward scattering. This will apply for a, b, and i

TABLE IV (cont.)

#	Definition given in Table II or III not used, but similar parameter listed, such as i_1/α^3, K/α
4s	Number of significant digits listed (in this case 4, e.g. $3.456\ 10^{-3}$)
5p	Number of places after decimal point (in this case 5, e.g. 0.00023)
F	Footnote, appearing at end of table

III. NONABSORBING SPHERES

3.1 List of existing computations for a_m, b_m, S_1, S_2, i_1, i_2 and K for monodisperse isotropic spherical particles (Dielectric and perfectly conducting particles).

The following table requires some explanation. The first column lists a running number. This number agrees with the number in the references, section 3.3. The second column lists the "name of the author(s) and the year of publication". The symbol U after the year means that the data are unpublished. The third column contains the refractive index n.

The fourth column lists the "range of α values". The accepted system is used: 1(1)10(10) 100 means steps of $\Delta\alpha = 1$ are used between $\alpha = 1$ and $\alpha = 10$ and steps of $\Delta\alpha = 10$ are given between $\alpha = 10$ and $\alpha = 100$. In the older literature we find that uneven steps of α have been taken and therefore we report the results in the following system: 0.5-5.7 [18] which means the lowest α value is 0.5, the highest is 5.7 and 18 α values between these limits are listed.

The fifth column lists the "tabulated quantities". This is the most difficult column to devise and a simplified system similar to that of v.d. Hulst and Penndorf is used. To use single space typing all subscript are avoided if possible. The order of symbols (see here especially Table IV for abbreviations is a, b, then S, then K, then i or I, followed by less frequent symbols, such as R, P. The listing "i θ = 0(10)(180)" means i_1 and i_2 are listed for scattering angle $\theta = 0^\circ$ to 180° in steps of $\Delta\theta = 10^\circ$. Without exception, $\theta = 0$ represents forward scattering, irrespective of what the author has used as definition for the forward angle. This is done so that the reader looking for a specific scattering angle can use the table. The symbol ● in the next column has to be watched. It indicates that the author has used another definition for forward scattering, mostly $\Theta' = 180^{\circ}$. In those cases where only i_1 is given it has been spelled out. If completely different quantities are listed like the angular distribution coefficient (references 48, 49), the quantity has been spelled out.

The sixth column lists the form of "data presentation". First the symbol @ indicates that the definition $\Theta' = 180$ for forward scattering is used by the author; hence, first the tables for i_1 and i_2 have to be watched and secondly the tables for a_m and b_m have to be converted according to equation 4. The symbols T and G tell you whether the author gives the results in tabular or graphical form or both. Finally, the letter N indicates that the source has not been checked. This applies to reports which I have not seen, but the results have been quoted from another source, which I believe to be reliable.

The seventh and last column indicated the "type of computation", namely, whether a desk computer was used or an electronic computer or any other type. This might be helpful because the program may be available to those having similar machines at their disposal. The older computations are naturally all "hand-made" or by a simple desk computer and liable to errors of computation. In those computations the term "reliable" or "unreliable" indicate what we know about the result in order to warn the user. The term unreliable comes up whenever the author did not use enough term of the series (equation 2.5). The electronic computer should lead to correct results, but errors can creep in, namely the program may contain a slight error, or the machine used an old tape at a certain portion of the computation or the programs were written for one machine but used by another machine. Those errors are hard to find by just looking at tables; however, graphical representations will show it. The content and accuracy of tabulated values is also indicated by such symbols as "4s" or "5p". "4s" means 4 significant digits are given; whereas, "5p" means 5 places are given after the decimal point. If all the digits listed are correct is impossible to check without knowing the details of the program. They are never published. Thus, the term "accuracy" applies only in the sense that the author has given a certain number of digits and he should know if the last digits have any significance or not.

The remark "based on (5)" means that the author used data, mostly a_m and b_m, from reference #5.

No.	Author(s) Year of Publ.	Refractive Index	Range of α Values	Tabulated Quantities	Data Presentation	Type of Comp Accuracy
1	**Rayleigh** 1910	1.5	1,1.5,1.75,2,2.25 (?)	i θ=0,60,90,120,180 (?)	@, #, T	D, 4p, Fl
2	**Schirrmann** 1918	2.0	0.7-1.49 [8] #	i θ=0(20)180,90	@, T, G	D, 4p
3	**Ray** 1921/23	1.333 1.466	12	a,b,i 25 angles a,b,i 13 angles		N
4	**Shoulejkin** 1924	1.32	1,3, ∞	i#,P θ=0(20)60,90 120(20)180,P	@	unrel.
5	**Blumer** 1925/26	1.20 1.25 1.333 1.4661 1.5 ∞	1,1.5 0.01-8 [19] 0.1,1.5,3,12 5 0.01-4 [9] 0.01-10 [7]	a,b,i θ=0(10)90P (P for a few values only)	@, T, G	rel. except for large and inter-pol. values 4p
6	**Stratton** **Houghton** 1931	1.33	0-40	1/2K	G	rel. only for $\alpha < 11$
7	**Schoenberg** 1932	∞	0-20	1/2K	T	rel. for $\alpha < 6$ based on (5)
8	**Casperson** 1933	1.50 1.56 1.63	0.2-2.8 [17] #	a,b,K P,i θ=0(45)180	@, T, G @, G	unrel.
9	**Goetz** 1935	∞	0-6 [18]	1/2K	T	rel.
10	**Engelhard** **Friess** 1937	1.44	0.4,4,6,8 1(0.5)3	i θ=0(5)180 i θ=0(10)180	@, G	rel. for $\alpha < 4$, 4p
11	**Greenstein** 1937	2.0	0.1,0.3,0.5,0.8, 1(1)5	1/2K	T	unrel.
		∞	0.1,0.4(0.2)1.0, 2(1)5,10	1/2K	T	rel.
12	**Paranjpe** **Naik, Vaidy** 1939	1.33	4(1)10,12,20,30	i θ=0(10)180	@	unrel. N

No.	Author(s) Year of Publ.	Refractive Index	Range of α Values	Tabulated Quantities	Data Presentation	Type of Comp Accuracy
13	**Ruedy** 1943/44	1.33	$\alpha \leq 6$ $\pi/8, \pi/4, 3\pi/8, \pi/2$ $3\pi/4, \pi$	K (#),a,b I θ=0-180 P θ=50-130	G G G	D, F(2) D, F2
14	**Langmuir** 1943	1.50 2.0	1-10 [24] 1-5 [21]	1/2K 1/2K	T	rel; D, 4s see ref.(19)
15	**Rubinstein** **Pellam** 1943	∞	0-10	R θ=180		N
16	**Peary, Scott** 1944	∞	0-25	R θ=180		N
17	**van de Hulst** 1946	1.0 ∞	$\rho < 23$ 0.5-10 [5]	K K	T T	rel; D
18	**Holl** 1946/48	4/3	0.3-18 [40] 0.3-6 [16] 6.6-18 [24]	1/2K, a, b, i,P θ=0(10)180 i, θ=0(90)180 π, ϵ θ=0(10)180	T,G ● T,G ● T ● T	rel;D; 4,5p 4p 4p 4p
19	**Barnes, Kenyon** **Zaiser, LaMer** 1947	1.50	0.5-12 [28]	K,listing of ref. (14,23)	T,G	rel; D, 4s
20	**Ryde** 1947	1.75		R θ=180		
21	**Spendley** 1947	∞	0-12	R θ=180		N
22	**Houghton** **Chalker** 1949	4/3	9-24 [50]	K	T,G	rel; ex- trapol. to α = 50; D
23	**Lowan** 1949	1.33 1.44 1.50 2.0 1.50 1.44(0.1) 1.55	0.5-6.0 [15] same same same 6.5-12 [10] 0.5-7.0 [20]	a,b,K,i θ=0(10)180 K K	●, T	rel; D, 3s(K) 4s or 6p (a,b,i)
24	**Riley** 1949	1.486	0.5(0.1)3	a,b,K,i θ=0(5)180	●	rel; D

No.	Author(s) Year of Publ.	Refractive Index	Range of α Values	Tabulated Quantities	Data Presentation	Type of Comp Accuracy
25	**Anderson** 1950	0.8333	0.2(0.2)4.6,5-15 4.2,8 [18]	K i θ=0-180	G	rel; D
		1.25	0.2(0.2)16 4.0,8	K i θ=0-180	G	partly unrel; D
26	**Hamren** 1950	∞	12.8	R θ=180		N
27	Durbin 1951	1.20	0.1(0.1)0.6(0.2) 1.2	K,i θ=20,30,40,140, 150,160	@	
28	Gumprecht Sliepcevich 1951	1.20,1.33 1.4,1.44, 1.5,1.6	1(1)6,8,10(5)100 (10)200(50)400	a,b,K,i θ=90	@ T	rel; C,ENIAC 4s(K,i) 6p (a,b)
29	Chu 1952	∞	0.05(0.05)0.5(0.1) 1.5(0.25)5	K,R θ=180	T	rel.
30	De Bary 1952	4/3	4.8-15 [18]	i θ=30,60,120,150 π,τ θ=0(10)180 and m = 1 to 25	@ T	rel; D based on (18) 4p
31	Gumprecht, Sung Chin, Sliepcevich 1952	1.33	6,8,10(5)40	S,i θ=0(1)10(10)180	@ T	rel; C, IBM 602A, ENIAC, 4s
32	Gumprecht Sliepcevich 1953	1.20 1.33	20,80 20,30,40,60,80, 100,200,400	a,b,I θ=0(0.2) 1.4 θ=0(0.2)1.4 for $\alpha \geqslant 100$ θ=0(1) 3 or 7, for $\alpha \leqslant 80$	@ T	rel; C, IBM 602A, ENIAC, 4s
		1.44	20,80,150			
33	Goldberg 1953	1.33	0.1(0.1)30	K	G	rel; IBM 701
34	Kerker, Perlee 1953	2.0	1.3-2.8 [12]	a,b,S,i $P(\phi)$ θ=90	T, G	rel; D, 4p
35	Penndorf Goldberg 1953, U	1.33 1.4,1.44 1.486,1.5	0.1(0.1)30 0.1(0.1)30	a,b,K,i θ=0(1)10(5) 180 a,b,K,i θ=0(1)10(10) 180	@, T	rel; C, IBM 701 105 (rel. to 4-5s) for large α

No.	Author(s) Year of Publ.	Refractive Index	Range of α Values	Tabulated Quantities	Data Presentation	Type of Comp Accuracy
36	Boll Gumprecht Sliepcevich 1954	0.8 0.9 0.93 ∞	1 - 110 [18] 1 - 200 [20] 1 - 200 [25] 1 - 90 [27]	K K K K	T, G	rel; C, ENIAC; 4s
37	Gucker 1954	1.33	3.3-18.5 [53]	a,b,K	G, T	rel.
38	Heller Pangonis 1954	1.05,1.1, 1.15,1.2, 1.25,1.3	0.2(0.2)7	K# (spec. turbidity)	T	rel; D, 5s
39	Johnson Terrell 1955	1.29 ∞	1 - 19.3 [22] 0.5-25 [7]	K K	T T	rel; D, 3s
40	Johnson 1955	1.33,1.44 1.55 2.0	0.5-6 [15] #	R θ=180 R	T, G	rel; 3s, based on (23)
41	Kerker 1955 1955U(L.C.)	2.0	3.3-12.5 [11] 3.3-4.6 [5] 3.3-12.5	i θ=40 a,b a,b,i	T T	rel; D, 6s rel; D, 6p N, D
42	Penndorf 1956	1.0 2.0 ∞	α=0.2(0.2)15, 16-39.22 [13] 1.3-12.5 [23] 0.3-3.2 [14]	 K K K	T T T	rel; D, 4p D, 4p D, 4p
43	Penndorf 1956	1.33	0.1(0.1)8(0.2)30 (1)45	K (smoothed)	T	rel; C IBM 701; 3s based on (44)
44	Penndorf Goldberg 1956	1.33,1.4, 1.44,1.486 1.5	0.1(0.1)30	a,b,K	G T,G	rel; C, IBM 701 10s (a,b) 5s(K)
45	Hey, Stewart, Pinson, Price 1956	∞	0.0(0.01)10,#	S_1 for R θ=180 R#	G T G	rel; C 5p
46	Logan 1956, U	∞	1.1(0.6)9,5,10,20	R θ=180		N
47	Siegel 1956, U	∞	0.32,0.532,1.06, 1.60,2.36,3.94, 3.20,4.78	S, R θ=0 - 180		N

No.	Author(s) Year of Publ.	Refractive Index	Range of α Values	Tabulated Quantities	Data Presentation	Type of Comp Accuracy
48	Clark, Chu, Churchill 1958	1.33	1(1)6(2)10(5)30	"an" angl. distr. coeff.	T	5p, C, MIDAC based on (23) (28), (50)
		same	25	F	G	
49	Chu, Clark, Churchill 1957	0.90	1(1)5(2)10,20,25	K, "an" angl. distr. coeff.	T	C, MIDAC based on (23) (28), (50) K(5s), an (5p)
		0.93	1(1)5(5)30			
		1.05	1(1)6			
		1.10	1(1)6			
		1.15	1(1)6(2)10.15			
		1.20	1(1)6(2)10(5)30			
		1.25	1(1)6(2)10,15			
		1.30	1(1)6			
		1.33,1.40, 1.44,1.50, 1.60	1(1)6(2)10(5)30			
		1.55,2.00	1(1)6			
50	Pangonis Heller Jacobson 1957	1.05	0.2(0.2)7(1)15,22 (2)32,39(1)41	a,b,K	G, T	5s(K),6p (a,b) D for α<7;C, n DEC for α>7
		1.10	0.2(0.2)7(1)21			
		1.15	0.2(0.2)7			
		1.20	0.2(0.2)15.2			
		1.25	0.2(0.2)7			
		1.30	0.2(0.2)7			
51	Walter 1957	1.33	6(2)12,15,18,25, 30,45,60	S,i θ=0,90,170(1)180	T	D,C,IBM 650, 4s
			90,120,180,250	S,i θ=0,90,170(2)180		
		∞	6,10,18,30,60	a,b,S,i θ=0,90,168 (2)180	T	D,C,IBM 650, 4p(a,b)4s (S,i)
52	Mori Kikuchi 1957	1.33	5.5(0.5)10	S,i θ=0(5)180	T	D, 3s
	1958	1.33	10(0.5)18.5,10.2, 10,8,14.3,14.7, 16.2,16.3	S,i θ=0(2.5)180	T	D, 4s rel. for 3s, α= 18.5 somewhat off
53	Ashley, Cobb 1958	1.20	1,2,3,5,8,10,15, 20,30,35	S,i,I θ=0(10)180	G,T,G	D,4s, rel. based on (28)
	1958,U	1.20	same	same, but θ=0(1)180	T	D, 4s

No.	Author(s) Year of Publ.	Refractive Index	Range of α Values	Tabulated Quantities	Data Presentation	Type of Comp Accuracy
54	Boll, Leacock Clark Churchill 1958	0.6,0.7 0.75,0.8, 0.9,	1(1)10(2)20(5)100 (10)160(20)200	a,b,K	●, T	C,IBM 650; 5s(K)6p (a,b)
		0.93	1(1)5(5)80,95(20) 135,160,200	same		
55	Walter 1959	1.33	10(5)80	i θ=0(10)170(1)180	T	C,IBM 650; 5s
			85(5)100(10)200 (50)400	i θ=0,90,150,160, 170(1)180	T	
56	Meehan Beattie 1959	1.75	0.1-4.0 [22]	K	T	4s, 3s; D?
			0.6-3.6 [10]	i_1 θ=0,40,90,140 i_2 θ=90	T	4s, D partly inter. pol. acc =?
57	Heller Nakagaki Wallach 1959	1.05(0.05) 1.30	1(1)15	i θ=0;i(0)/i(180)	T, G	D, based on (50); 6s
		1.20	0.2(0.2)15.2			
58	Pfleiderer 1959	1.33	85(10)100(10)200 (50)400	i θ=10(10)140	T	C, IBM 650 5s
59	Giese 1959	1.33	10 1,3,14 1-15	i θ=0(10)180 i θ=0-180 P θ=60	T G G	C,Siemens 2002; 4s
	1959U	1.33	0.5(0.5)7(1)15	i θ=0(2)180	T	N
60	Kerker Matijevic 1960	1.20	0.2(0.2)7,1.06, 2.02,3.91,4.26	S, i θ=45,90,145	T	IBM 704 Based on (28) (50)
61	Chromey 1960	0.5(0.25) 3.0	0.2(0.2)2.0	K #	T	C,IBM, 4p
		0.5,0.75	0.2	a,b	T	4p
		3.0	2.0	a,b	T	4p
	U(L.C.)	same	same	a,b	T	N
62	Murley 1960	1.831	0.5 - 5	K	G	D ?

No.	Author(s) Year of Publ.	Refractive Index	Range of α Values	Tabulated Quantities	Data Presentation	Type of Comp Accuracy
63	Gucker Rowell 1960	1.486,1.5	18.4	I θ=0(1)180	G	C
		1.486	18.5(0.1)19.1	i θ=50(1)100	G	
	1960,U	1.486	0.1(0.1)30	S,i,I θ=(1)180	T	C, IBM; based on (35)
		1.5	10.12,15.8,18.2, 20.5,20.9	i θ=0(2.5)180	T	
			18.4(0.1)18.8; 20.2(0.1)20.6	i θ=0(1)180	T	
64	Pangonis Heller 1960	1.05 (0.05) 1.30	0.2(0.2)7	i,I θ=0(5)180	θ,#,T	C,Univac 1, 5p
65	Meehan Beattie 1960	1.55, 1.70, 1.76	0.5-2.6 [12]	N#;i# θ=0,40,140; P θ=80/90	#, G	Interpol. based on (23, 28,56) partly unrel.
66	Penndorf 1960	1.33,1.4, 1.44,1.5	0.1(0.1)30	i#, I θ=0	G, T	C; based on (35,31,57)
		1.05(0.05) 1.30,	1(1)15	i#, I θ=0	G, T	
		1.33	35 - 400	i#, I θ=0	G, T	
67	Meehan, Hugus 1961	1.65(0.05) 1.85,1.831	0.5(0.1)1.5(0.05) 2.4,2.5,2.6(0.2)6	K,i_1 θ=0	T	C, Univac 1103, 5s
68	Giese 1961U	1.33 1.5	1(1)40 1(1)80(2)150	i θ=0(2)180 same	T	Siemens 2002
		1.33,1.5	25,50	i θ=0(0.5)180	T	
		1.33,1.5	100	i θ=0(0.2)180	T	
	1961a	1.33	1,5,10,15	i θ=0 to 180	G	
	1961b	1.33,1.5	25,50,100	i θ=0 to 180	G	
69	Deirmendjian Clasen Viezee 1961, U	1.29	0.5(0.5)30	K,i θ=0(5 or 2.5) 180	T selected G in publications	C, IBM 704, 7090
		1.315	0.5(0.5)40			
		1.34	0.5(0.5)70			
		1.44	0.5(0.5)70			
	1961	1.525	0.5(0.5)70			

No.	Author(s) Year of Publ.	Refractive Index	Range of α Values	Tabulated Quantities	Data Presentation	Type of Comp Accuracy
70	Kerker Matjevich 1961	2.105	0.2(0.4)6(0.2)15 0.2(0.4)5.8	K,i θ=0(10)180 i θ=45,135 for 5.8	●, T, G	IBM 704
71	Penndorf 1961	1.33	0.5(0.5)15	i, θ=0-180	G	based on 35
72	Kerker, Kratovil Matigevic 1961U(L.C.)	1.60(0.04) 2.08	0.1(0.1)10	K, 5, i θ=0(10)180	T	C, IBM 704, for α > 15 7090 N
		1.4821; 2.1050	10.1(0.1)230(2) 53	K	T	N
	1961	1.481, 2.105 2.0	0-53 0-10	K K	G G	
73	Kerker Kratovil Matijevich 1962	1.4821, 2.1050	0.1(0.1)23(1)53 0.1(0.1)23(1)53	K	T	based on (72)
74	Napper Ottewill 1962	1.7067	0.1(0.1)10	K,i θ=35(5)145	T,G	C, IBM 1620
75	Giese, de Bary Bullrich Vinnemann 1962	1.5	0.2(0.2)159	K,i θ=0(1)10(10)180	T, G (samples)	4s Siemens 2002
	1962U			i θ=149(2)171		
76	Penndorf 1962a	1.33 1.33	0.1 - 400 0.1 - 30	I θ=0,90,180 I θ=0,10,20,40	G	based on (35)(66)
	1962b	1.33	0 - 30	I#, θ=0	G	based on (35)(66)
	1962c	1.33,1.4 1.44,1.5	0.1-30	i#, I θ=0	G	based on 66
		1.05(0.05) 1.3	1 - 15	i#, I θ=0	G	
		1.55,1.75, 2.0	1 - 7	i#, I θ=0	G	

No.	Author(s) Year of Publ.	Refractive Index	Range of α Values	Tabulated Quantities	Data Presentation	Type of Comp Accuracy
77	Remy-Battian 1962	1.25	0.1(0.1)10(1)20 (3)50	i θ=0(2)10(10) 170(2)180	T	C, Bull Gamma et ordinator 4s, 5s
			same	P	G	
78	Rowell 1962	1.486	18 to 24	position of max and min θ=0 to 180	G	based on (63)
79	Deirmendjian 1963	1.29, 1.315, 1.525 1.44 1.54,1.55 1.56 2.2	0.5(0.5)15 0.5(0.5)15 0.5(0.5)15 0.5(0.5)7 0.5(0.5)10 0.5(0.5)10 0.5(0.5)10,12(4) 40	s θ=0, 180	T	C,IBM 7090 4s
80	Penndorf 1962	1.5	0-15	i,I,θ=0-180	G	based on (35)
81	Wakeshima Takata 1963	1.33	0(0.1)30	i θ =90	T	C,Facom 128,8s

Footnotes

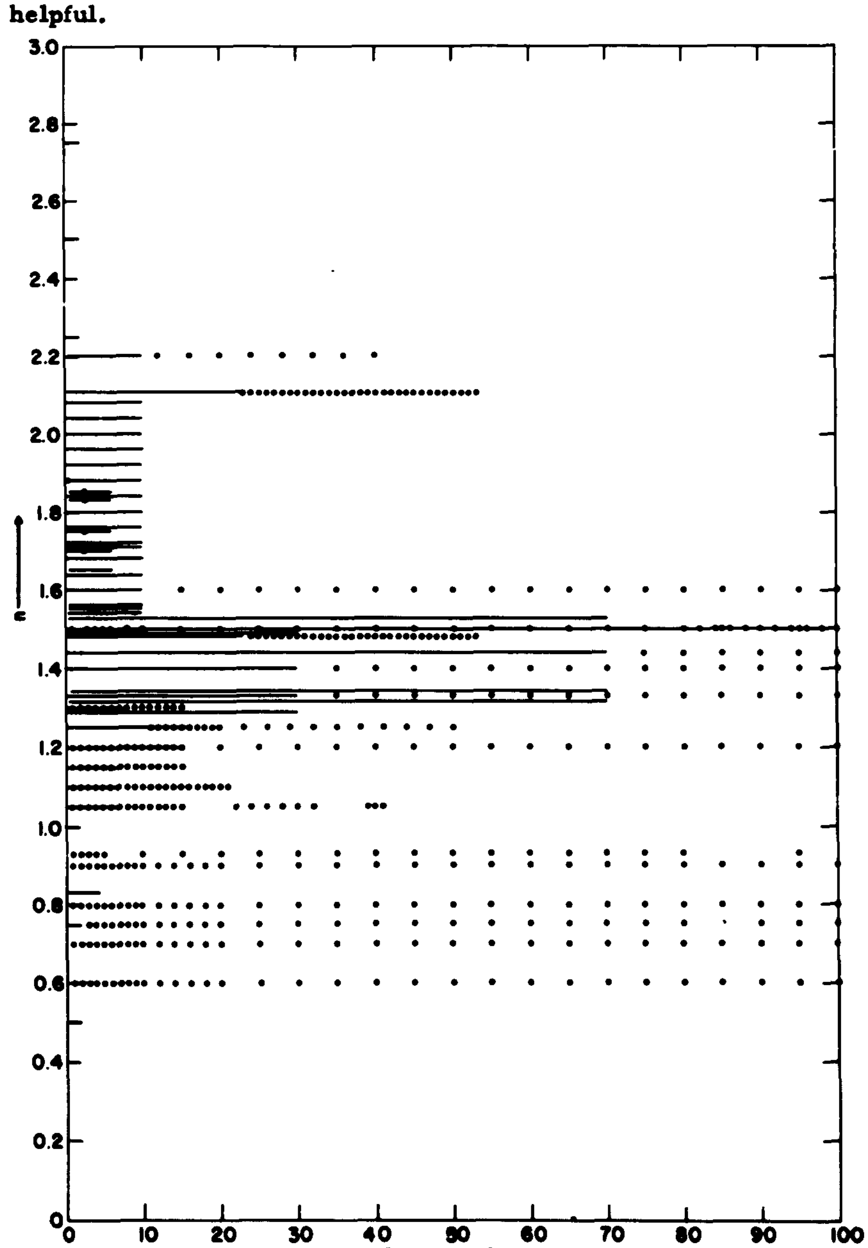
#1 Information copied from v.d. Hulst. Seems doubtful. I believe Rayleigh used radius r not α and log cos θ. No thorough check has been made because the results have only historic interest.

#2 It is not quite clear what refractive index has been used. In the first paper $\widetilde{n} = \frac{4}{3}(1-0.03i)$ is mentioned, in the other papers no value for n is given. I believe, however, that a_1, a_2, a_3, b_1, b_2 are identical in all three papers. The value of α is not correctly stated in v.d. Hulst (p. 168), because Ruedy used $\alpha' = 2r/\lambda$.

3.2 List of Computations arranged according to Refractive Index

This table has been prepared to list the computations for increasing refractive indexes only; the author number for which data exist has been included. Those being considered not reliable have been excluded.

In addition, a figure shows the range of α values for all n for which data exist. Naturally it gives only a general picture, but even that might be helpful.



n	Reference	n	Reference
0.5	61	1.54	79
0.6	54	1.55	40,76,79
0.7	54	1.56	8,79
0.75	54,61	1.6	28,72
0.8	36,54	1.63	8
0.8333	25	1.64	72
0.9	36,49,54	1.65	67
0.93	36,49,54	1.68	72
1.0	17,42,61	1.7	65,67
1.05	38,49,50,57,64,66,76	1.7067	74
1.1	38,49,50,57,64,66,76	1.72	72
1.15	38,49,50,57,64,66,76	1.75	20,56,61,67,76
1.2	5,27,28,32,38,49,50,53, 57,60,64,66,76	1.76	65,72
1.25	5,25,38,49,50,60,61,64, 66,76,77	1.8	67,72
1.29	39,69,79	1.831	62,67
1.3	38,49,50,57,64,70,73,76	1.84	72
1.315	69,79	1.85	67
1.33	3,5,6,12,13,18,22,23,28, 30,31,32,33,35,37,40,43, 44,48,49,51,52,55,58,59, 66,68,71,76,81	1.88	72
1.34	69	1.92	72
1.4	28,35,44,66,76	1.96	72
1.44	10,23,32,35,40,44,66,69 76,79	2.0	2,11,14,23,34,41,42,61,72,76
1.466	3	2.04	72
1.4661	5	2.08	72
1.4821	72,73	2.105	70,72,73
1.486	24,35,44,63,78	2.2	79
1.5	1,5,8,14,19,23,28,35,44,59, 61,63,66,68,75,76,80	2.25	61
1.525	69,79	2.5	61
		2.75	61
		3.0	61
		∞	5,7,9,11,15,16,17,21,26,29, 36,39,42,45,46,47,51

3.3 References

The number in this reference list agrees with the number in the tables in sections 3.2 and 3.3.

1. Rayleigh, Lord, "The incidence of light upon a transparent sphere of dimension comparable with the wavelength". Proc. Roy. Soc. 84A, 25 (1910).

2. Schirmann, M.A., "Dispersion and Polychroismus des polarisierten Lichtes, das von Einzelteilchen von der Groessenordnung der Wellenlaenge des Lichtes abgebeugt wird". Sitzungsber. Akad. Wien. Math. naturw. Klasse, IIa 127, 1559 (1918).

3. Ray, B., "On the color and polarization of the light scattered by sulfur suspensions". Proc. Ind. Assoc. 7, 40 (1921), 8, 23 (1923).

4. Shoulejkin, W., "Scattering of light by very big colloidal particles". Phil. Mag. (6) 48, 307 (1924).

5. Blumer, H., "Strahlungsdiagramme kleiner dielektrischer Kugeln". Z. Physik 32, 119 (1925); 38, 304 (1926).

6. Stratton, J. and H. Houghton, "A theoretical investigation of the transmission of light through fog ". Phys. Rev. 38, 159 (1931).

7. Schoenberg, E., "Das Gesetz der Verfaerbung bei Diffusion und einige astronomische Anwendungen derselben". Mitt. Univ. Sternw. Breslau #3 p. 53, (1932).

8. Casperson, T., "Zur Kenntnis der Optik weisser Sole". Kolloid Z. 60, 151 (1932); 65, 162 (1933).

9. Goetz, F.W.P., "Das Gesetz der Verfaerbung bei Diffusion". Astron. Nachr. 255, #6100 (1935).

Goetz, F.W.P., "Extinktionskoeffizient a_λ fuer undurchsichtige Kugeln". In: F. Linke: Meteorol. Taschenbuch 4. Ausg. Leipzig, Akad. Verl. p. 240 (1943).

10. Engelhard, H. and H. Friess, "Ueber die Brauchbarkeit des Mieeffektes zur Teilchengroessenbestimmung weisser Aerosole, insb. grobdisperser weisser Aerosole". Kolloid. Z. 81, 129 (1937).

11. Greenstein, J.L., "The theory of interstellar absorption". Harvard Circular #422 (1938).

12. Parandjpe, G.R., Y.G. Naik and P.B. Vaidya, "Scattering of light by large water drops". Proc. Indian Acad. Sci. A9, 333 (1939).

13. Ruedy, R., "Absorption of light by small drops of water". Can. J. Res. A21, 79 (1943).

"Scattering of light by small drops of water". Can. J. Res. A21, 99 (1943).

"Transmission of light by water drops of 1 to 5 μ in diameter", Can. J. Res. A22, 53 (1944).

14. Langmuir, G., In La Mer, V.K., "Verification of Mie Theory". O.S.R.D. Report #1857 (1943).

15. Rubinstein and Pellam, (M.I.T.)

16. Pearcy, T. and I.M.C. Scott, "The radar echoing power of conducting spheres". RRDE; ADRDE Research Report No. 228 (1944).

17. Van de Hulst, H.G., "Optics of spherical particles". Recherches Astron. de l'Obs. d'Utrecht 11, part 1 (1946), part 2 (1949).

18. Holl, H., "Lichtstreuung an dielektrischen Kugeln vom Brechungsindex n = 4/3". Optik 1, 213 (1946), 4, 173 (1948).

19. Barnes, M.D., A.S. Kenyon, E.M. Zaiser and V.K. La Mer "Monodispersed sulfur sols IV". J. Coll. Science 2, 349 (1947).

20. Ryde, J.W., "The attenuation and radar echoes produced at centimeter wavelengths by various meteorological phenomena", in "Meteorological factors in radio wave propagation". Phys. Soc., London, p. 169, 1947.

21. Spendley, W., "Scattering by metallic spheres", ASRE, Ref. IRE (1947).

22. Houghton, H.G. and W.R. Chalker, "The scattering cross section of water drops in air for visible light". J. Opt. Soc. Amer. 39, 955 (1949).

23. Lowan, A.N., "Tables of scattering functions for spherical particles". U.S. Nat. Bur. Standards, Appl. Math. Series #4 (1948).

24. Riley, J.D., "Calculations of light intensity functions". U.S. Nav. Res. Lab., Radio Div. III, ORB Info. Bull. #9 (1949).

25. Anderson, V.C., "Sound scattering from a fluid sphere". J. Acoust. Soc. Amer. 22, 426 (1950).

26. Hamren, S.D., "Scattering from spheres". U. of Cal. Antenna Lab. Techn. Rep. #171 (1950).

27. Durbin, NACA Note 2441 (1951).

28. Gumprecht, R.O. and C.M. Sliepcevich, "Light scattering functions for spherical particles". Univ. of Mich. Press (1951), 594 p.

29. Chu, C.M., "Scattering and absorption of water droplets at millimeter wavelengths", Thesis, U. of Michigan (1952).

30. De Bary, E., "Lichtstreuung an dielektrischen Kugeln vom Brechungsindex n = 4/3". Optik 9, 319 (1952).

31. Gumprecht, R.O., N.L. Sung, J.H. Chin, and C.M. Sliepcevich, "Angular distribution of intensity of light scattered by large droplets of water". J. Opt. Soc. Amer. 42, 226 (1952).

32. Gumprecht, R.O. and C.M. Sliepcevich, "Scattering of light by large particles". J. Phys. Chem. 57, 90 (1953).

33. Goldberg, B., "New computation of the Mie scattering functions for spherical particles". J. Opt. Soc. Amer. 43, 1221 (1953).

34. Kerker, M. and H.E. Perlee, "Additional light scattering functions for spherical particles of refractive index 2". J. Opt. Soc. Am. 43, 49 (1953).

35. Penndorf, R. and B. Goldberg. (unpublished manuscript) (1953).

36. Boll, R.H., R.O. Gumprecht, and C.M. Sliepcevich, "Theoretical light-scattering coefficients for relative refractive indices less than unity and for totally reflecting spheres". J. Opt. Soc. 44, 18 (1954).

37. Gucker, (unpublished manuscript) (1954).

38. Heller, W. and W.J. Pangonis, "Light scattering of spherical colloidal particles". J. Chem. Phys. 22, 948 (1954).

39. Johnson, J.C. and J.R. Terrell, "Transmission cross section for water spheres illuminated by infrared radiation". J. Opt. Soc. Amer. 45, 451 (1955).

40. Johnson, J.C., "The back-scattering coefficient of a spherical homogeneous mixture of ice and air at wavelengths between 1 and 10 centimeters". J. Meterol. 12, 188 (1955).

41. Kerker, M. and A.L. Cox, "Eleven additional light scattering functions for spherical particles of refractive index 2". J. Opt. Soc. Amer. 45, 1080 (1955); L.C. Document 4677.

42. Penndorf, R., "Total Mie scattering coefficients for real refractive indices". In: "New Tables of Mie scattering functions for spherical particles". Geophys. Res. papers No. 45, part 6 (1956).

"Total Mie scattering coefficients for spherical particles of refractive index 2.0". J. Opt. Soc. Am. 46, 1001 (1956).

"Total Mie scattering coefficients for spherical particles of refractive index $n \sim 1.0$". J. Opt. Soc. Am. 47, 603 (1957).

43. Penndorf, R., "Mie scattering coefficient for water droplets in air". J. Meteorol. 13, 219 (1956).

44. Penndorf, R. and B. Goldberg, "New tables of Mie Scattering functions for spherical particles". Geophys. Res., paper #45, part 1-6, (1956).

"New tables of total Mie scattering coefficients for spherical particles of real refractive indices". J. Opt. Soc. Am. 47, 1001 (1957).

45. Hey, J.S., G.S. Stewart, J.T. Pinson and P.E.V. Prince, "The scattering of electromagnetic waves by conducting spheres and discs". Proc. Phys. Soc., London 69B, 1038 (1956).

46. Logan, N., AFCRC (unpublished manuscript ?) (1956).

47. Siegel, K. (unpublished manuscript, 1956). see: Weil. 4, et al. Studies in Radar cross sections X, Eng. Res. Inst. Univ. of Michigan (1956).

48. Clark, G.C., C.M. Chu and S.W. Churchill, "Angular distribution coefficients for radiation scattered by a spherical particle". J. Opt. Soc. Amer. 47, 81 (1957).

49. Chu, C.M., G.C. Clark and S.W. Churchill, "Tables of angular distribution coefficients for light-scattering by spheres". Eng. Res. Inst. Univ. of Michigan 58 pgs. (1957).

50. Pangonis, W.J., W. Heller, A. Jacobson, "Tables of light scattering functions for spherical particles". Wayne State Univ. Press, Detroit 116 pgs. (1957).

Heller, W., W.J. Pangonis, "Theoretical investigations on the light scattering of colloidal spheres". J. Chem. Phys. 28, 498 (1957).

51. Walter, H., "Streulichtintensitaet grosser sphaerischer Partikel". Optik 14, 130 (1957).

52. Mori, N. and H. Kikuchi, "Scattering of light by spherical particles". (In Japanese) Bull. Electrotech. Lab. (Tokyo) Vol. 21, No. 8, 561, (1957), Vol. 22, No. 3, 209, (1958).

53. Ashley, L.E. and C.M. Cobb, "Single particle scattering functions for latex spheres in water". J. Opt. Soc. Am. 48, 261 (1958).

54. Boll, R.H., J.A. Leacock, G.C. Clark, and S.W. Churchill, "Tables of Light scattering functions". Univ. of Michigan Press, Ann Arbor, 360 p. (1958).

55. Walter, H., "Streulichtintensitaet grosser sphaerischer Partikel II". Optik 16, 401 (1959).

56. Meehan, E.J. and W.H. Beattie, "Light scattering functions for spherical particles with m = 1.75". J. Opt. Soc. Am. 49, 735 (1959).

57. Heller, W., M. Nakazaki and M.L. Wallach, "Theoretical investigations on the light scattering of colloidal spheres, V. forward scattering". J. Chem. Phys. 30, 444 (1959).

58. Pfleiderer, J., "Streulichtintensitaet grosser sphaerischer Partikel III". Optik, 16, 409 (1959).

59. Giese, R.H., "Lichtstreuung an sphaerischen Metallpartikeln", Z. f. Naturf. 14a, 1085 (1959).

60. Kerker, M. and E. Matijevich, "Light scattering of monodispersed polystyrene latexes". J. Cpt. Soc. Am. 50, 722 (1960).

61. Chromey, F.C., "Evaluation of Mie equations for colored spheres". J. Opt. Soc. Am. 50, 730 (1960); Document 6247 L.C.

62. Murley, R.D., "Mie theory of light scattering - limitations on accuracy of approximate methods of computation". J. Phys. Chem. 64, 161 (1960).

63. Gucker, F.T. and R.L. Rowell, "The angular variation of light scattered by single dioctyl phthalate aerosol droplets". Disc. Faraday Soc. No. 30, 185 (1960).

64. Pangonis, Wm. J. and W. Heller, "Angular scattering functions for spherical particles". Wayne State Univ. Press, Detroit, (1960), 222 pgs.

65. Meehan, E.J. and W.H. Beattie, "Determination of particle size in silver bromide sols by light scattering". J. Phys. Chem. 64, 1006 (1960).

66. Penndorf, R., "Mie scattering in the forward area". AFCRC-TN-60-285. RAD-TR-60-10. AVCO/RAD (1960) 86 pgs.

67. Meehan, E.J. and Z.Z. Hugus, "Light scattering functions for spherical particles with m = 1.65 (0.05) 1.85". J. Opt. Soc. Am. 51, 260, (1961).

68. Giese, R.H., "Streuung elektromagnetischer Wellen an absorbierenden und dielektrischen kugelfoermigen Einzelteilchen und an Gemischen solcher Teilchen". Z. f. Astrophys. 51, 119 (1961).

"Beispiele berechneter Streufunktionen von groesseren kugelfoermigen Teilchen mit den Brechungsindizes m = 1.33 und m = 1.5". Met. Rundsch. 14, 48 (1961).

69. Deirmendjian, D., R.J. Clasen and W. Viezee, "Mire scattering with complex index of refraction". J. Opt. Soc. Amer. 51, 620 (1961).

Deirmendjian, D. and R.J. Clasen, "Light scattering on partially absorbing homogeneous spheres of finite size". Rand Co., Report R-393-PR (1962).

70. Kerker, M. and E. Matijevich, "Mie scattering functions for refractive index of 2.105". J. Opt. Soc. Am. 51, 87 (1961).

71. Penndorf, R., "Atlas of scattering diagrams for n = 1.33 AFCRL - 1044, RAD-TR-61-32. AVCO/RAD, Wilmington, Mass., (1961) 69 pgs.

72. Kerker, M., J.P. Kratohvil and E. Matijevic, "Light scattering functions for $m = 1.60$ (0.04) 2.08 and $\alpha = 0.1(0.1)$ 10. I. Mie scattering coefficients". J. Phys. Chem. 65, 1713 (1961), Document 6603 L.C.

73. Kerker, M., J.P. Kratohvil and E. Matijevic, "Light scattering functions for concentric spheres. Total scattering coefficients $m_1 = 2.1050$, $m_2 = 1.481$". J. Opt. Soc. Am. 52, 551 (1962).

74. Napper, D.H. and R.H. Ottewill, "Tabulated Mie functions i_1 and i_2 (and K) for $m = 1.7067$". (Print) Dept. Colloid Science Cambridge, England, (1962).

Napper, D.H., and R.H. Ottewill, "Studies on the light scattering of Silver Bromide particles". J. Photogr. Science 11, 84 (1963)

75. Giese, R.H., E. de Bary, K. Bullrich and C.D. Vinnemann, "Tabellen der Streufunktionen i_1 (φ) und i_2 (φ) und des Streuquerschnittes K (α,m) homogener Kuegelchen nach der Mie'schen Theorie." Abh. Deutsche Akad. der Wissensch. Berlin. Klasse Math. Phys. u. Technik. Jahrg. 1961, Nr. 6. Berlin (1962) 175 pgs.

76. Penndorf, R., "Angular Mie scattering". J. Opt. Soc. Am. 52, 402 (1962).

Penndorf, R., "Approximation formula for forward scattering". J. Opt. Soc. Am. 52, 797 (1962).

Penndorf, R., "Mie scattering in the forward area". Infrared Phys. 2, 85 (1962).

77. Remy-Battian, L., "Mie scattering functions for spherical particles of refractive index $m = 1.25$". Mém. Soc. Roy. de Sciences de Liège. (4) I, Tome 2, Fasc 7, (1962).

78. Rowell, R.L., "Extreme contour diagrams for intensity functions computed from the Mie theory." Sc. Rep. #1, Univ. of Mass. (1962).

79. Deirmendjian, D., "Tables of Mie scattering cross sections and amplitudes". R-407-PR, Rand Co., (1963).

80. Penndorf, R., "Atlas of scattering diagrams for $n = 1.5$, AFCRL-62-1131, RAD-TR-63-9. AVCO/RAD, Wilmington, Mass. (1963).

81. Wakeshima, H., and K. Takata, "A new table of Mie intensity functions of light scattered sideward by a water droplet." Rev. Kobe Univ of Mercantile Marine, Part II. Nav., Mar. Eng., and Sc. section. No 10, 81(1963)

IV. ABSORBING SPHERES

4.1 List of existing computations for a_m, b_m, S_1, S_2, i_1, i_2, K^e, K^s, K^a and related functions or coefficients for monodisperse isotropic spherical particles (dielectric particles).

The following table requires again some explanation. The first column lists the running number. This number agrees with the number in the reference, section 4.3. The second column lists the "name of the author(s) and the year of publication". The symbol U after the year means that the data are unpublished or deposited as document in the Library of Congress. The third column contains the refractive index $\tilde{n}$, according to Eq. (1). Values from authors who used Eq. (2) and (3) have been transformed to our system. In some cases, if it has not been stated accurately, no data are given in this column.

The fourth column lists the type of compound to which it applies, such as Au in H_2O, or carbon, or water droplets in air (H_2O).

The fifth column shows the appropriate spectral range for which this value of $\tilde{n}$ and the compound apply. V (visible), IR, (infrared), R (radar) are the symbols used. They seem to be clear. Sometimes the wavelength is also listed for V and IR the numbers are given in μ, for R in cm.

The sixth column shows the "range of α values". This system is the same as in section 3.1. The seventh column shows the "tabulated quantities". The eighth and ninth column is again similar to those in section 3.1.

Data for some water and ice mixtures and pure water and ice are given in section VI.

No.	Author Year	Refractive Index	Compound	Spectral Range	α Size Parameter	Tabulated Quantities	Presen-tation	Type of Comp.
1	Mie 1908	1.27-1.27i 1.30-1.29i 0.83-1.51i 0.59-1.67i 0.43-1.84i 0.28-2.22i 0.31-2.65i	Au in H_2O	V	0.x-1.58 0.x-1.41 0.x-1.41 0.x-1.41 0.x-1.41 0.x-1.41 0.x-1.41	K/2	TΘ,G	D
2	Schirrmann	$\tilde{n}$ var. with λ	Au in H_2O Au in air Hg in air	V(0.45-0.65μ) V(0.5-0.65μ) V(0.35-0.65μ)	0.74-1.1 0.81-1.3 0.81-1.9	a,b,i Θ=0(20)180,P,Θ	TΘ,G	D,4p; used a1, a2,b1 only
3	Senftleben Benedict 1919	1.95-0.66i	C	V(0.491μ)	1.12	IΘ,P	TΘ,G	D,3s
4	Feick 1925	$\tilde{n}$ var. with λ	Ag	V(0.42,0.525 0.65,0.75μ)	0.2(0.2)1.0(0.5) 2.5,4	K	T,G	D,2p, used a1, a2,b1 only
			Hg in H_2O	V(0.42,0.45, 0.5,0.525,0.55 0.6,0.65,0.75μ)	0.2(0.2)1.0(0.5) 3(1)5,8	K	T,G	D,2p,
5	Ryde	$\tilde{n}$ after Saxton	H_2O Ice Hail	R(1-10cm)	small and var.	R#, K(for clouds, rain)#.	G	D,Sum- mary article
6	Schalén 1939/45	1.16-1.27i 1.27-1.37i 1.34-1.45i 1.38-1.50i 1.51-1.63i 1.70-1.84i	Fe	V	0.x-3.98 0.x-3.57 0.x-3.57 0.x-3.18 0.x-2.78 0.x-2.50	K	T	D,2p.
		1.36-2.30i 1.46-2.68i 1.44-2.88i 1.50-3.10i 1.58-3.42i	Ni	V	0.x-2.39 0.x-2.14 0.x-1.00 0.x-1.71 0.x-1.00	K	T	D

No.	Author Year	Refractive Index	Compound	Spectral Range	α Size Parameter	Tabulated Quantities	Presentation	Type of Comp.
6		1.74-3.80i	Ni	V	0.x-1.00	K	T	D
		0.84-2.91i 0.93-3.18i 1.05-4.49i 1.41-4.10i 1.93-4.66i 2.62-5.08i	Zn	V	0.x-2.39 0.x-3.57 0.x-3.57 0.x-1.99 0.x-2.78 0.x-1.71	K	T	D
		1.17-1.76i 1.14-2.05i 1.10-2.34i 1.00-2.28i 0.89-2.23i 0.65-2.43i 0.62-2.63i 0.56-3.01i	Cu	V	0.x-2.39 0.x-2.14 0.x-1.00 0.x-1.00 0.x-1.00 0.x-1.00 0.x-1.00 0.x-1.00	K	T	D
		0.06-1.84i 0.05-2.21i	Na	V	0.x-2.14 0.x-1.71	K	T	D
7	Lowan 1949	3.41-1.94i 4.21-2.51i 5.55-2.85i 7.20-2.65i	H_2O in air	R(0.3cm) R(0.5cm) R(0.9cm) R(1.7cm)	0.1(0.05)1(0.1)5 0.1(0.05)1(0.1)3 0.1(0.05)1(0.1)2 0.1(0.025)1(0.05) 1.3	a,b,Ke,@	TG	D,3p(K); 4p(a,b)
		8.18-1.96i 8.90-0.96i		R(3cm) R(10cm)	0.1(0.025)1 0.1(0.01)0.3(0.005) 0.43(0.01)0.6			
8	Aden 1951	9.01-0.43i	H_2O in air	R(16.2cm)	0.6-6.0	R,Θ=180	G	D, compared with exp. results
9	Chu 1952	$\tilde{m}$ after Saxton (?)	H_2O in air	R(0.1cm) R(0.3cm) R(0.5cm) R(0.75cm) R(1 cm)	0.05(0.05)0.5(0.1) 1.5(0.25)5	K,i Θ=180	T	D,4s

No.	Author Year	Refractive Index	Compound	Spectral Range	α Size Parameter	Tabulated Quantities	Presentation	Type of Comp.
10	Kennough Sloan 1952	3.41-1.94i	H_2O in air	R(0.28cm)	0.1(0.05)1(0.1) 5	R,θ=180	?	N
		4.21-2.51i		R(0.45cm)	0.1(0.05)1(0.1) 3			
		5.55-2.85i		R(0.8cm)	0.1(0.05)1(0.1) 2			
		7.20-2.65i		R(1.6cm)	0.1(0.025)1 (0.05)1.3			
		8.18-1.96i		R(2.8cm)	0.1(0.025)1			
		8.90-0.69i		R(10cm)	0.1(0.01)0.3 (0.005)0.43(0.01) 0.6			
11	Johnson Terrell 1955	1.29-0.064i			0-18 [17]	K	T,G	D,2p
		1.29-0.129i			0-7 [6]			
		1.29-0.322i			0-25 [22]			
		1.29-0.645i			0-6.6 [8]			
		1.29-1.161i			0-1.0 [1]			
		1.29-1.290i			0-6 [7]			
		1.29-1.419i			0-1 [1]			
		1.29-2.234i			0-6 [5]			
		1.29-2.580i			0-2 [3]			
		1.29-2.902i			0-2 [3]			
		1.29-5.160i			0-2 [3]			
		1.29-129i			0-2 [3]			
		1.14-0.114i	H_2O in air	IR(11μ)	1-17.5 [23]	K	T	D,2p
		1.17-0.210i		IR(11.9μ)	1-17.5 [23]			
		1.22-0.061i		IR(10.0μ)	1-17.5 [23]			
		1.28-0.051i		IR(8.5μ)	1-17.5 [23]			
		1.28-0.294i		IR(12.6μ)	1-17.5 [23]			
		1.33-0.013i		IR(4.6μ)	1-17.5 [23]			
		1.33-0.040i		IR(7.0μ)	1-17.5 [23]			
		1.33-0.399i		IR(13.5μ)	1-17.5 [23]			
		1.42-0.014i		IR(3.6μ)	1-17.5 [23]			
12	Kerker 1955	1.46-4.30i	Hg	V	0.2,0.4,0.5	i θ=130,140, 150	T	D,4s
	1955,U				0.2-5(23)	i θ=30(10)150		D
13	Chromey 1960	m=0.5(0.25)3 K=0(0.1)1			0.2(0.2)2	Ke,Ks,ϕ,	T	C,IBM?, 4s

No.	Author Year	Refractive Index	Compound	Spectral Range	α Size Parameter	Tabulated Quantities	Presentation	Type of Comp.
14	Stull Plass 1960	$\tilde{m}$ var. for 0.4 to 13μ at 2250K	C at high temp.	IR(1-20μ)	0.075-1.5 for λ = 0.4μ 0.0015-0.01 for λ = 20μ	Kφ,Kφ	G	C, IBM 650
15	Stephens 1960	3.95-2.32i	H_2O in air	R(0.43cm)	0.04-5.2	R θ=180	T,G	C, IBM 650, 4s
		5.43-2.84i		R(0.86cm)	0.04-2.8			
		5.82-2.81i		R(1 cm)	0.3-2.5			
		7.50-2.47i		R(2 cm)	0.15-1.3			
		8.51-1.57i		R(4 cm)	0.08-0.65			
		8.67-1.30i		R(5 cm)	0.04-1.5			
		8.77-1.11i		R(6 cm)	0.04-0.42			
		8.83-0.96i		R(7 cm)	0.04-0.38			
		8.87-0.85i		R(8 cm)	0.04-0.33			
		8.89-0.76i		R(9 cm)	0.04-0.28			
		9.01-0.43i		R(16.23cm)	0.02-0.65			
		6.4 -2.8i		R(1.24cm)	0.04-2.0			
		7.1 -2.89i	H_2O in air 0°C	R(3.2cm)	0.2-4.0			
		8.0-2.33i	10°C	R(3.2cm)	0.2-0.8			
		8.3-1.90i	18°C	R(3.2cm)	0.04-4.0			
		8.33-1.72i	30°C	R(3.2cm)	0.2-0.8			
		8.9-0.69i	18°C	R(10cm)	0.2-0.75			
		1.78-0.0024i	Ice	all	0.03-4.0			
16	Havard 1960	$\tilde{m}$ var. with λ	H_2O in air	IR(3.6-13.5μ)	r=1,2,4,9,12,15μ	Ke,Ka,i θ=0 (10)180	T	C, IBM, 650
17	Herman Battan 1961	1.78-0.0024i	Ice, 0°C	R(1-10cm)	0.1(0.1)3(0.2)30 (10)100(50)200, 500	a,b,R θ=180	T,G	C, IBM, 650, 4p
		7.14-2.89i	H_2O in air	R	0(?)10	R θ=180	G	
		2.00-0.048i	Hail	R	1-6	R θ=180	G	
		3.00-0.048i			1-6			
		4.00-0.048i			1-6			
		5.00-0.048i			1-6			
		8.99-0.048i			1-6			
		8.99-0.081i		R	1-6	R θ=180	G	
		8.99-0.090i			1-6			
		8.99-0.222i			1-6			
		8.99-0.45i			1-6			

No.	Author Year	Refractive Index	Compound	Spectral Range	α Size Parameter	Tabulated Quantities	Presentation	Type of Comp.
18	Olaf Robock 1961, U	n=1.5,1.75,2.0			1(2)10	1 Θ=0(2)180,●	T	C, IBM 704
		K=0,0.125,0.25			12(?)30	1 Θ=0(1)180,●	T	
	1961	1.75-0.125i			2,8,14,20,26	1 Θ=0-180,●	G	
		2.0-0.25i			same			
19	Deirmendjian, Clasen Viezee 1961, U	1.111-0.1831i	H_2O	IR(11.5μ)	0.5-10	a,b,S,i Θ=0 (?)130,Ke,Ks	T, selected	C, IBM 704
		1.212-0.0601i		IR(10.0μ)	0.5-10			
		1.29-0.0472i		IR(8.15μ)	0.5-12		G for publ.	IBM 7090
		-0.0645i	-	-	0.5-18			
		-0.472i		-	5.25-10			
		1.315-0.0143i	H_2O	IR(1.61μ)	0.5-15			
		-0.4298i		IR(6.05μ)	0.5-10			
		-0.0137i		IR(5.30μ)	0.5-15			
		-0.0059i		IR(15.0μ)	0.5-20			
		1.353-0.0059i		IR(3.90μ)	0.5-20			
		1.44-0.4000i		IR(16.6μ)	0.5-7			
		1.525-0.0050i	-	-	0.5-7			
		-0.100i	-	-	0.5-7			
		-0.0682i	H_2O	IR(3.07μ)	0.5-25			
		1.28-1.37i	Fe	V(0.441μ)	0.1-7			
		1.51-1.63i	Fe	V(0.589μ)	0.1-7			
		1.70-1.84i	Fe	V(0.668μ)	0.1-7			
20	Giese 1959 U	1.27-1.37i	Fe	V(0.44μ)	0.25(0.25)1.5 (0.5)7(1)16	Ke,Ks,Kpr,i Θ=0(2)180,P	T	C, Siemens 2002, 4s
		1.38-1.5i	Fe	V(0.58μ)	1(1)4(2)16	Ke,Ks,Kpr,i Θ=0(10)180,P		
		1.7-1.84i	Fe	V(0.668μ)	same			
		1.41-4.1i	Zi	V(0.508μ)	same			
		1.33-6i	-	-	1,2,4,6			
	1959	1.33-0.5i			10	a,b,S,i Θ=0 (10)180	T, samples	
		1.27-1.37i	Fe		10			
		1.38-1.5i	Fe		10			
		1.7-1.84i	Fe		10			
		1.41-4.1i	Zi		10			
		1.27-1.37i	Fe		1,3,14	1 Θ=0-180	G	
		1.27-1.37i			1-15	P	G	

No.	Author Year	Refractive Index	Compound	Spectral Range	α Size Parameter	Tabulated Quantities	Presentation	Type of Comp.
20	1961	1.27-1.371	Fe		0.25,1(1)6(2)10 (5)20,30	i θ=0-180	Ö	
		1.33-Ki κ=0,0.5,1,1.33 2,3,4			0.5(0 5)7(1)15	i θ=0-180	G	

No.	Author Year	Refractive Index	Compound	Spectral Range	α Size Parameter	Tabulated Quantities	Presentation	Type of Comp
21	Stephens 1961	1.338-0.0015i	H_2O	IR (4 μ)	0.75(0.05)1.1(0.1) 2.6(0.2)5.2,6,7,8	Ke,Ks,Ka, e, s, a	T	C,IBM
		1.343-0.0096i		IR (4.5 μ)	0.65(0.05)1.1(0.1) 2.6(0.2)5.2,6,7,8			4p
		1.330-0.010i		IR (5 μ)	0.60(0.05)1.1(0.1) 2.6(0.2)5.2,6,7,8			
		1.300-0.0097i		IR (5.5 μ)	0.55(0.05)1.1(0.1) 2.6(0.2)5.2,6,7,8			
		1.324-0.1341i		IR (6 μ)	0.50(0.05)1.1(0.1) 2.6(0.2)5.2,6,7,8			
		1.334-0.0969i		IR (6.5 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)5.1			
		1.327-0.0224i		IR (7 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)4.6			
		1.31-0.0228i		IR (7.5 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)4.2			
		1.293-0.0232i		IR (8 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)4.0			
		1.28-0.0243i		IR (8.5 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)3.8			
		1.264-0.0261i		IR (9 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)3.6			
		1.24-0.0289i		IR (9.5 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)3.4			
		1.196-0.0368i		IR (10 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)3.2			
		1.187-0.2221i		IR (12 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)2.8			
		1.30-0.413i		IR (14 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)2.4			
		1.38-0.472i		IR (16 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)2.0			
		1.505-0.4189i		IR (18 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)1.8			
		1.53-0.3599i		IR (20 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)1.6			
		1.53-0.3072i		IR (22 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)1.5			

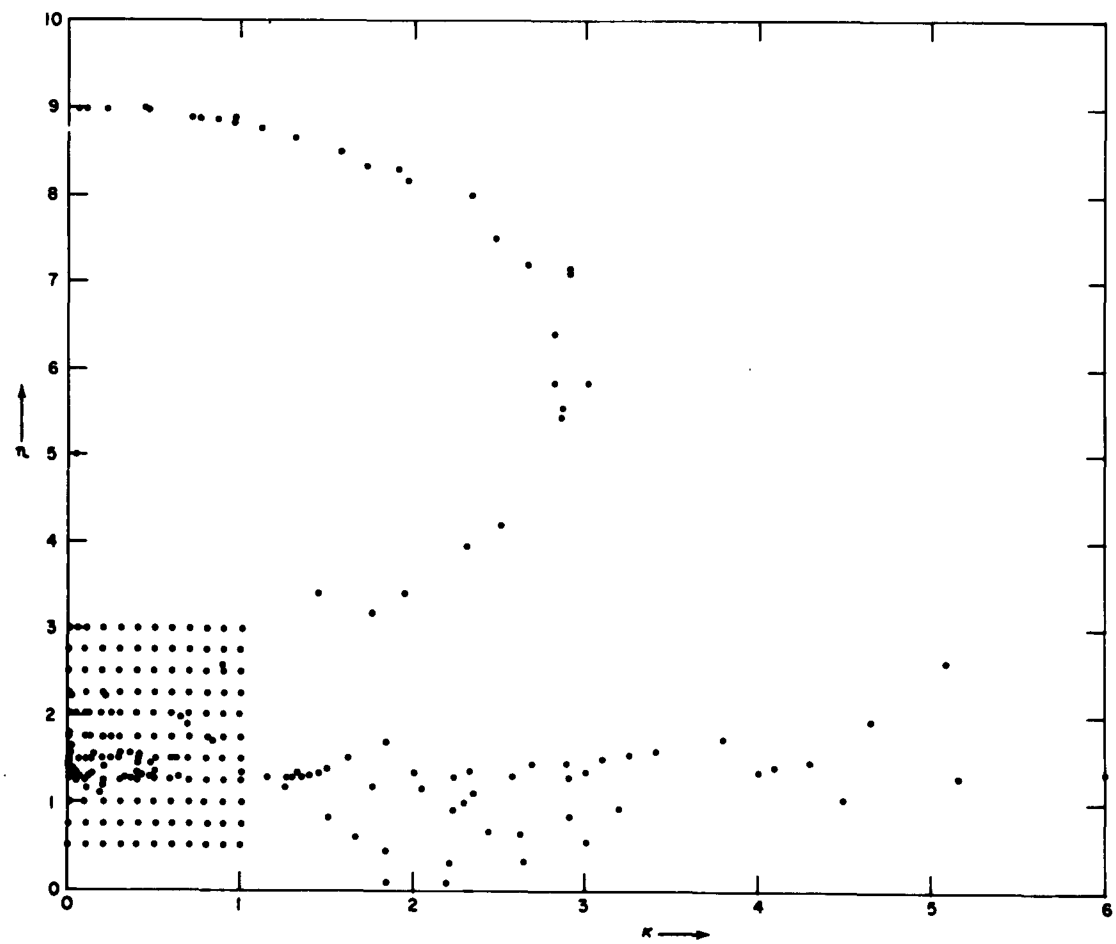
No.	Author Year	Refractive Index	Compound	Spectral Range	α Size Parameter	Tabulated Quantities	Presentation	Type of Comp
21		1.49-0.2608i		IR (24 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)1.4			
		1.41-0.2272i		IR (26 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)1.3			
		1.39-0.2114i		IR (28 μ)	0.01(0.01)0.5 (0.05)1.1(0.2)1.2			
		1.38-0.1874i		IR (30 μ)	0.01(0.01)0.5 (0.05)1.1			
		1.36-0.160i		IR (35 μ)	0.01(0.01)0.5 (0.05)0.95			
		1.36-0.1752i		IR (40 μ)	0.01(0.01)0.5 (0.05)0.8			
		1.36-0.2606i		IR (45 μ)	0.01(0.01)0.5 (0.05)0.8			
		1.36-0.3921i		IR (50 μ)	0.01(0.01)0.5 (0.05)0.75			
		1.36-0.4797i		IR (55 μ)	0.01(0.01)0.5 (0.05)0.7			
		1.37-0.5145i		IR (60 μ)	0.01(0.01)0.5 (0.05)0.65			
		1.38-0.5026i		IR (65 μ)	0.01(0.01)0.5 (0.05)0.5			
		1.40-0.4823i		IR (70 μ)	same			
		1.41-0.4645i		IR (75 μ)	same			
		1.41-0.4632i		IR (80 μ)	same			
		1.41-0.4641i		IR (85 μ)	same			
		1.41-0.4815i		IR (90 μ)	same			
22	Stephens	same $\tilde{n}$	H_2O	same IR region; 35 λ between 4 and 90 μ; T = 253 to 298° K	r = 0.5(0.25)5 μ	$\overline{Ke}$, $\overline{Ks}$, $\overline{Ka}$	T, G	C, IBM 4p

No.	Author Year	Refractive Index	Compound	Spectral Range	α Size Parameter	Tabulated Quantities	Presentation	Type of Comp
23	Stephens 1961	same $\tilde{n}$	H_2O	IR, (5,6.2,7, 8,10,12,13,15, 18 μ)	0.05(0.05)1.1 . (0.1)2.0(0.2)5.0	Ka, #	T, G	C, IBM
24	Herman 1962	1.338-0.0015i 1.330-0.0098i 1.324-0.1345i 1.327-0.0224i 1.293-0.0231i 1.264-0.0260i 1.196-0.0368i 1.187-0.2220i 1.300-0.4131i 1.389-0.4721i 1.505-0.4179i 1.480-0.3596i 1.454-0.3069i 1.429-0.2606i	H_2O	IR(4μ) IR(5μ) IR(6μ) IR(7μ) IR(8μ) IR(9μ) IR(10μ) IR(12μ) IR(14μ) IR(16μ) IR(18μ) IR(20μ) IR(22μ) IR(24μ)	0.1-30	Ke,Ka,Ks, Example for absorption by cloud of droplets.	G	C,IBM 650
25	MacDonald 1962	1.9-0.684i	C	V(0.436μ)	0.2(0.2)1(0.5)2, 4,8	N#, Ke,Ks,Ka,#	G	C,IBM 650
		2.0-0.66i	C	V(0.623μ)	same			
26	Adler Johnson 1962	1.45957-0.01897i 1.628-0.01953i 1.98511-0.02481i 2.01284-0.03923i 8.18-1.96i ∞	Teflon Lucite Bakelite Bakelite H_2O	R (Range R 5518-5800 R Mc/s) R R	0.2(0.1)5 0.4(0.1)10	R θ=180	T,G	C,IBM 709 4s
	1962, U	same set			0.2(0.01)5			
27	Deirmendjian 1963	1.111-0.1831i 1.212-0.0601i 1.28-1.37i	H_2O Fe	IR(11.5μ) IR(10.0μ) V(0.44μ)	0.5(0.5)10 0.5(0.5)10 0.1(0.1)1(0.25) 2(0.5)10	Ks,S1(0,180) " "	T	C,IBM 7090
		1.29-0.0472i -0.0645i -0.4720i	H_2O	IR(8.15μ) - -	0.5(0.5)15 same same	" " "		

No	Author Year	Refractive Index	Compound	Spectral Range	α Size Parameter	Tabulated Quantities	Presentation	Type of Comp.
27		1.315-0.0143i	H_2O	IR(5.30μ)	same	Ks		
		-0.1379i		IR(6.05μ)	same	Ks		
		-0.4298i		IR(15.0μ)	same	Ks		
		1.353-0.0059i		IR(3.90μ)	1(1)20	Ks		
		1.44-0.4000i		IR(16.6μ)	0.5(0.5)7	Ks,81(0,180)		
		1.51-1.63	Fe	V(0.589μ)	0.1(0.1)1(0.25) 2(0.5)10	"		
		1.525-0.0682i	H_2O	IR(3.07μ)	1(1)25	Ks		
		1.55-0.0155i			2(2)12(4)40	Ks,81(0,180)		
		-0.1550i			same	"		
		1.70-1.84i	Fe	V(0.668μ)	0.1(0.1)1(0.25) 2(0.5)10	"		
		1.78-0.0024i	Ice	R(0.2-0.5cm)	0.5(0.5)4(2)16	"		
		2.20-0.0220i			0.5(0.5)10,12, (0.4)40	"		
		-0.2200i			same	"		
		2.5604-0.8947i	H_2O	R(0.2cm)	0.5(0.5)4(2)16	"		
		3.1918-1.7657i	H_2O	R(0.5cm)	0.25(0.25)1(1)7	"		
		5.8368-3.0046i	H_2O	R(2 cm)	0.1(0.1)1(0.5)2	"		

4.2 List of Computations arranged according to Refractive Index

Instead of a listing, we have prepared a figure in which the ordinate represents μ, the real part and the abscissa κ, the imaginary part. Each point in the plane represents then a refractive index $\tilde{n}$ for which the same type of computation has been accomplished.



4.3 References

1. Mie, G., Beiträge zur Optik trüber Medien - speziell kolloidaler Metallösungen", Ann. Physik (4), 25, 377 (1908).

2. Schirmann, M.A., "Dispersion und Polychroismus des polarisierten Lichtes, das von Einzelteilchen von der Groessenordnung der Wellenlaengc des Lichtes abgebeugt wird". Sitzungsber. Akad. Wien. Math. naturw. Klasse, IIa 127, 1559 (1918).

3. Senftleben, H. and E. Benedict, "Ueber die Beugung des Lichtes an den Kohlenstoffteilchen leuchtender Flammen", Ann. Physik (4), 60, 297 (1919).

4. Feick, R., "Ueber die Farbe von kolloidallem Silber und Quecksilber", Ann. Physik 77, 673 (1925).

5. Ryde, J.W., "The attenuation of radar echoes produced at centimeter wavelengths by various meteorological phenomena". In:"Met. factors in Radiowave propagation". Phys. Soc. London, p. 169, (1946).

6. Schalén, C., "Beitraege zur Theorie der interstellaren Absorption" and "Studies of reflection nebulae", Uppsala Astron, Obs. Ann. 1, No. 2 (1939) and 1, No. 9 (1945)

   Schalén, C. and G. Wernberg, "Einige Berechnungen des Strahlungsdruckes auf absorbierende Teilchen", Meddelande Uppsala Astron. Obs. No. 83, (1941) and Arkiv, Met., Astr., Fysik 27A.

7. Lowan, A.N., "Tables of scattering functions for spherical particles". U.S. Nat. Bur. Standards, Appl. Math. Series 4, (1948).

8. Aden, A.L., "Electromagnetic scattering from spheres with sizes comparable to a wavelength". J. Appl. Phys. 22, 601 (1951).

9. Chu, C.M., "Scattering and absorption of water droplets at millimeter wavelengths". Thesis, U. of Michigan (1952).

10. Kennaugh, E. and R. Sloan, "Effects of Type of polarization on echo characteristics". Ohio State Univ. Res. Found. Rep. No. 389-15 AF28(099)-90 (1952).

11. Johnson, J.C. and J.R. Terrell, " Transmission cross sections for water spheres illuminated by infrared radiation". JOSA 45, 451, 1955.

    Johnson, J.C., R.G. Eldridge and J.R. Terrell, "Improved infrared transmissometer for cloud drop sizing". Scient. Rep. No. 4, AF19(122)-245, AFCRC-TN-54-255, Dept. Met. MIT (1954).

12. Kerker, J., "Scattering functions for spherical particles of refractive index of 1.46 - 4.30 i." J. Opt. Soc. Am. 45, 1081 (1955) Doc. 4678 L.C.

13. Cromey, F.C., "Evaluation of Mie equations for colored spheres". J. Opt. Soc. Am. 50, 730, (1960) Doc. 6247 L.C.

14. Stull, V.R. and G.N. Plass, "Emissivity of dispersed carbon particles". J. Opt. Soc. Am. 50, 121 (1960).

15. Stephens, J.J., "Radar cross-sections for water and ice spheres". J. Met. 18, 348 (1961).

16. Havard, J.B., "On the radiational characteristics of water clouds at infrared wavelengths." Thesis, U. of Washington (1960).

17. Herman, B.M. and L.J. Battan, "Calculations of Mie back-scattering of microwaves from ice spheres". Q. J. Roy. Met. Soc. 87, 223 (1961).

18. Olaf, J. and K. Robock, "Zur Theorie der Lichtstreuung an Kohle and Bergepartikeln". Staub 21, 495 (1961).

19. Deirmendjian, D., R.J. Clasen and W. Viezee, "Mie scattering with complex index of refraction". J. Opt. Soc. Amer. 51, 620 (1961).

20. Giese, R.H., "Lichtstreuung an sphärischen Metallpartikeln". Z. f. Naturf. 14a, 1085 (1959).

Giese, R.H., "Streuung elektromagnetischer Wellen an absorbierenden und dielektrischen kugelfoermiger Einzelteilchen und an Gemischen solcher Teilchen". Z. f. Astrophys. 54, 119 (1961).

21. Stephens, J. J., "Total attenuation, scattering and absorption cross-sections of water drops for infrared radiation" AFCRL 1011, Rep. 4-05, Electr. Eng. Res. Lab., Univ. Texas, 146 p, (1961).

22. Stephens, J. J., "Spectrally averaged total attenuation, scattering and absorption cross-section for infrared radiation" J. Met. 18, 822 (1961).

23. Stephens, J. J. and J. R. Gerhardt, "Absorption cross-section of water drops for infrared radiation" J. Met. 18, 818 (1961).

24. Herman, B.M., "Infrared absorption, scattering, and total attenuation cross-sections for water spheres". Q. J. Roy. Met. Soc. 88, 143 (1962).

25. McDonald, J.E., "Visibility reduction due to jet-exhaust carbon particles". J. Appl. Met. 1, 391 (1962).

26. Adler, S.B. and R.S. Johnson, "New backscattering computations and tables for dielectric and metallic spheres". Appl. Opt. 1, 655 (1963). Doc. S-289 Univ. Microfilms Inc., Ann Arbor, Mich.

27. Deirmendjian, D., "Tables of Mie scattering cross-sections and amplitudes". Rand Co. Report R-407-PR (1963).

V. HETERODISPERSE SYSTEMS OF ISOTROPIC SPHERICAL PARTICLES

5.1 List of existing computations

The list has been prepared in a form similar to 3.1 and 4.1. The important column here is the "size distribution" used by the author. We have cnly indicated the type of distribution function used; for detail one has to read the original paper. It seems impossible to list an exact formula for each case.

No.	Author Year	Refractive Index	Range of α Values	Tabulated Quantities	Size Distribution	Presentation	Type of Computations
1	Foitzik 1950	1.33	0.01-100	K	3 gaussian distrib.	T	graphical method
2	Gilbert 1956	1.33	0-30	K, i Θ=0 (10) 180	numerous gaussian type distrib.	G	C;ERA 1103 based on Penndorf (35) -not quite reliable
3	Bouziaux Battian 1958	1.33	0(0.4)52	i Θ=0(10)180	3 square distrib.	T	based on Lowan (23) Gumprecht (28) D
4	Deirmendjian 1959/1960	H_2O $\tilde{n}$ variable	λ =0.6-14μ	K^*	Haze and cloud model size distrib.	T,G	D
5	Stevenson Heller 1961	1.05(0.05)1.3	0(0.2)25.2	P Θ=90	$q=\alpha_o(s/r_o)$ s = half-width of distrib $f(r)=(r-r_o)\exp\left[-\left(\frac{r-r_o}{s}\right)^3\right]$	T	C,IBM 650, 704, based on
6	Giese 1961	1.27-1.371 1.33 1.55	4or 10 to 40	i P Θ=0-180	$f(\alpha)$=const α^{-k} k=2,2.5,3 (Junge distr.)	G	C, Siemens 2002
7	Deirmendjian 1962	H_2O, variable $\tilde{n}$	r=0.01 to 10 depending on λ	$I\phi$, Θ=0-180	Haze and cloud model distribution		IBM 7090
8	Bullrich et al 1962	1.5	0.2(0.2)159	i Θ=0(1)10(10) 180;149(2)171	Junge distrib. for aerosols	G	C, Siemens 2002
				Visibility for 0.5 to 20km; λ = 0.4 to 1.2μ			
9	Herman	$\tilde{n}$, variable IR, water	λ= 4(1)10(2) 24μ	$N\phi$	cloud model	T	C,IBM 650

5.2 References

1. Foitzik, L., "Zur meteorologischen Optik von Dunst und Nebel". Z. f. Met. 4, 321 (1950).

2. Gilbert, T.L., "A study of the particle size distribution of aerosols from light scattering measurements". Quart. Status Rep. #2, AF19(604)-1428; Armour Res. Found. (1956).

3. Houziaux, L. and L. Battian, "Diffusion de la lumière par des particles sphériques d'indice de réfraction m = 1.33". Mém. Acad. Roy. Belgique, Classe des Sciences II series, Tome 14, fasc. 4 (1958).

4. Deirmendjian, D., "The role of water particles in the atmospheric transmission of infra-red radiation". Q. J. Roy. Met. Soc. 85, 404 (1959).

   Deirmendjian, D., "Atmospheric extinction of infra-red radiation". Q. J. Roy. Met. Soc. 86, 371 (1960).

5. Stevenson, A.F. and W. Heller, "Tables of scattering functions for heterodisperse systems". Wayne State Univ., Detroit, 214 pgs. (1961).

6. Giese, R.H., "Streuung elektromagnetischer Wellen an absorbierenden und dielektrischen kugelfoermiger Einzelteilchen und an Gemischen solcher Teilchen". Z. f. Astrophys. 51, 119 (1961).

7. Deirmendjian, D., "Scattering and polarization properties of polydispersed suspensions with partial absorption". Rand Co., RM-3228 PR (1962).

8. Bullrich, K., E. de Bary, B. Braun and R. Eiden, "Research on atmospheric optical radiation transmission". Final Rep. AFCRL-62-622, U. of Mainz, Germany, 1962.

9. Herman, B.M., "Infrared absorption, scattering and total attenuation cross-sections for water spheres". Q. J. Roy. Met. Soc. 88, 143, (1962).

VI. TWO OR MORE CONCENTRIC SPHERES OF DIFFERENT REFRACTIVE INDEX

6.1 List of computations

The list has been prepared in a form similar to 3.1 and 4.1. The refractive indices n_i and n_a indicate the refractive index for the inner core and the outer coating, similarly the symbols α_i and α_a or r_i and r_a indicate the size parameter or radius for the core and the coating; i.e., r_a is the thickness of the center shell; $q = r_i/(r_i + r_a)$.

No.	Author Year	Refractive Index	Range of α Values	Tabulated Quantities	Presentation	Type of Computations
1	Alred 1947	$n_i = \infty$ n_a = variable and complex	$\alpha > 12$	R	?	N; based on simplifications see Weil (5)
2	Kerker Langleben Gunn 1951	n_i = ice = 1.75 n_a = water = 8.9 -1.5i λ = 0.9;3;10cm	q = 0.0 to 1.0	R		
3	Guettler 1953	$n_i = Fe = (0.786 + 8.021)(\lambda + 0.215)^2$ $n_a = H_2O = 1.33$ λ = 0.4(0.5)0.7	α = 0.045 -0.8 q = 0(H_2O) (0.2) 1.0 (Fe)	Ke,Ks	T,G	D, 5p
4	Scharfmann 1954	ϵ_i = metal ϵ_a = 2.56,4,8,10, 15,25,50,100, 10^3, 10^4	r_a/r_i = 1/6	R	T	N, see Weil (5)
5	Herman Battan 1961	n_i = 1.78-0.0024i n_a = 7.14-0.0566i (λ=3.21 cm) n_a = 7.95-0.0348i (λ=4.67 cm) n_a = 8.99-0.01821 (λ=10 cm)	r_a = 0.1 to 5 cm $r_a - r_i = 10^{-8}$ to 5 cm	R	G	C, IBM 650
		(ice/water)	α_a = 0.1(0.1)23(2.0)53	K	T,G	C, IBM 704 7090
6	Herman Battan 1961	n_i = 1.0(air) n_a = 7.14-2.44i(water) 3.21 cm	r_i = 0.1 to 2.5 cm r_a = 1 to 8 u	R	G	C, IBM 650

No.	Author Year	Refractive Index	Range of α Values	Tabulated Quantities	Presentation	Type of Computations
7	Kerker Kratohvil Matijevic 1962	ni = 2.1050 na = 1.4821	αa = 0.1(0.1)23(2.0) 53 ($\alpha i/\alpha a$) = 0(0.2) 0.8,0.9,0.95,0.99, 1.0	K	T,G	C, IBM 704 7090
8	Penn, Oser 1962	ni = 1.59-0.66i,C na = 1.33(H_2O)	ra/ri = 1,1.001, 1.005, 1.01, 1.05, 1.1,1.5,2,5,10, 15,20 α so that $2\pi ra/\lambda<40$	Ke,Ks,Ka, $I\theta=0(1)20$ (10)180	G,T for Ka	C, IBM 704 4p
9	Herman Battan 1962	$\tilde{n}$, homog.mixture of water and ice at 0°C; water mass 0,10,25,50 75,90,100% λ = 3.21 cm	0 to 10	Ke,Ks,Ka,R	G	?
		ice core,spongy shell	1-5 ra-ri = 0.01,0 1 cm	R	G	?

6.2 References

1. Alred, R.V., "Scattering of electromagnetic waves by metallic spheres covered with absorbing materials". Rep. XRE 4/47/5. Admiralty Signal Estab. Witley, England (1947).

2. Kerker, M., M.P. Langleben and K.L.S. Gunn, "Scattering of microwaves by a melting spherical ice particle". J. Met. 8, 424 (1951).

   Langleben, M.P. and K.L.S. Gunn, "Scattering and absorption of microwaves by a melting ice sphere". Sc. Rep. MW-5. McGill Univ. (1952).

3. Guettler, A., "Die Mie'sche Theorie der Beugung durch dielektische Kugeln mit absorbierendem Kern und ihre Bedeutung fuer Probleme der interstellaren Materie und des atmosphaerischen Aerosols". Ann. Phys. (6) 11, 65, (1953).

4. Scharfmann, H., "Scattering from dielectric-coated spheres in the region of first resonance". J. Appl. Phys. 25, 1352 (1954).

5. Hermann, B.M. and L.J. Battan, "Calculations of Mie backscattering from melting ice spheres". J. Met. 18, 468 (1961).

6. Battan, L.J., B.M. Herman, "Backscattering of 3.21 -centimeter radiation by water bubbles". J. Geophys. Res. 66, 3255 (1961).

7. Kerker, M., J.P. Kratohvil, E. Matijevic. "Light scattering functions for concentric spheres. Total scattering coefficients $m = 2.1050$, $m_2 = 1.481$". J. Opt. Soc. Am. 52, 551 (1962).

8. Fenn, R., H. Oser, "Theoretical considerations on the effectiveness of carbon seeding" U. S. Army Signal Res. and Dev. Lab., Fort Monmouth, USASRDL Techn. Rep. 2258, 30 p. (1962).

9. Battan, L. J., B. M. Herman, "The radar cross section of 'spongy' ice spheres." J. Geophys. Res. 67, 5139 (1962).

6.2 ATLAS OF SCATTERING DIAGRAMS FOR n = 1.1, 1.2, 1.33, 1.4, 1.44, and 1.5

The diagrams for n = 1.1 and n = 1.2 are based on numerical data published by Pangonis and Heller, the rest is taken from our unpublished listings. The diagrams for n = 1.33 and n = 1.5 are repeated from earlier reports.

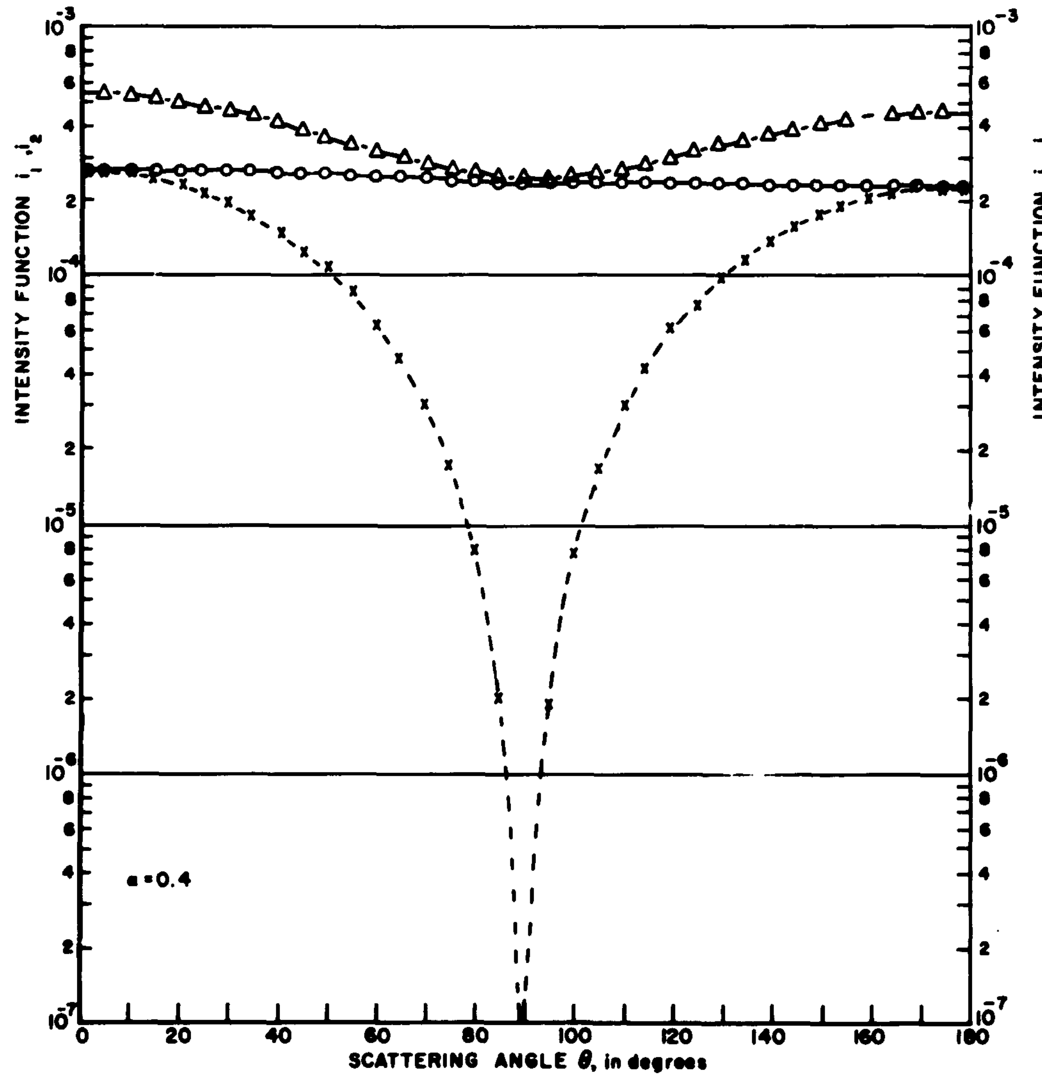
i_1 = ――――――

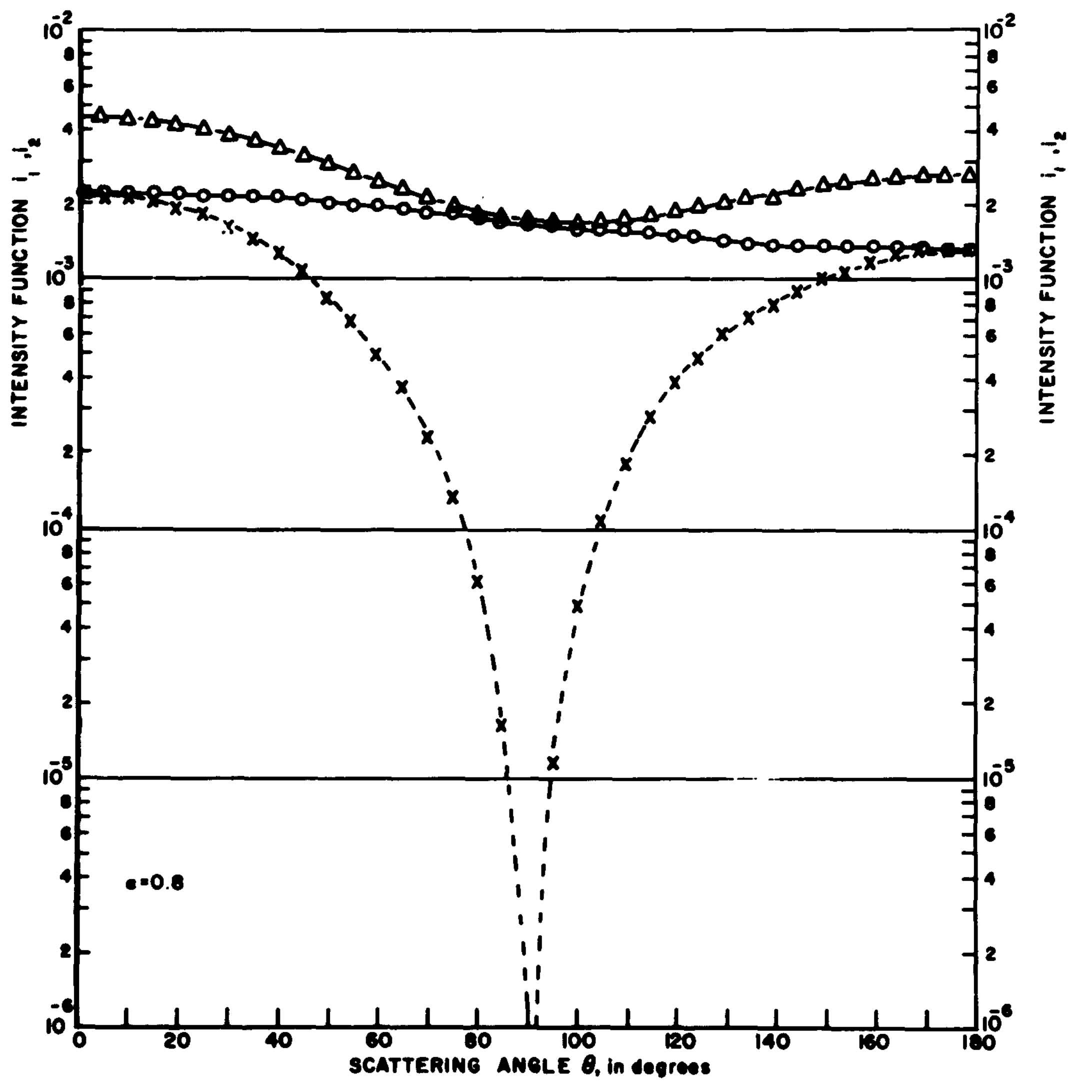
i_2 = — — —

$i_1 + i_2$ = — · — · —

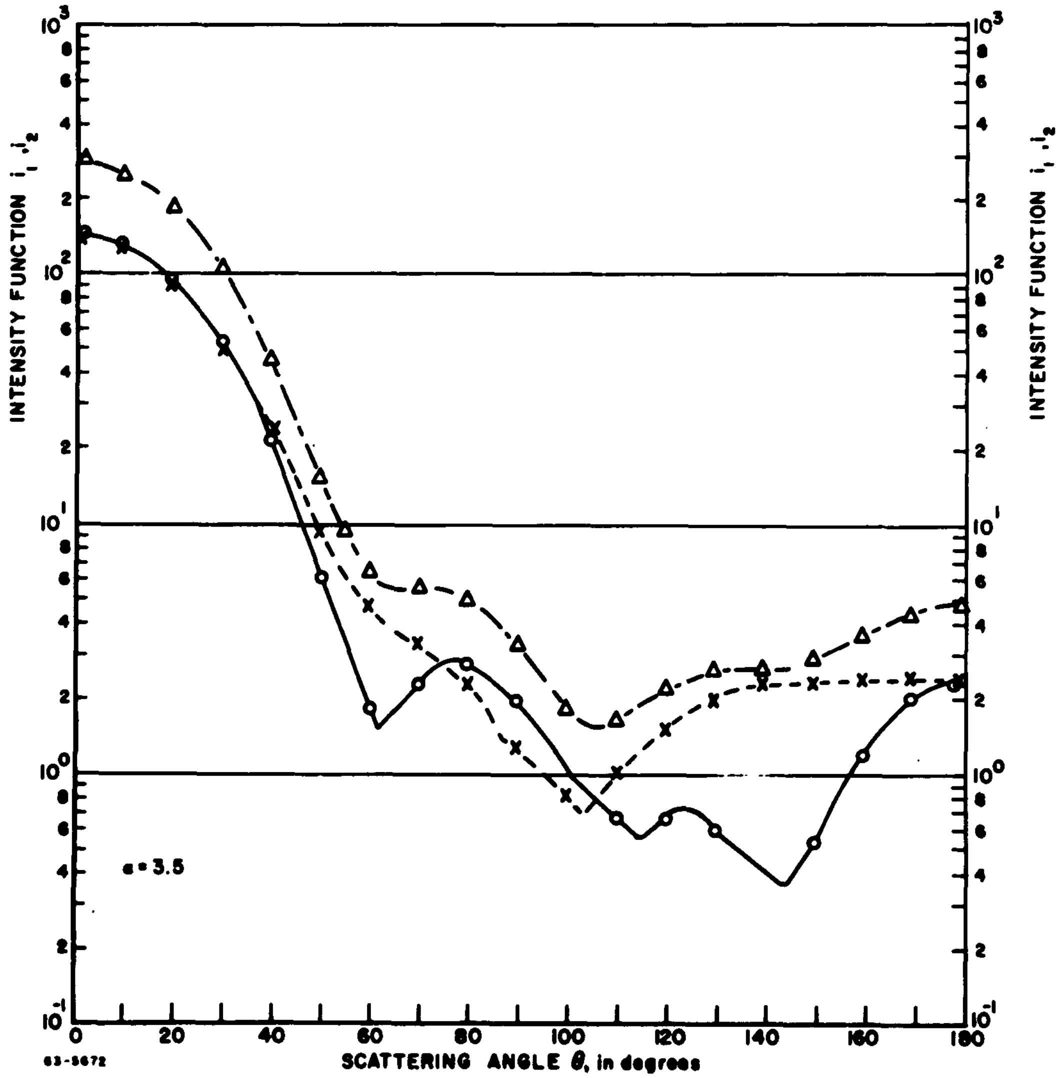
6.21 Atlas of scattering diagrams

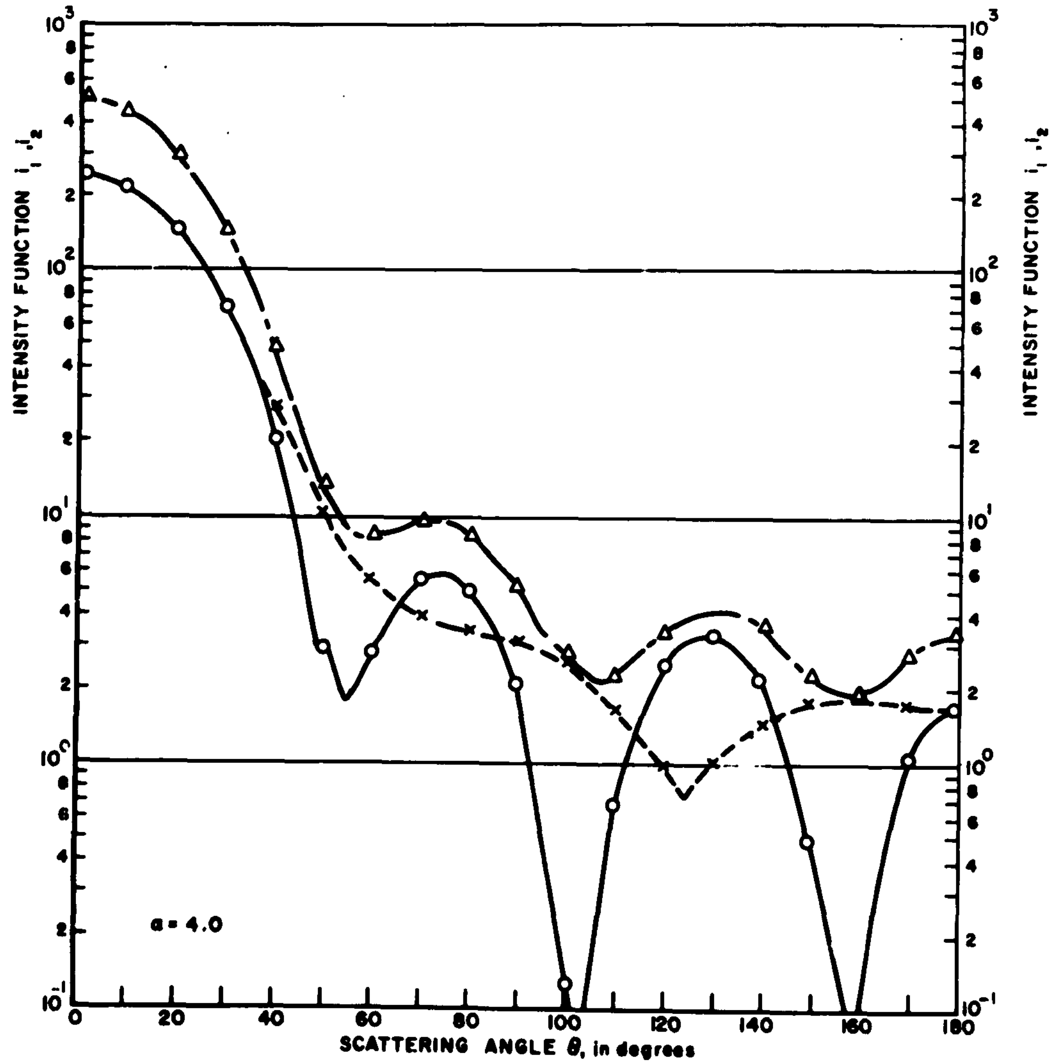
for $n = 1.1$

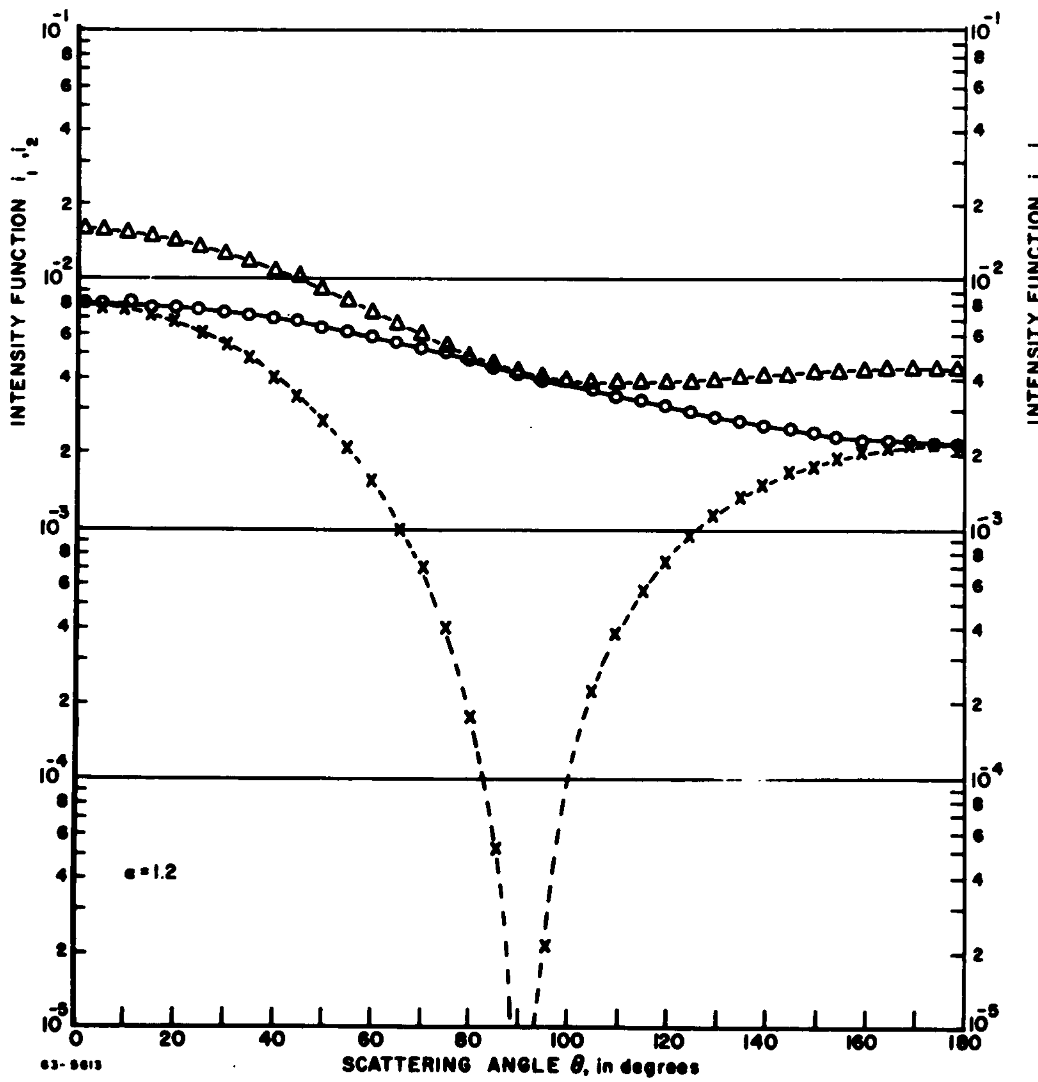

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 0.4$
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
20
40
60
80
100
120
140
160
180



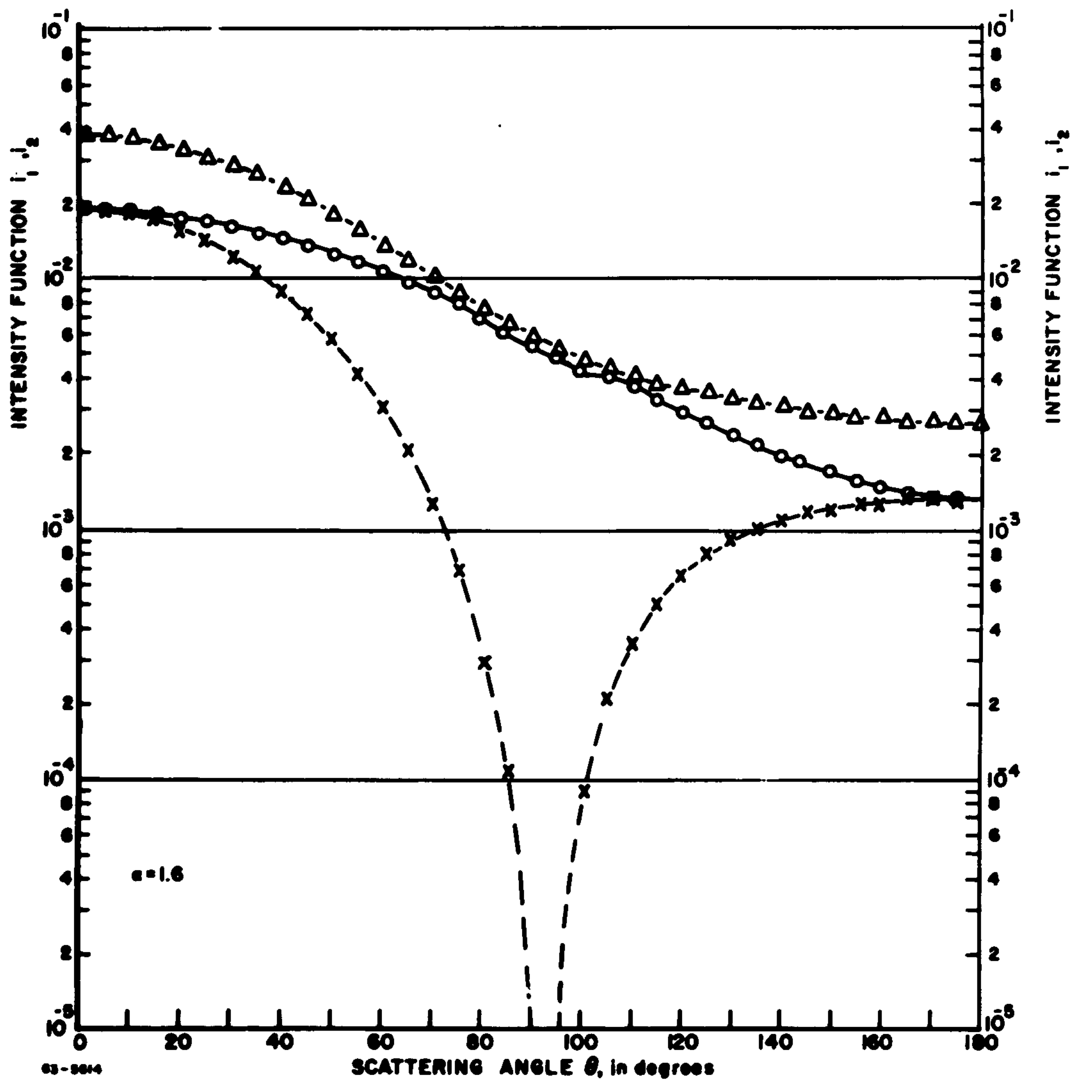
INTENSITY FUNCTION i_1, i_2
10^{-2}
8
6
4
2
10^{-3}
10^{-4}
10^{-5}
10^{-6}
α=0.8
0
20
40
60
80
100
120
140
160
180
SCATTERING ANGLE θ, in degrees

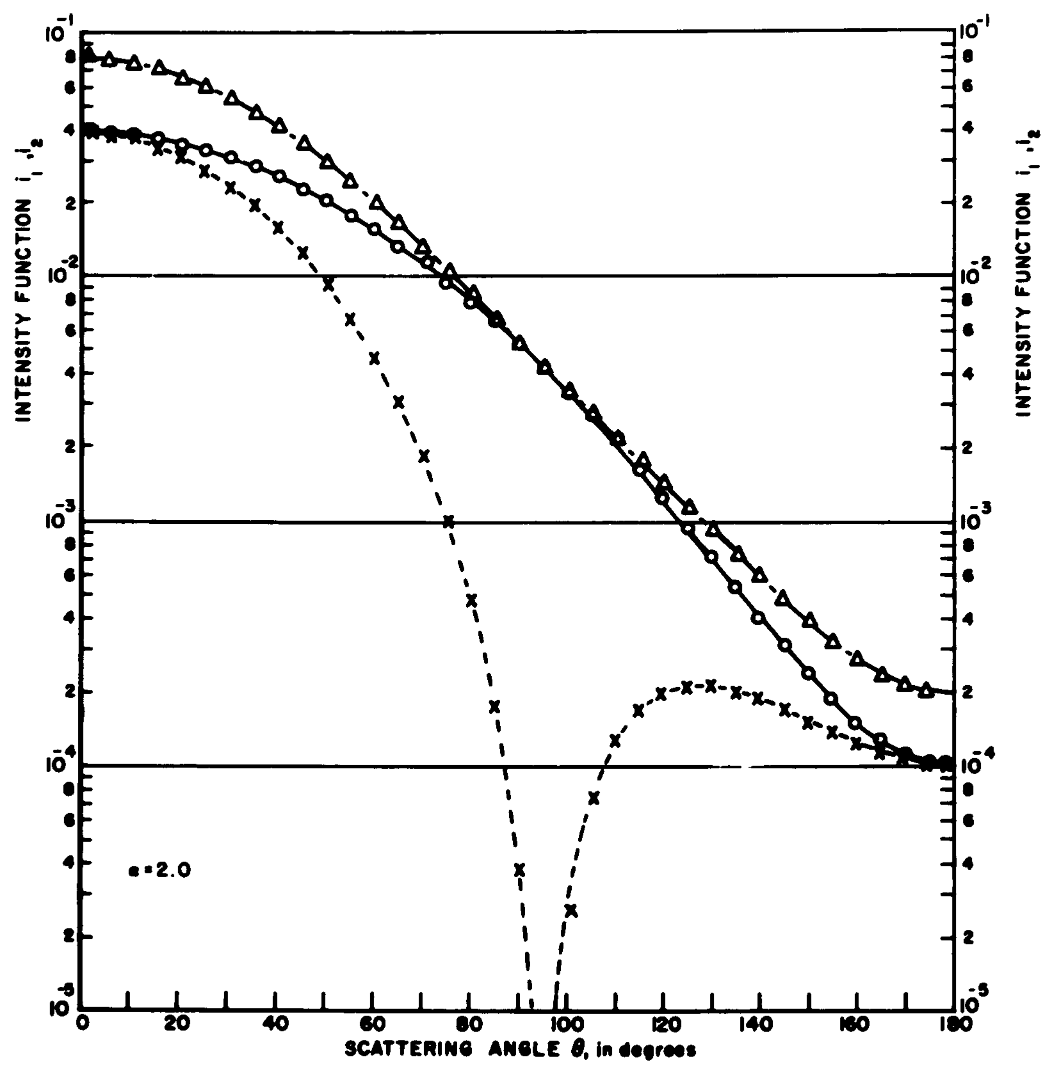

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
α = 3.5
63-3672


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 4.0$

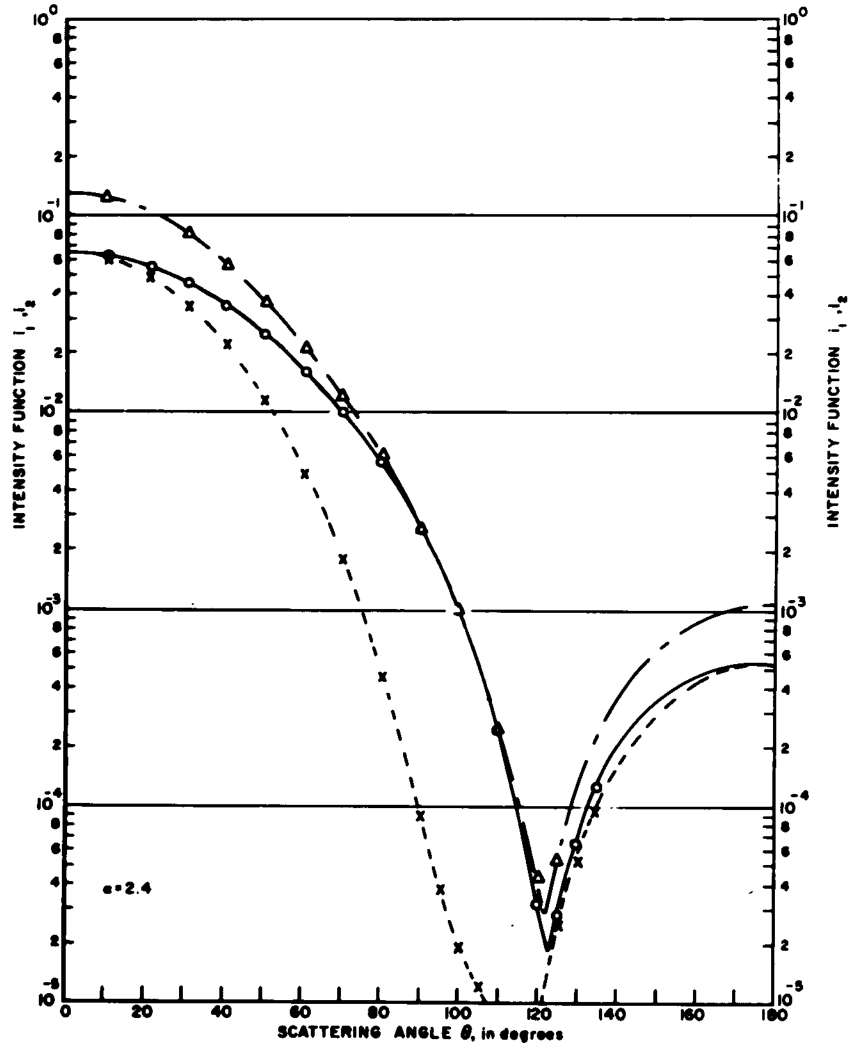


INTENSITY FUNCTION i_1, i_2
$\alpha = 1.2$
SCATTERING ANGLE θ, in degrees
63-5613

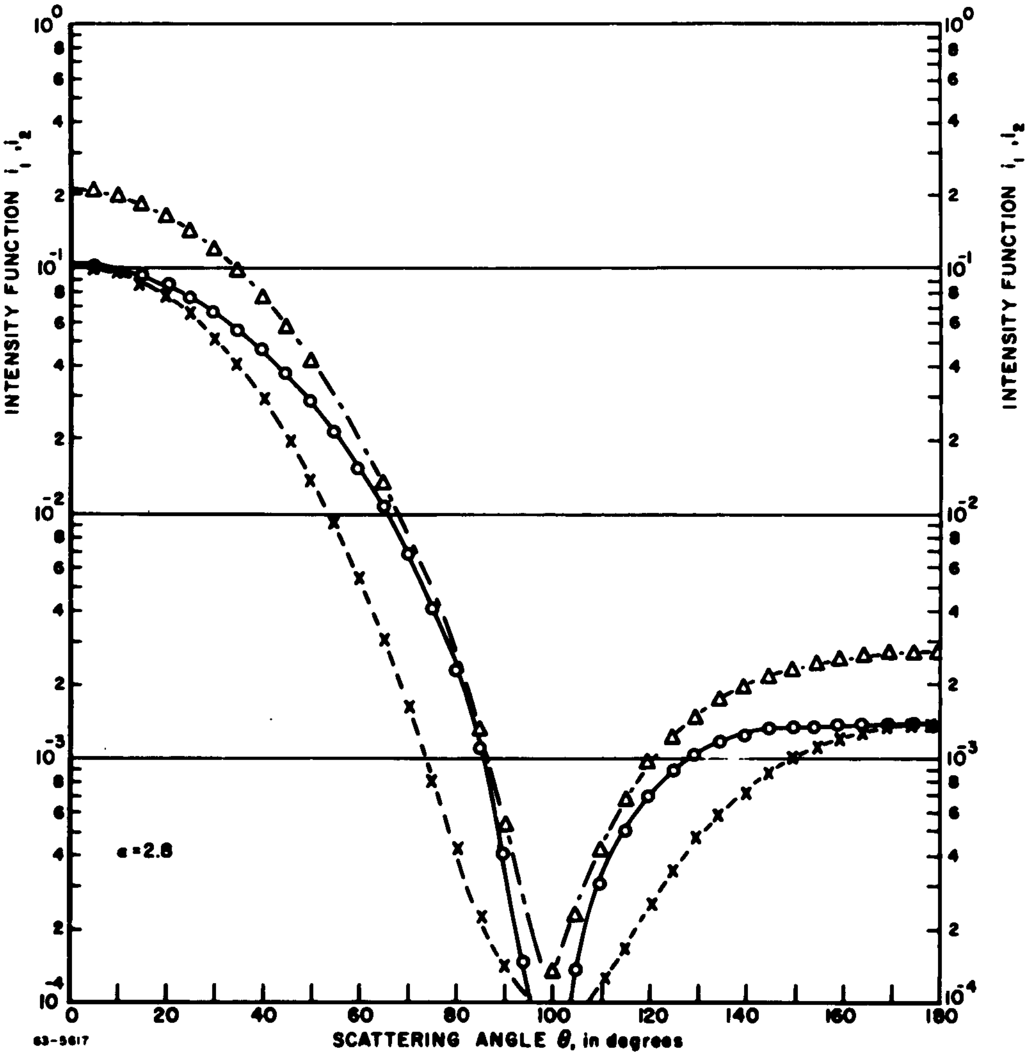

INTENSITY FUNCTION i_1, i_2
$\alpha = 1.6$
SCATTERING ANGLE θ, in degrees
63-3614

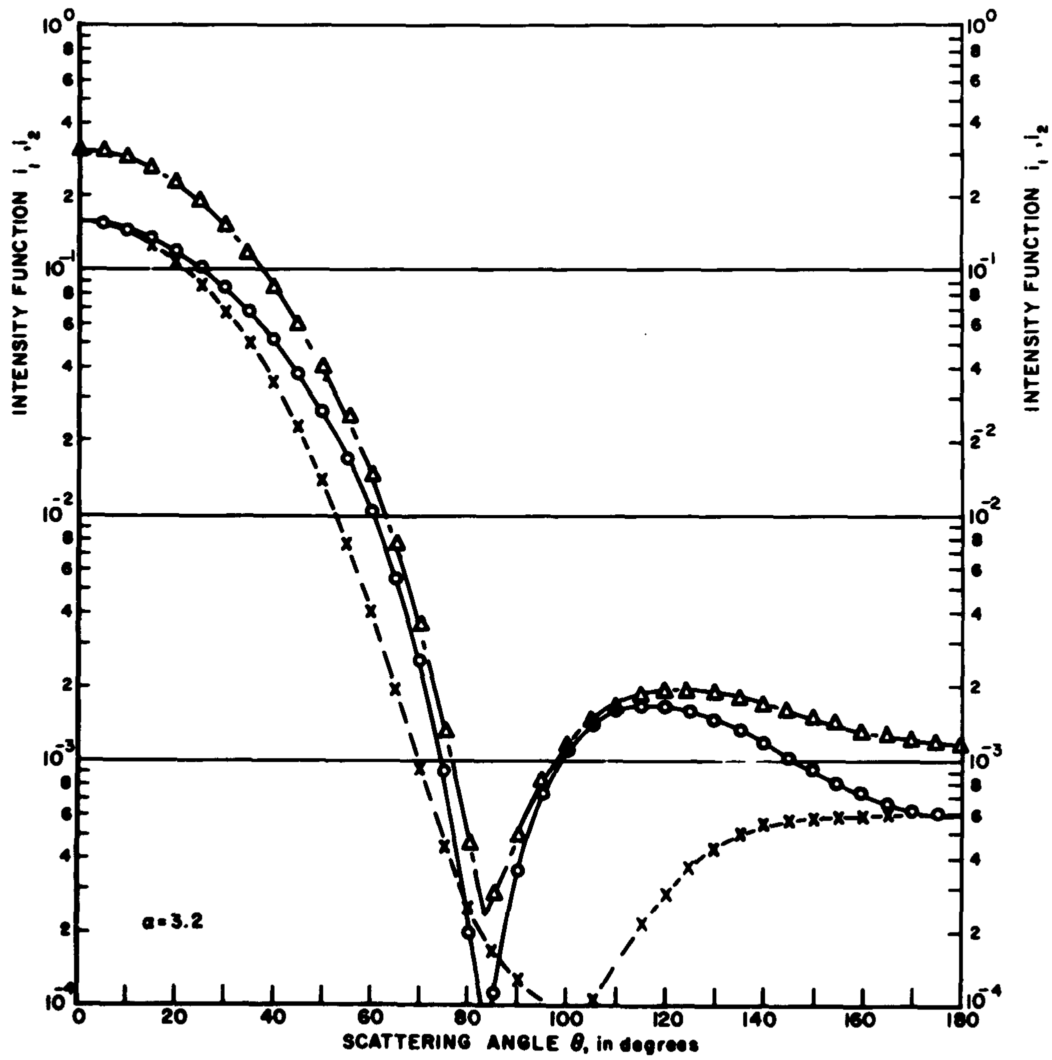


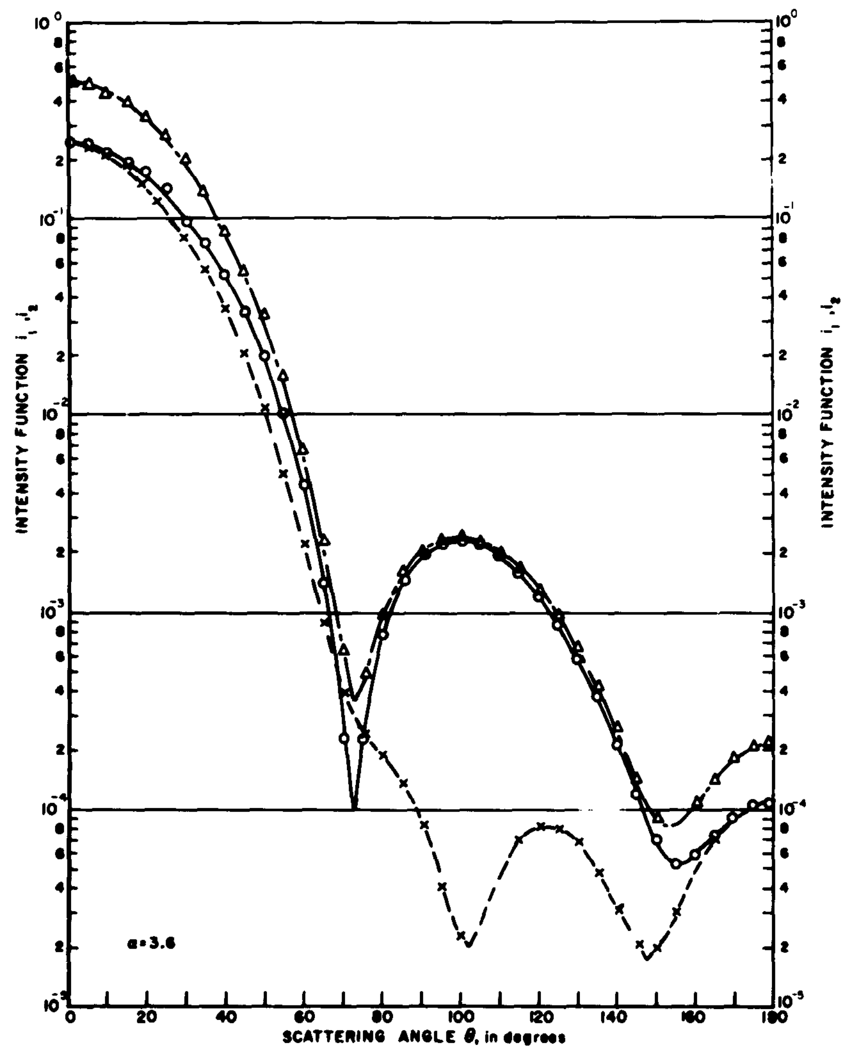
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 2.0$



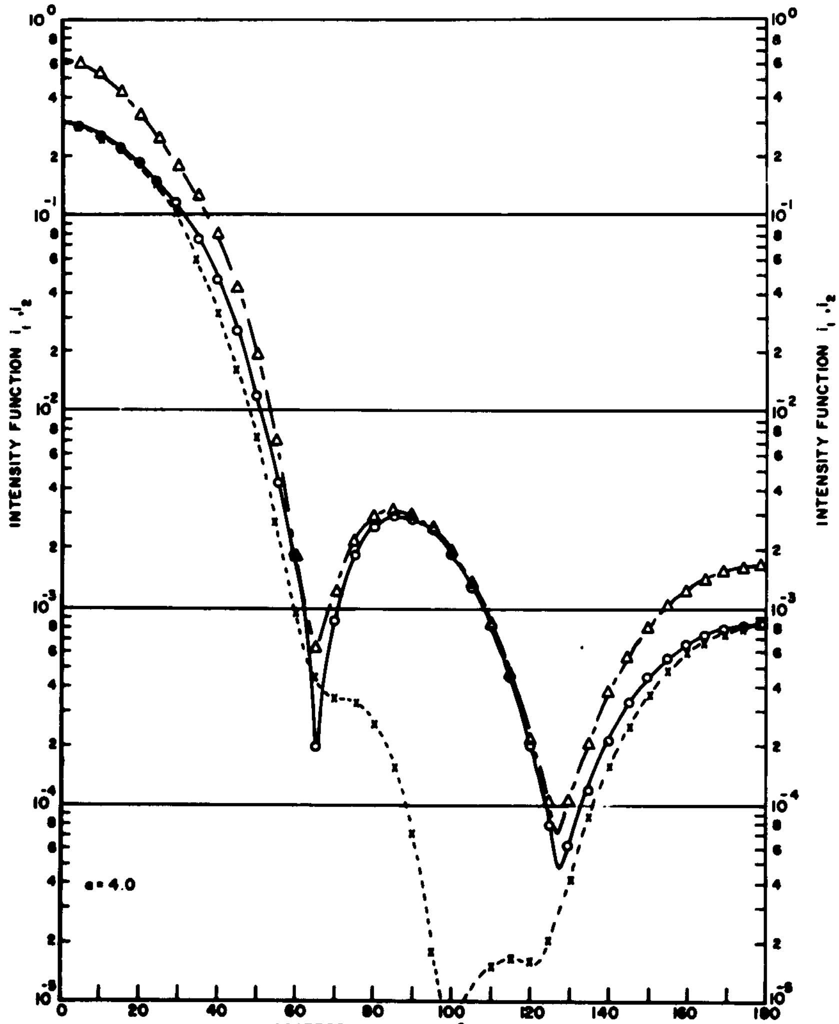
INTENSITY FUNCTION i_1, i_2
α = 2.4
SCATTERING ANGLE θ, in degrees
0
20
40
60
80
100
120
140
160
180


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 2.8$
63-5617

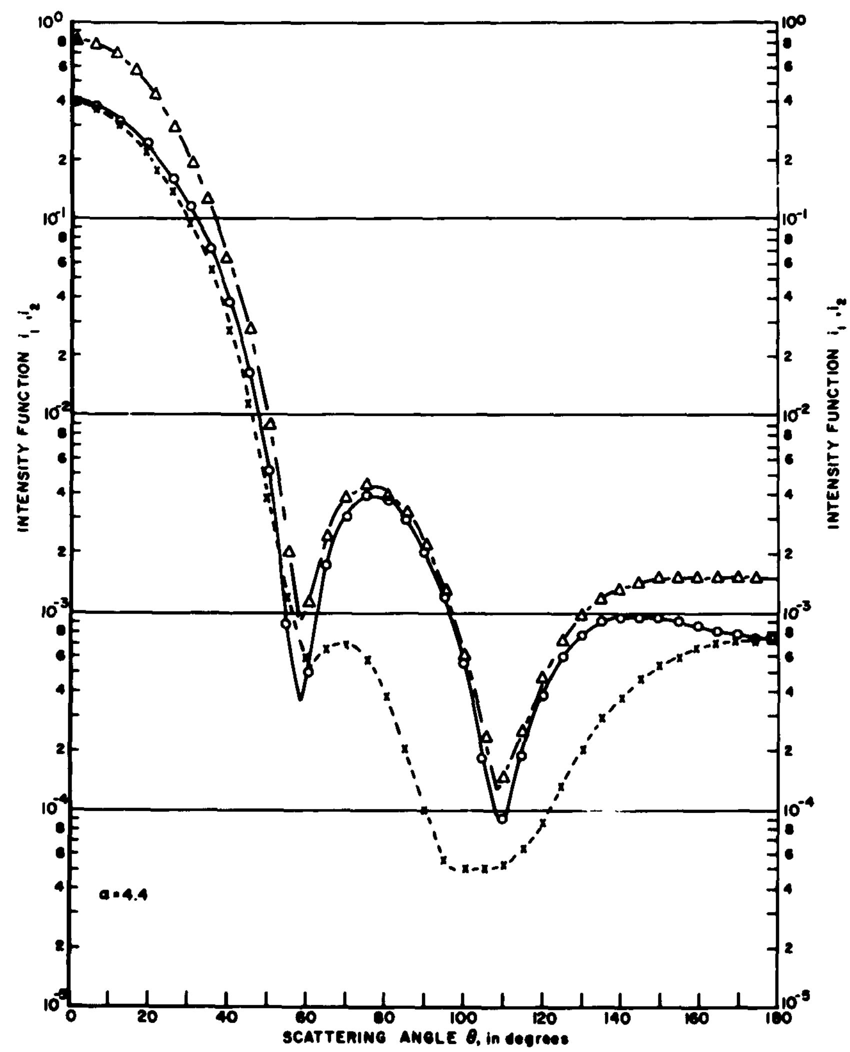

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 3.2$

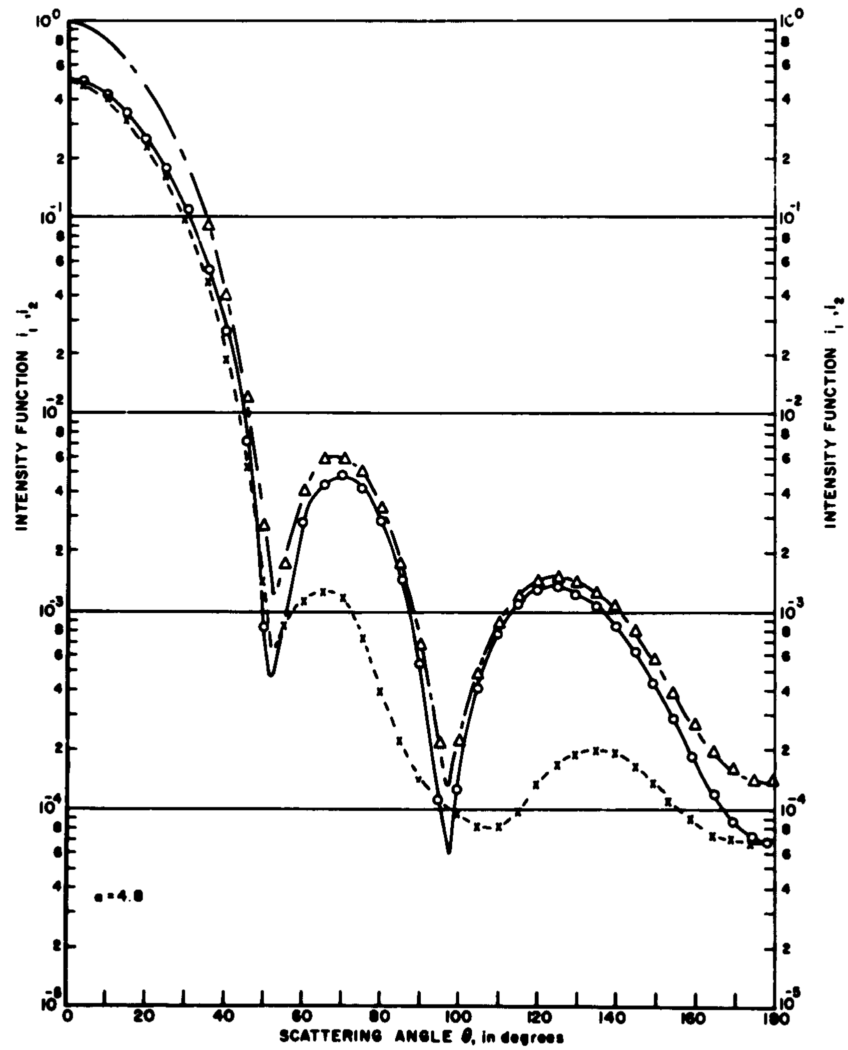


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 3.6$

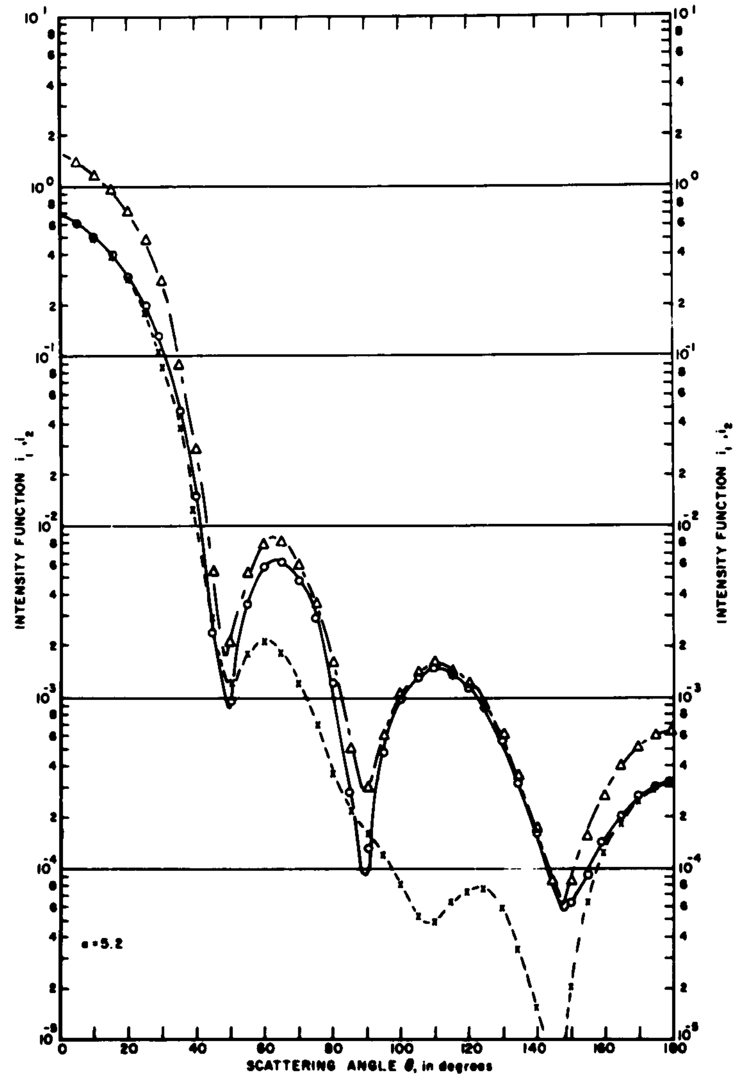


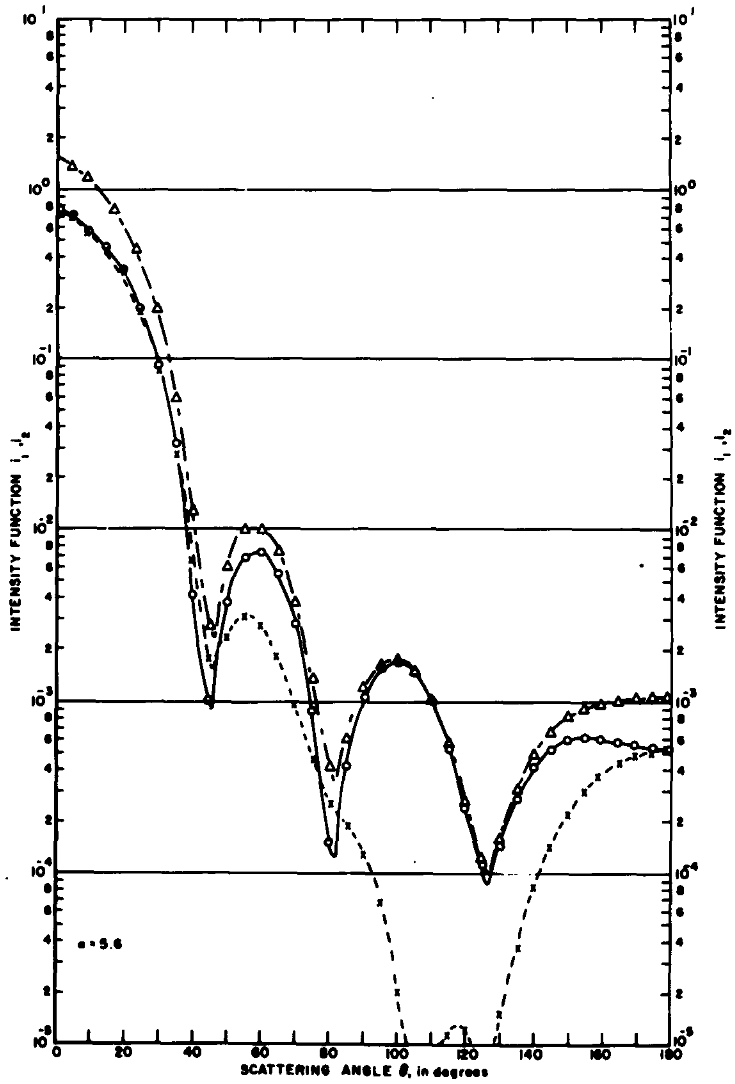
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
α = 4.0

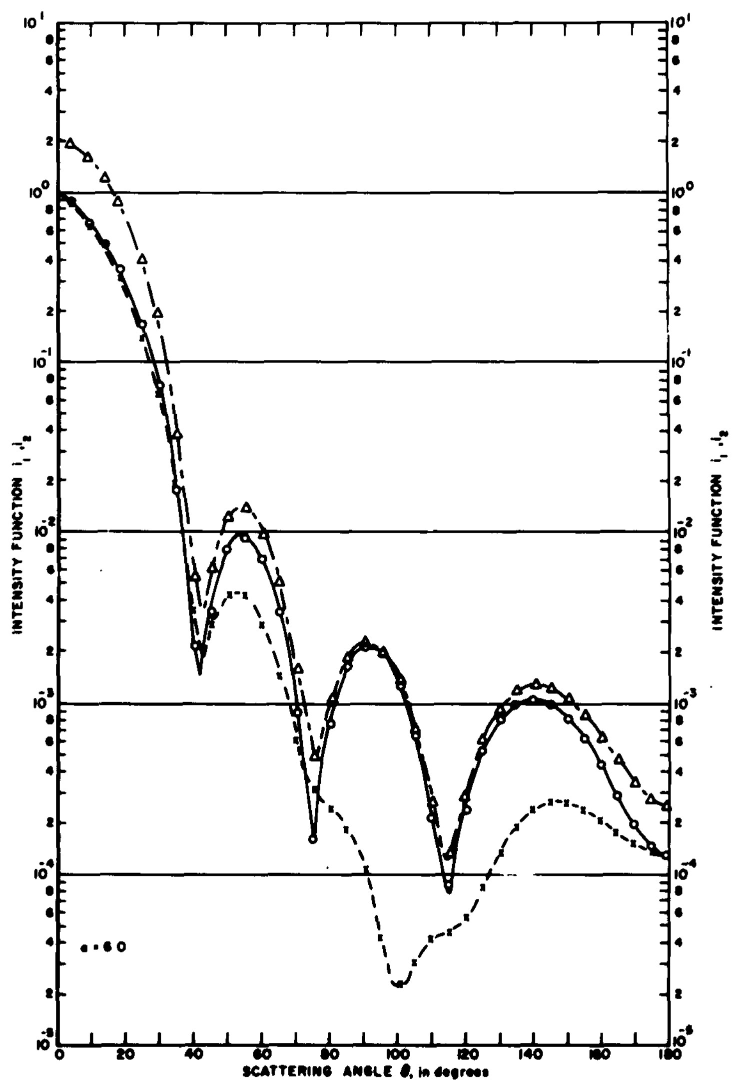

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 4.4$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
20
40
60
80
100
120
140
160
180



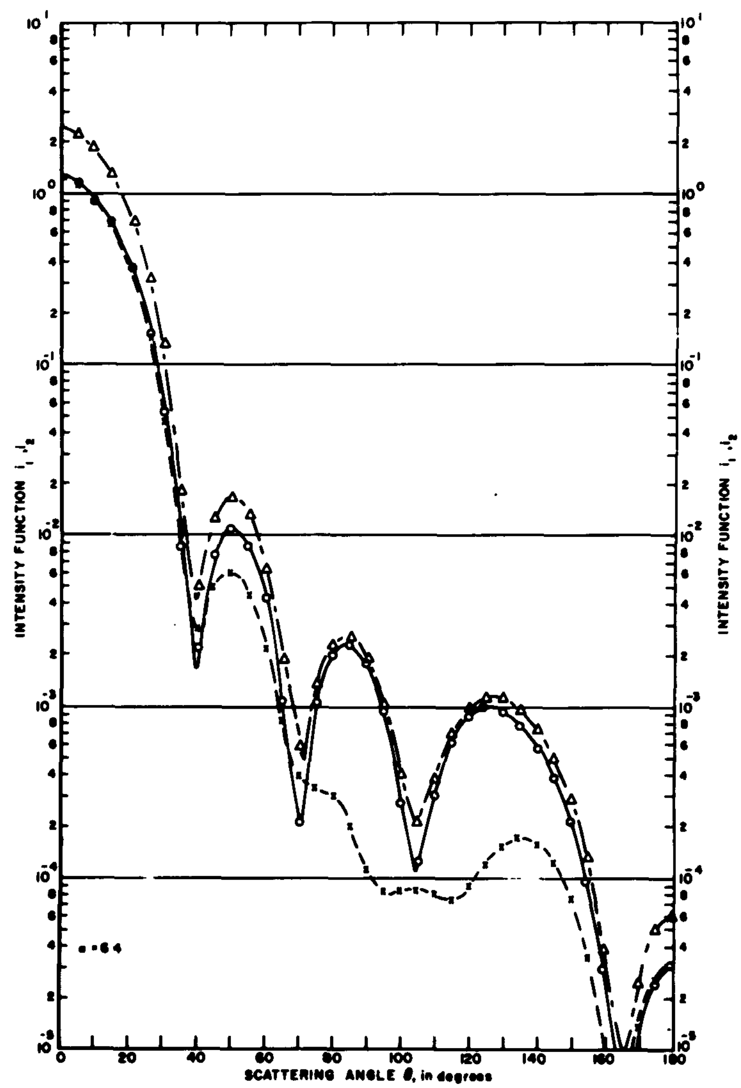
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 4.8$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 5.2$

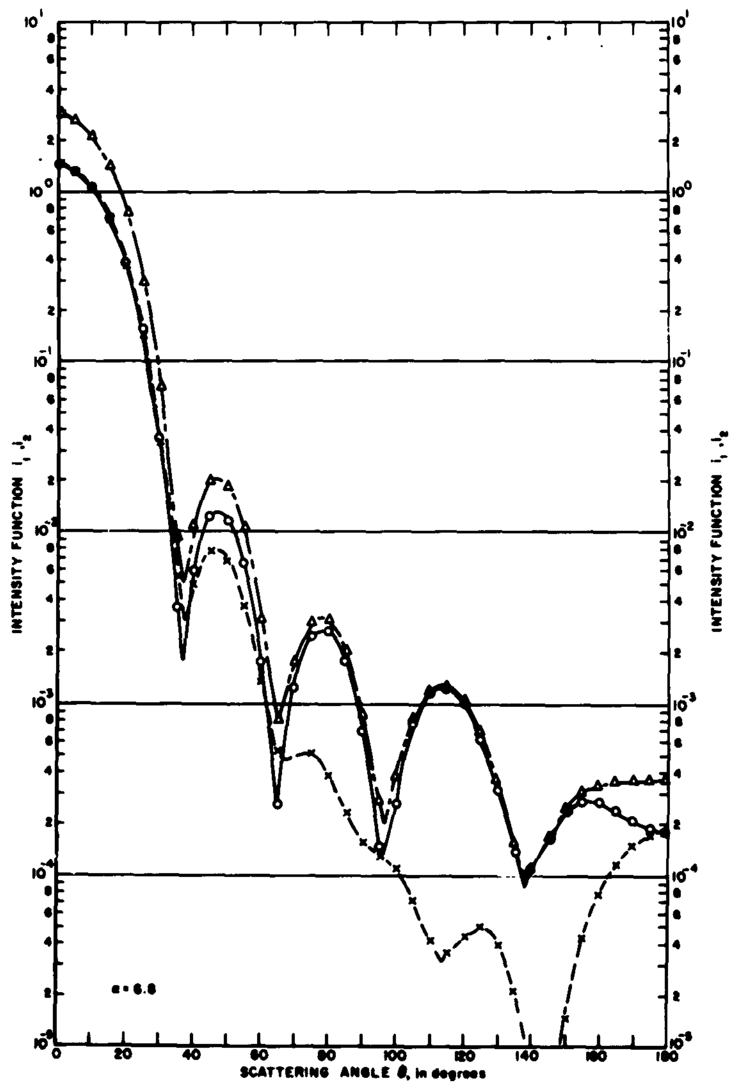

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 5.6$



INTENSITY FUNCTION $i_1 \cdot i_2$
SCATTERING ANGLE θ, in degrees

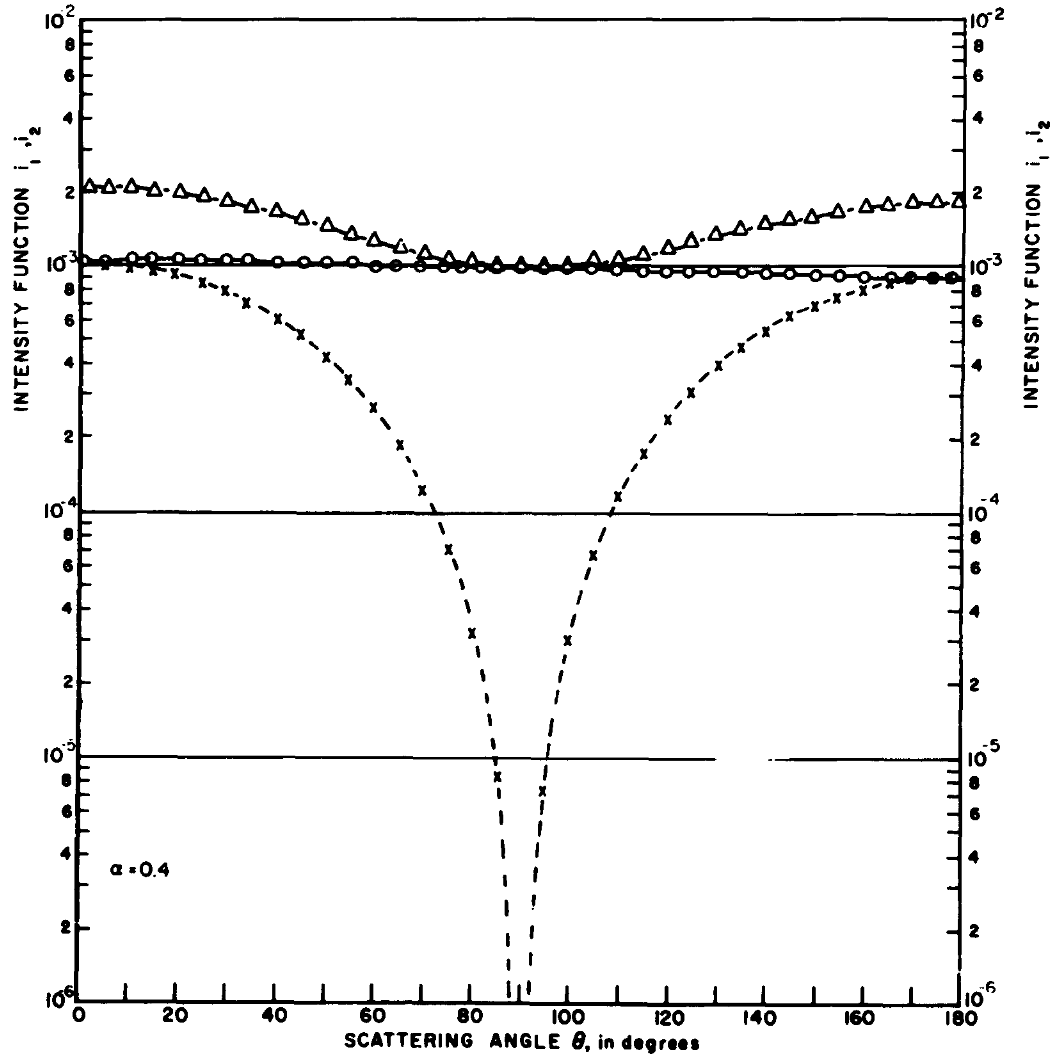


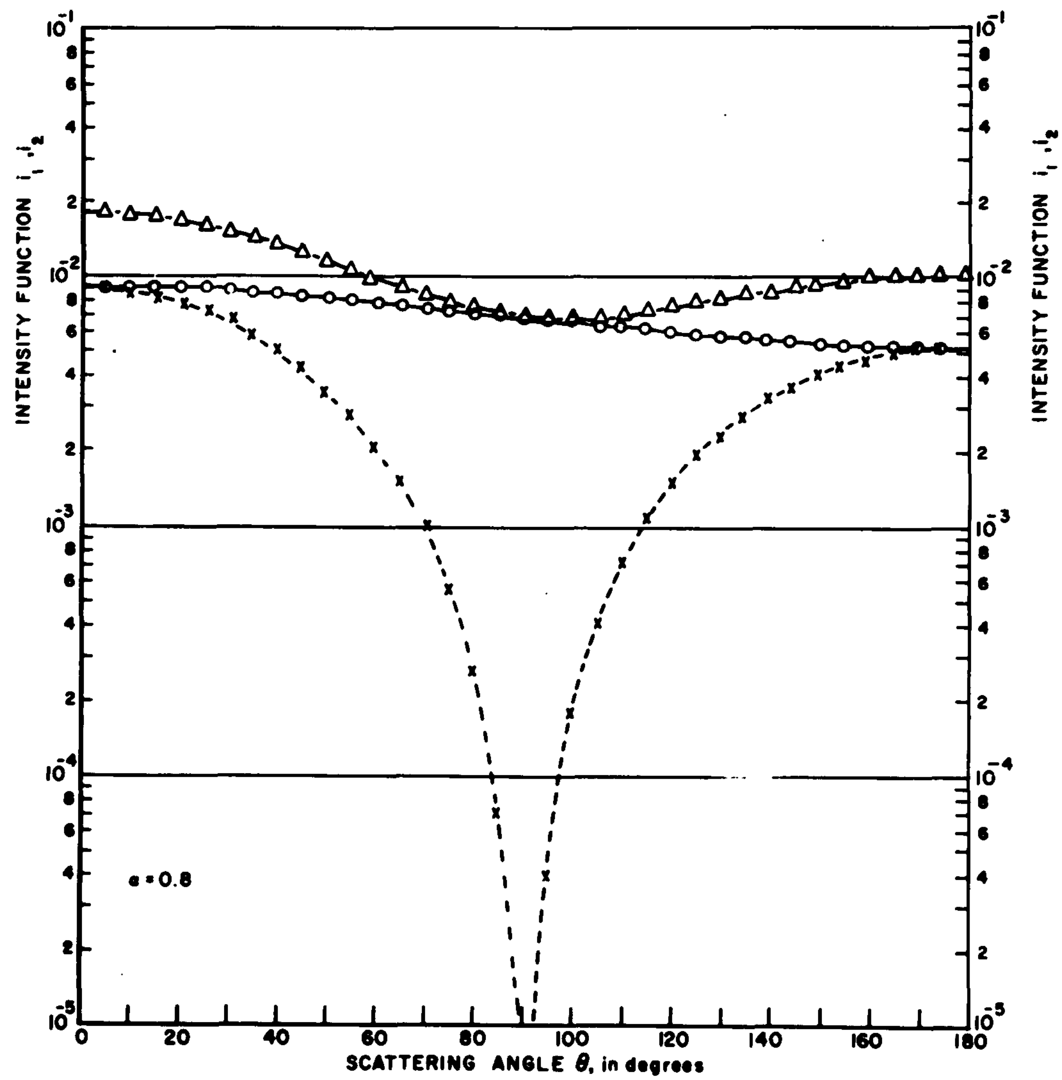
INTENSITY FUNCTION i_1, i_2
$\alpha = 6.4$
SCATTERING ANGLE θ, in degrees

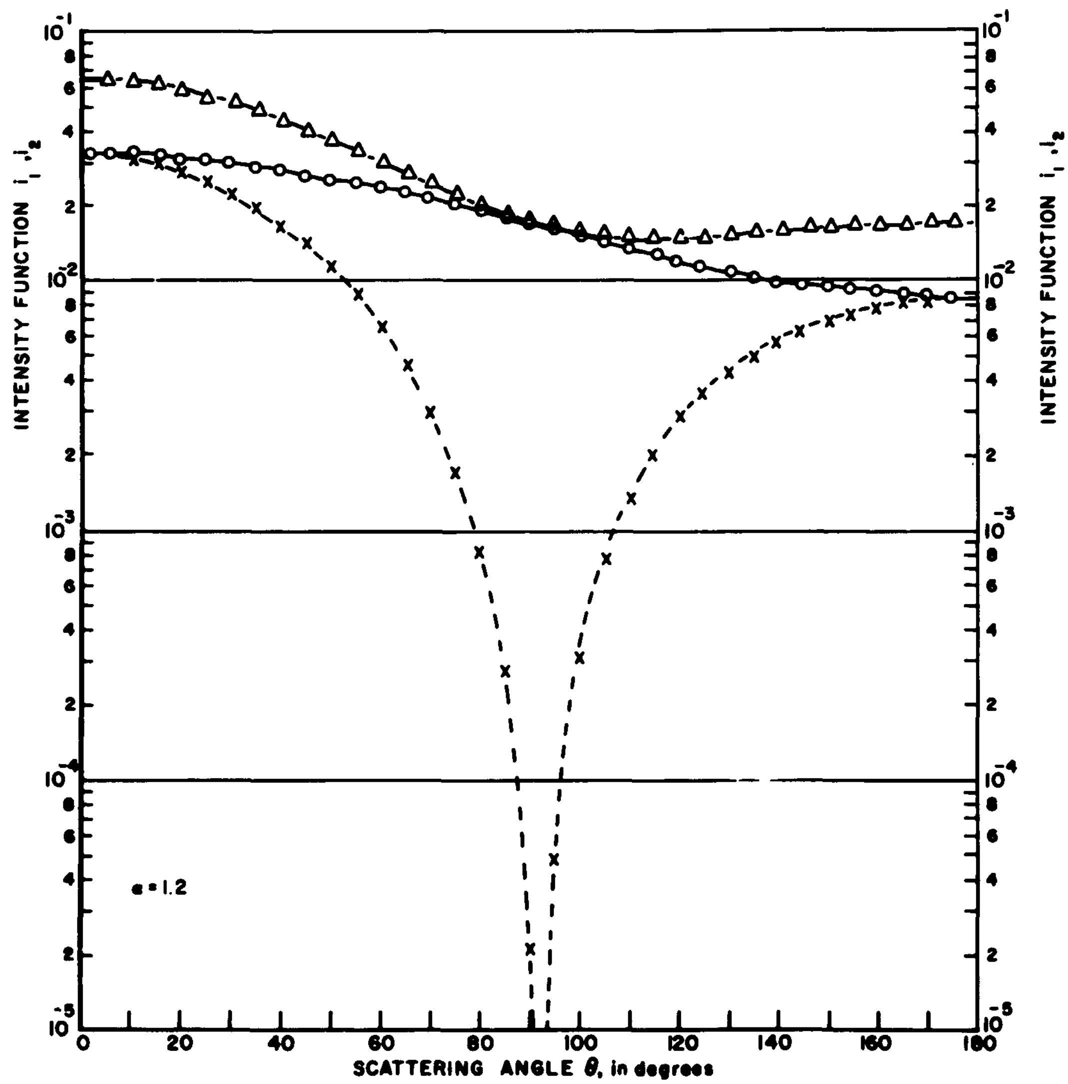

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 6.8$

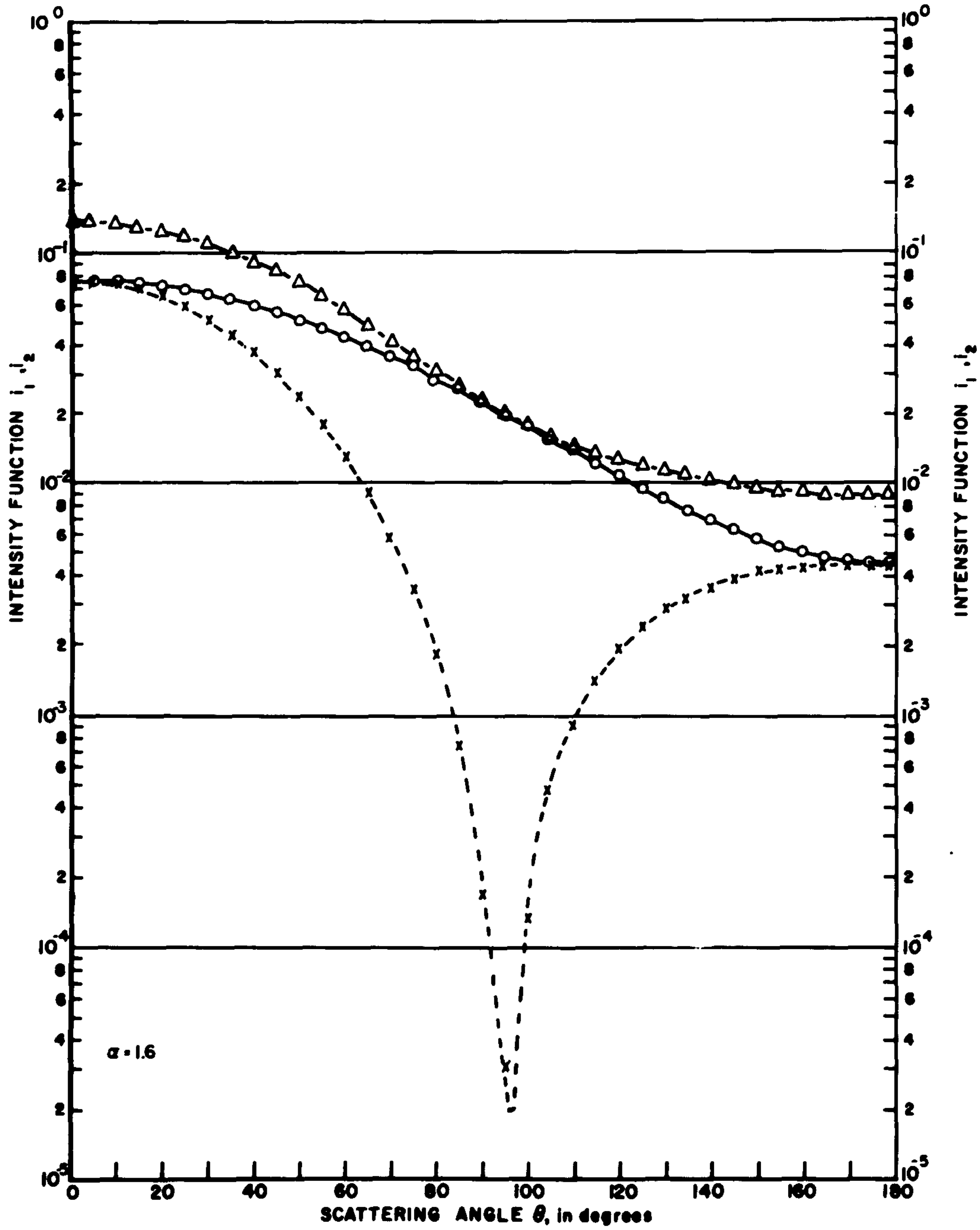
6.22 Atlas of scattering diagrams

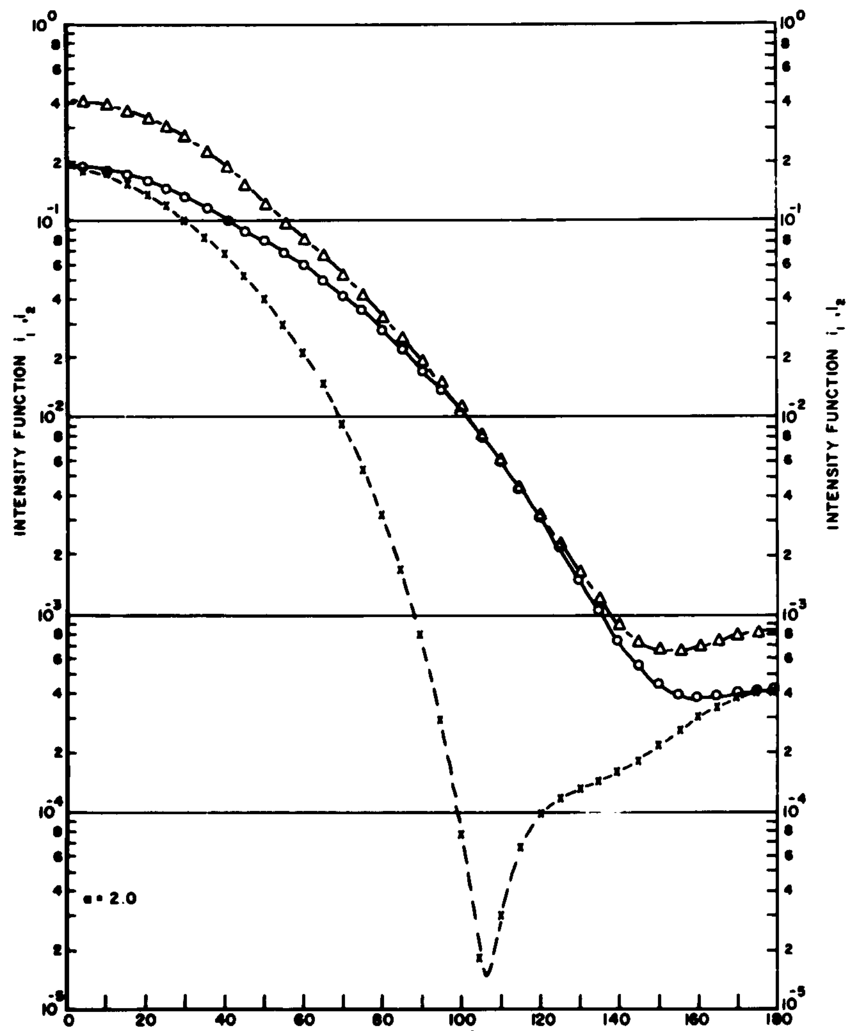
for $m = 1.2$

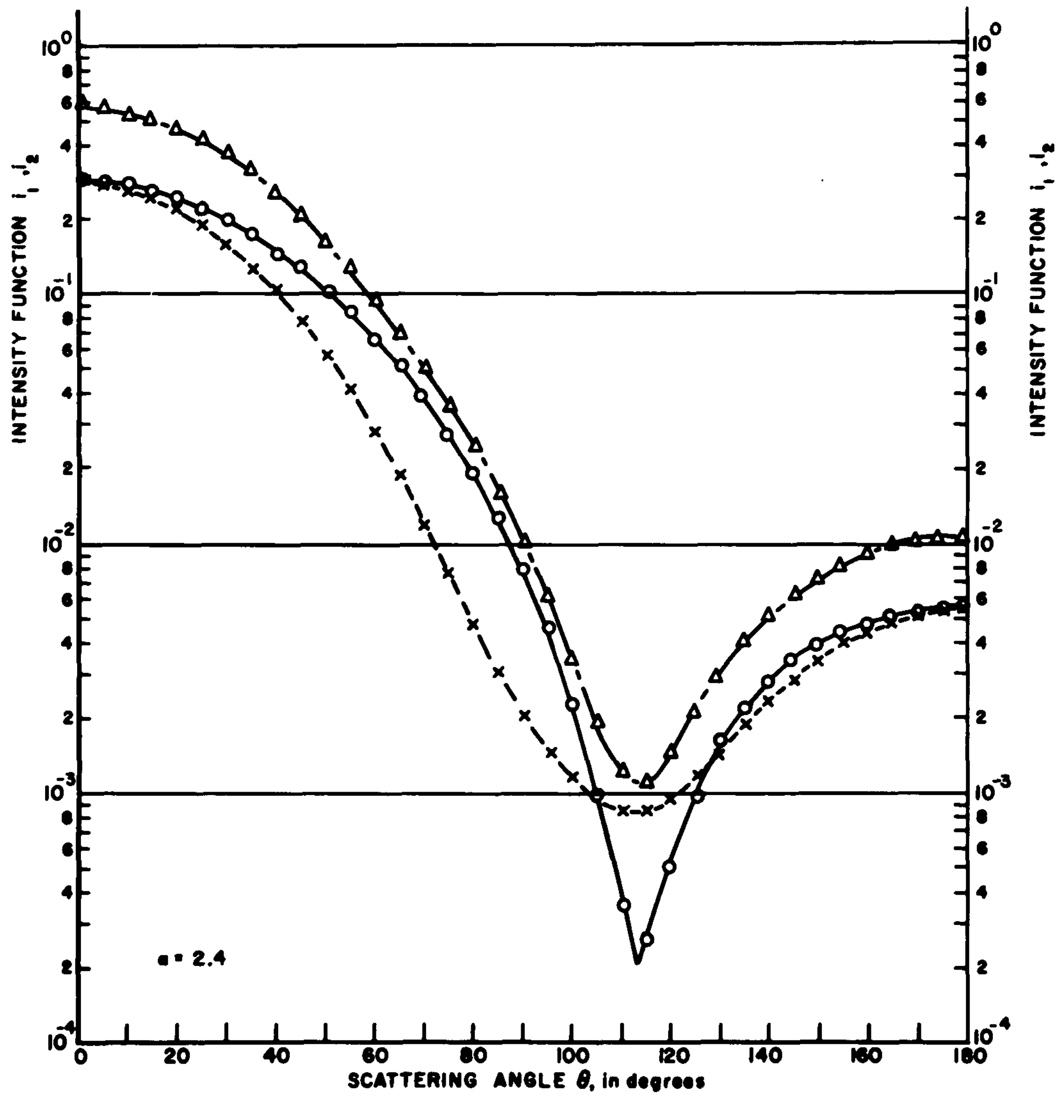

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 0.4$


INTENSITY FUNCTION $i_1 \cdot i_2$
SCATTERING ANGLE θ, in degrees
α = 0.8

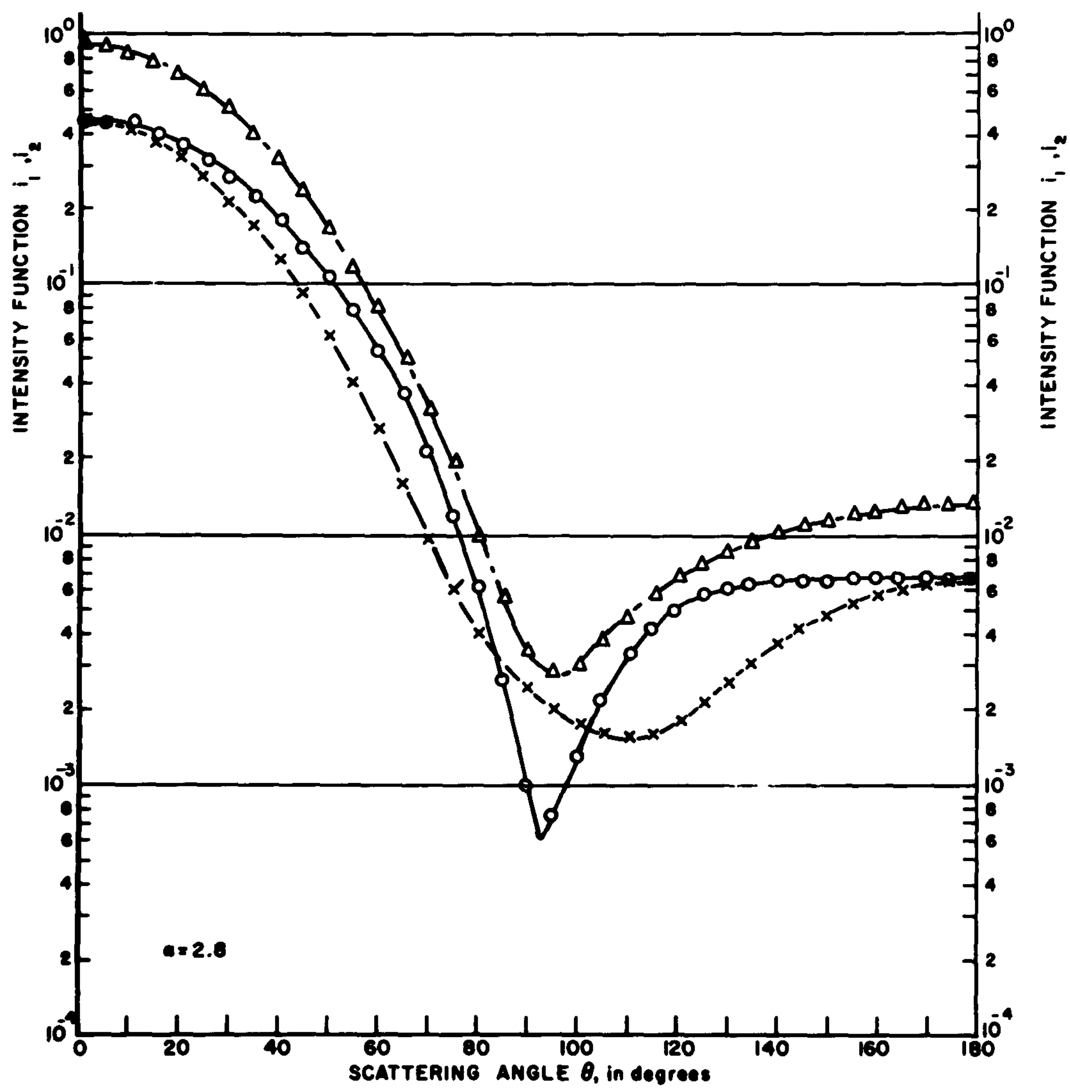

INTENSITY FUNCTION i_1, i_2
$\alpha = 1.2$
SCATTERING ANGLE θ, in degrees

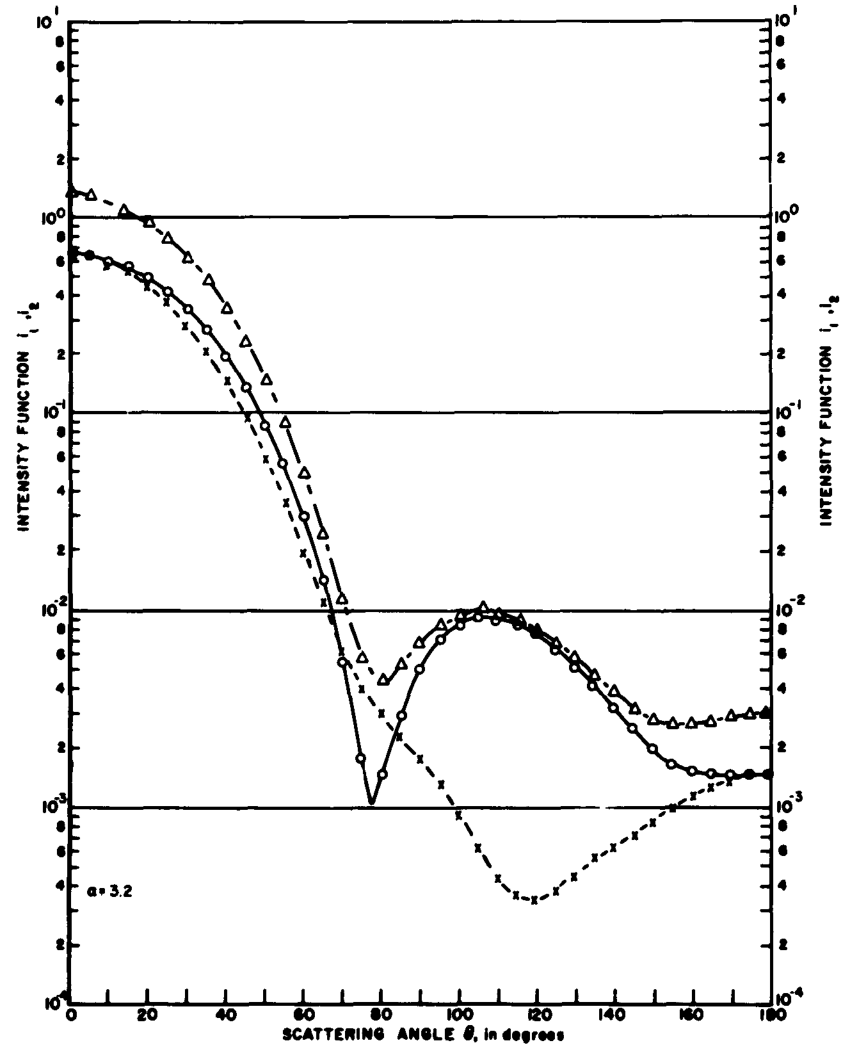

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 1.6$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 2.0$
0
20
40
60
80
100
120
140
160
180

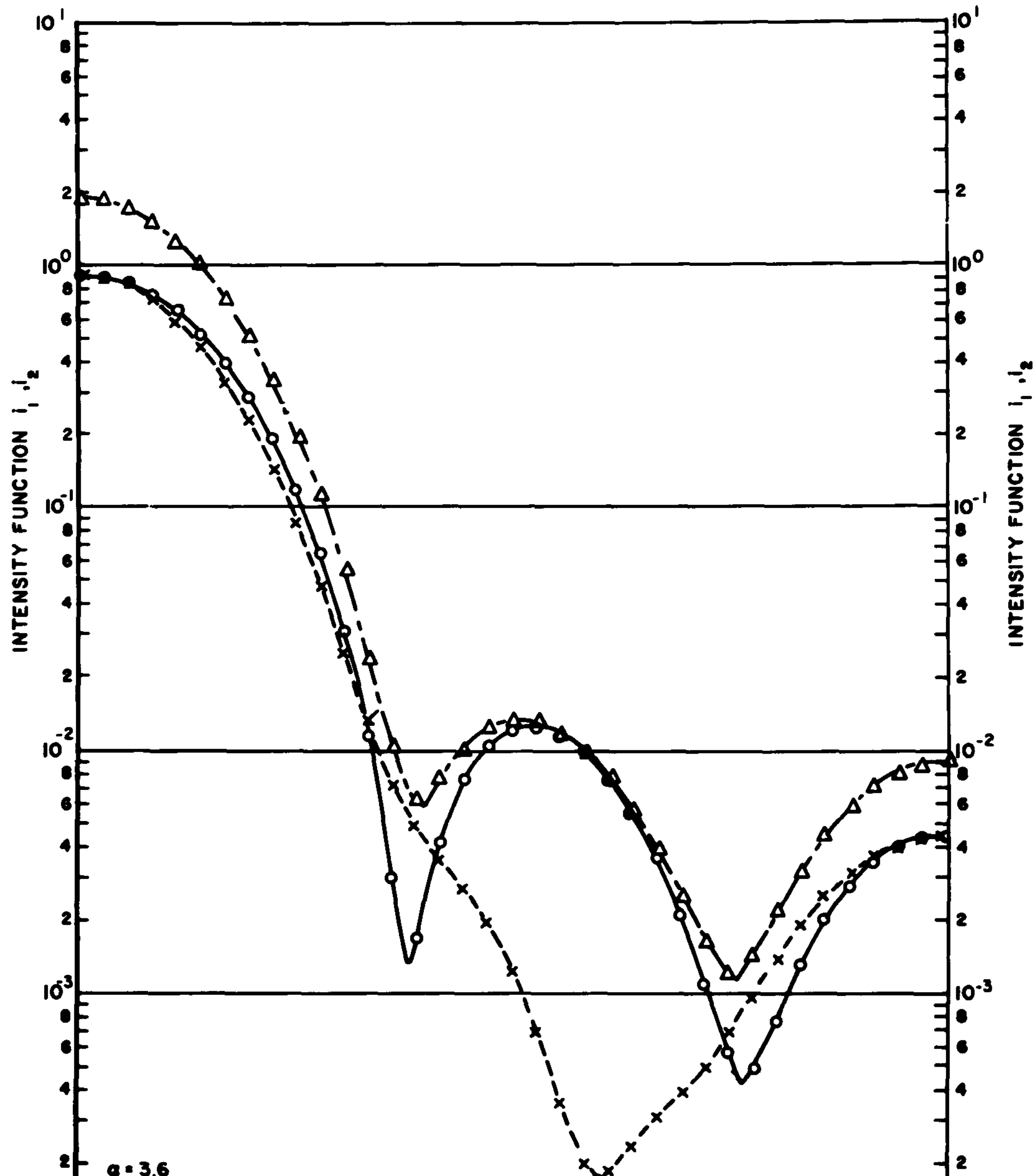


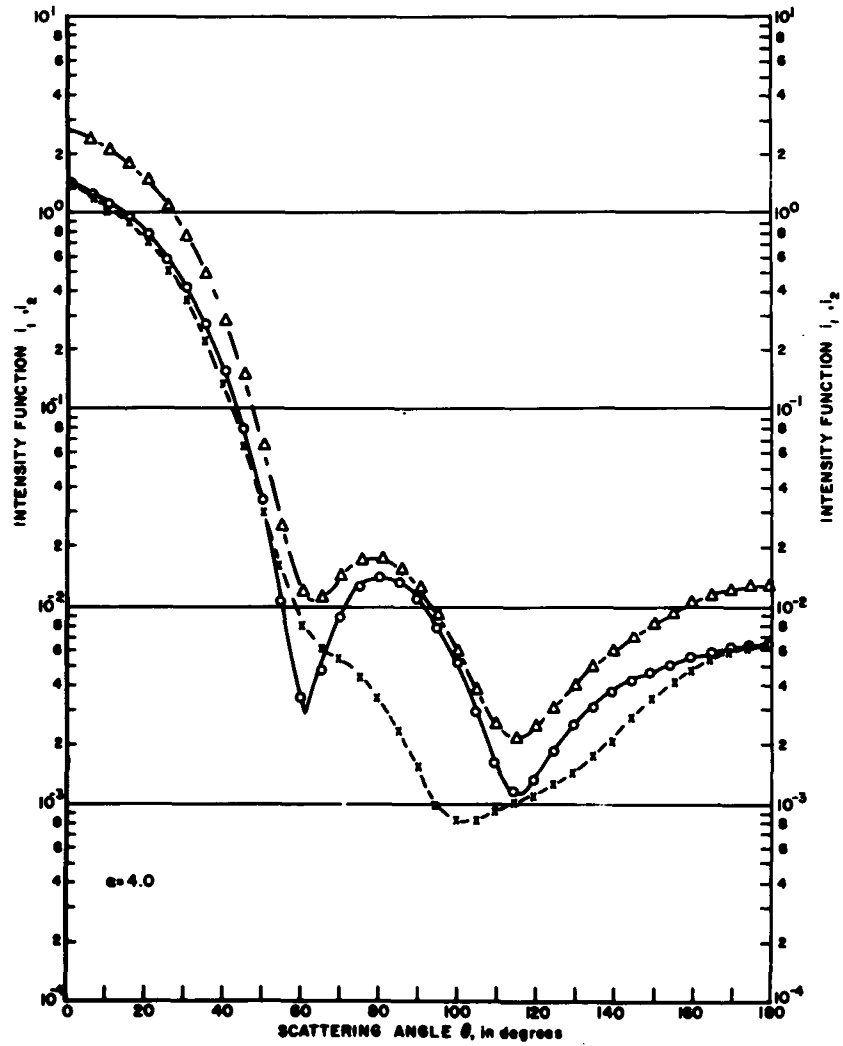
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 2.4$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 2.8$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0 20 40 60 80 100 120 140 160 180

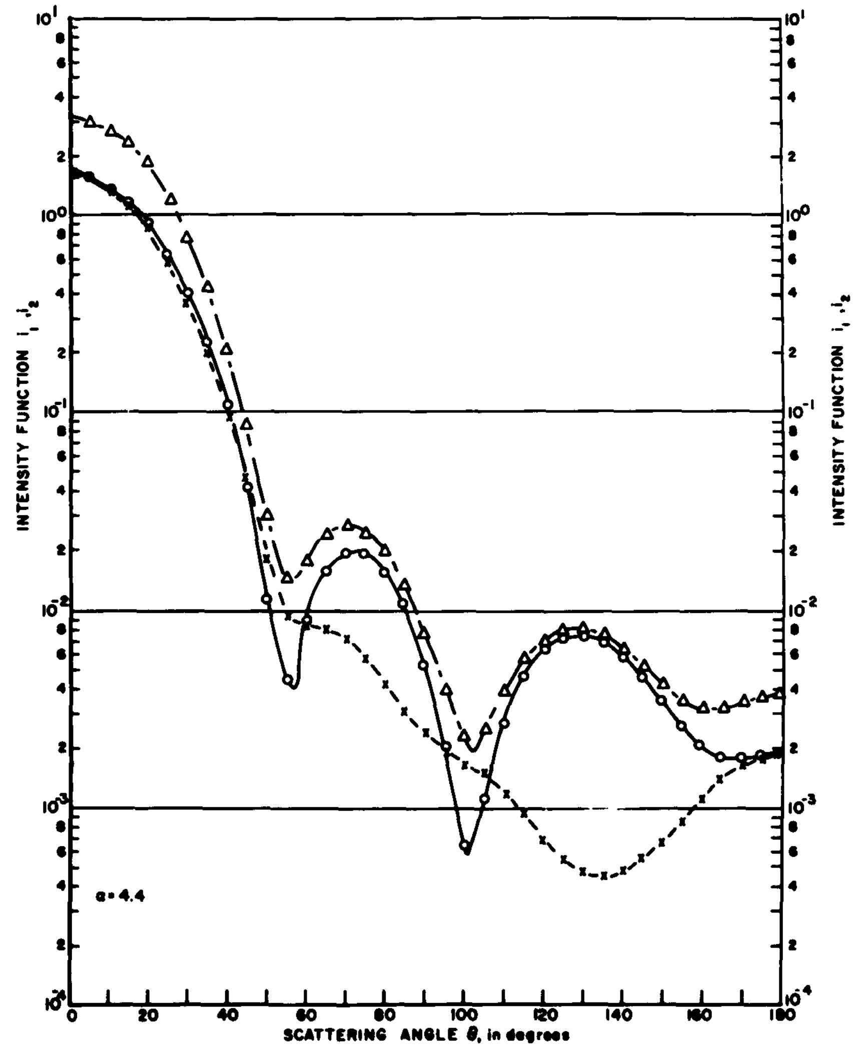


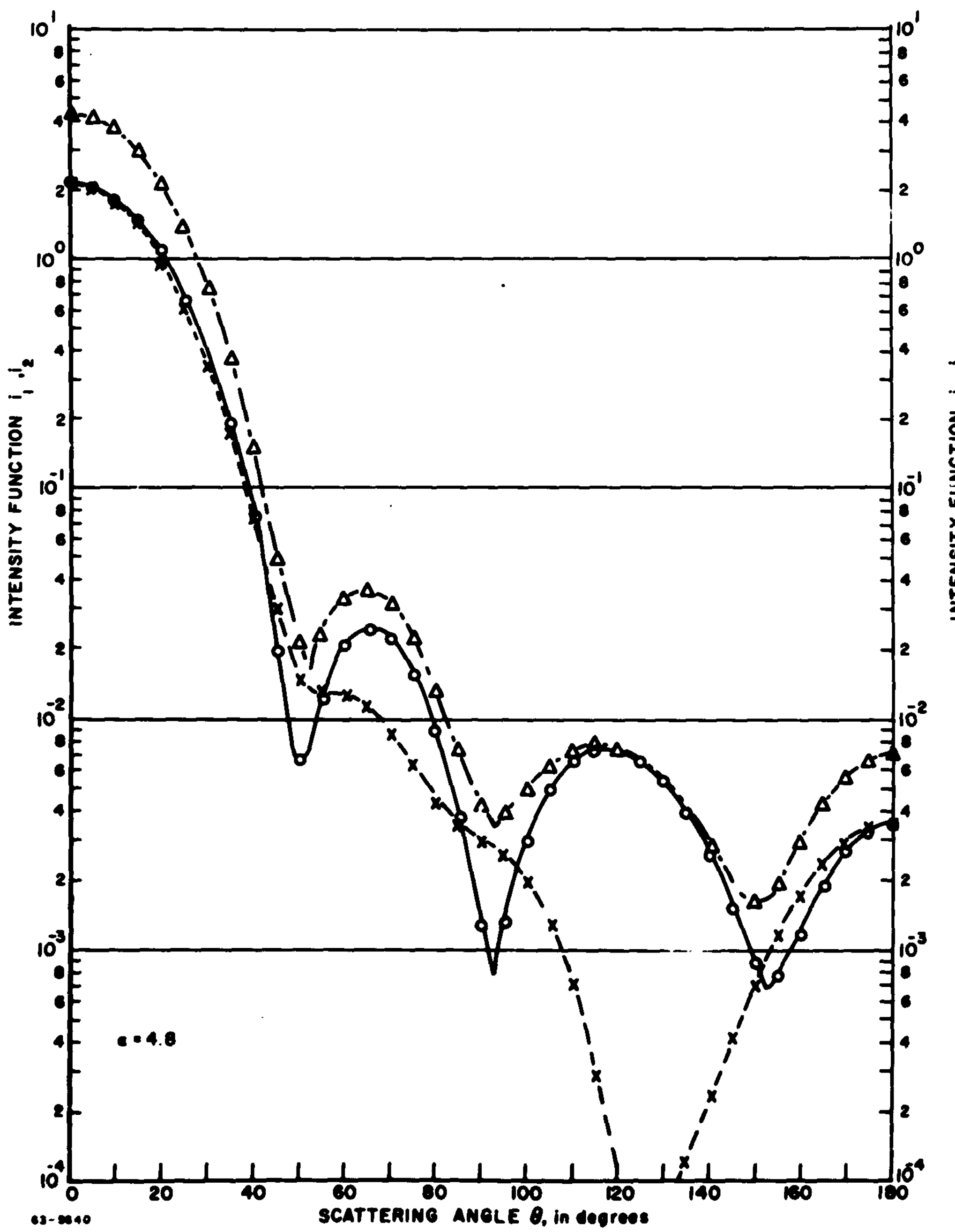
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 3.2$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 3.6$

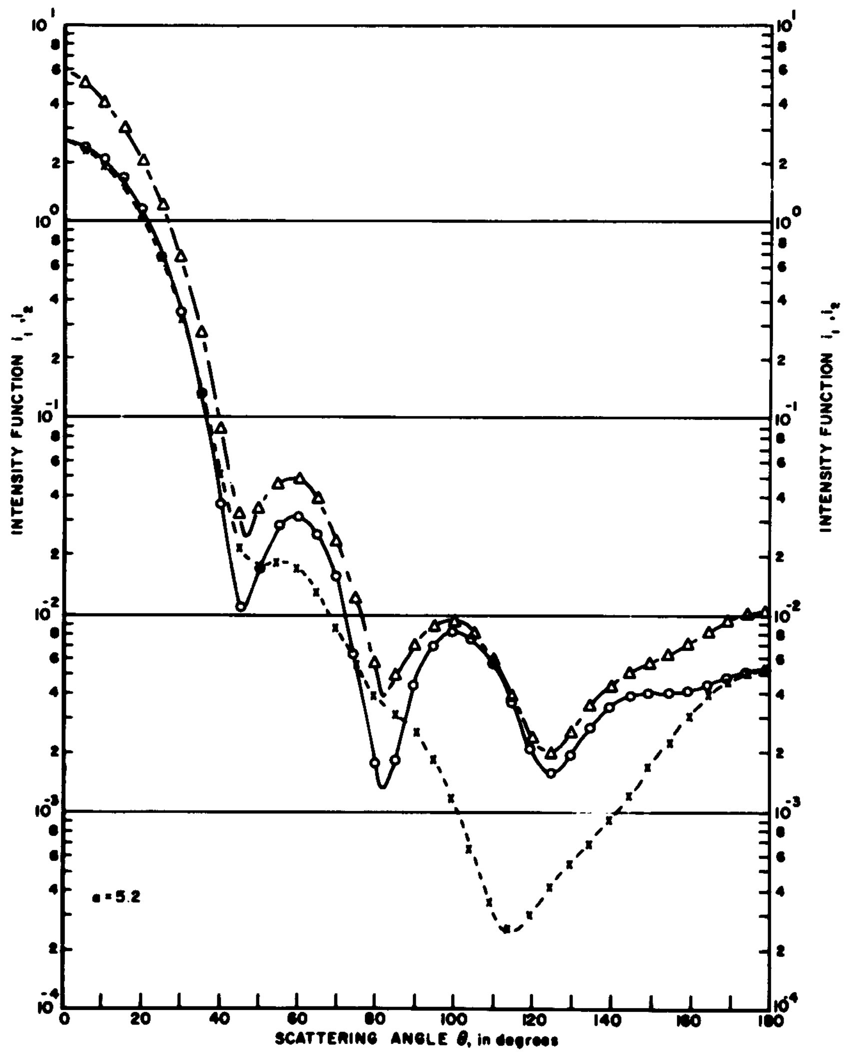


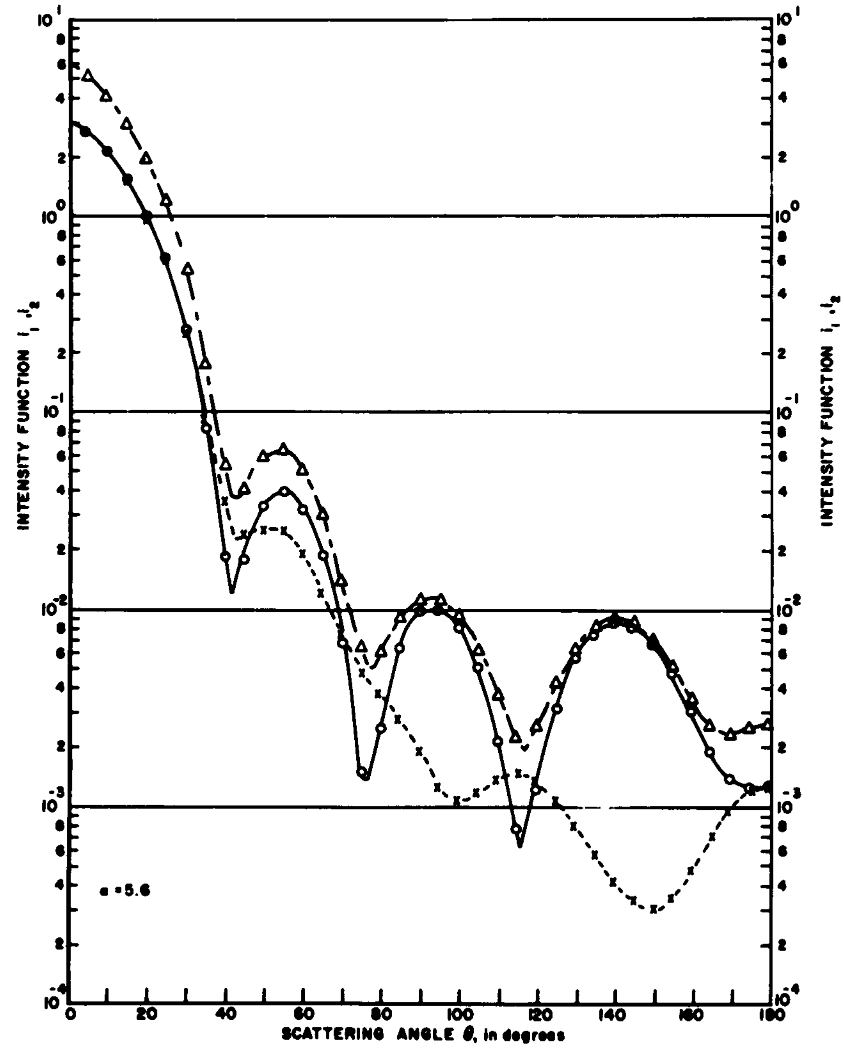
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 4.0$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 4.4$

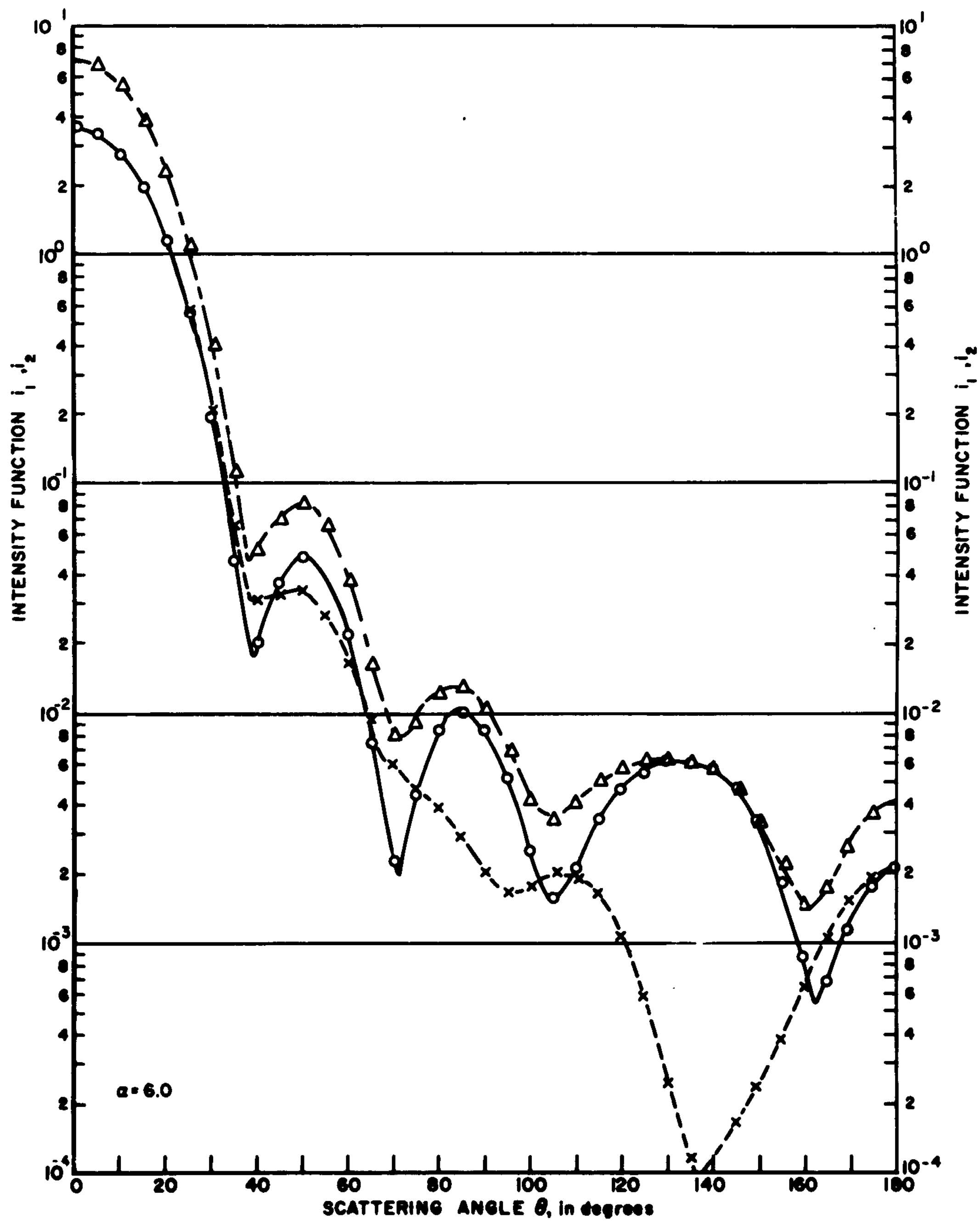


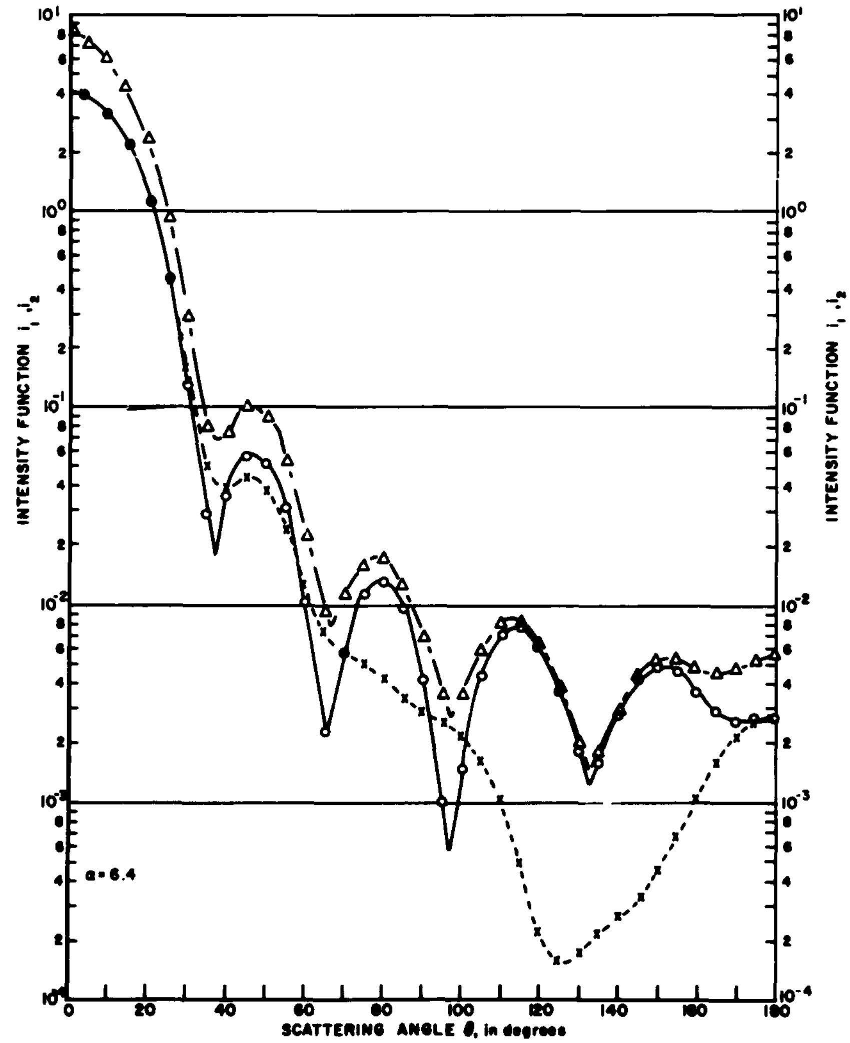
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 4.8$
63-9840

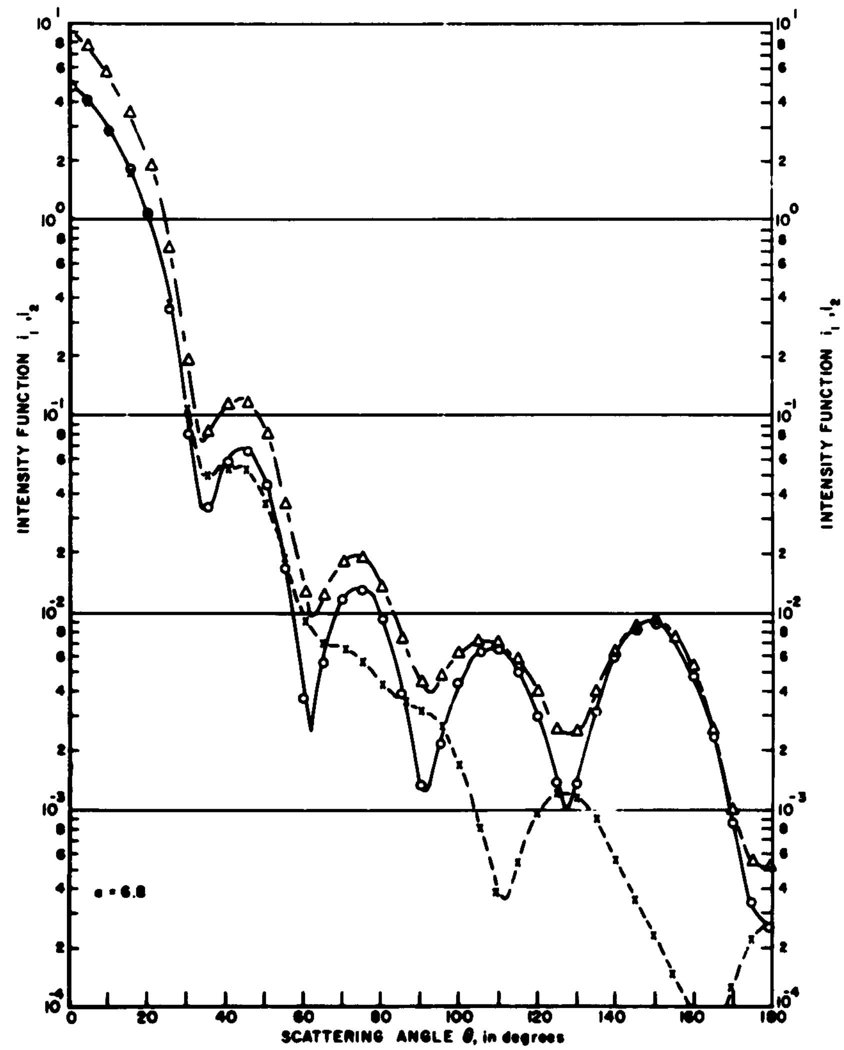

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
α = 5.2



INTENSITY FUNCTION i_1, i_2
$\alpha = 5.6$
SCATTERING ANGLE θ, in degrees


INTENSITY FUNCTION i_1, i_2
α = 6.0
SCATTERING ANGLE θ, in degrees

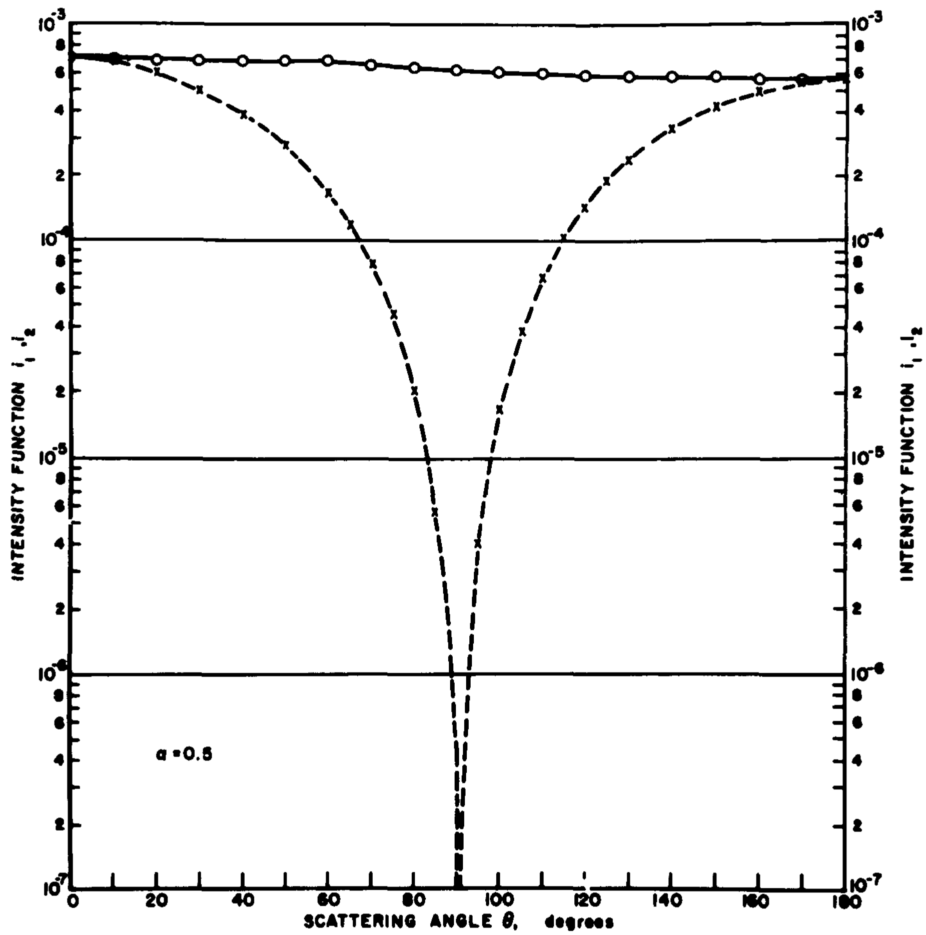

INTENSITY FUNCTION i_1, i_2
$\alpha = 6.4$
SCATTERING ANGLE θ, in degrees



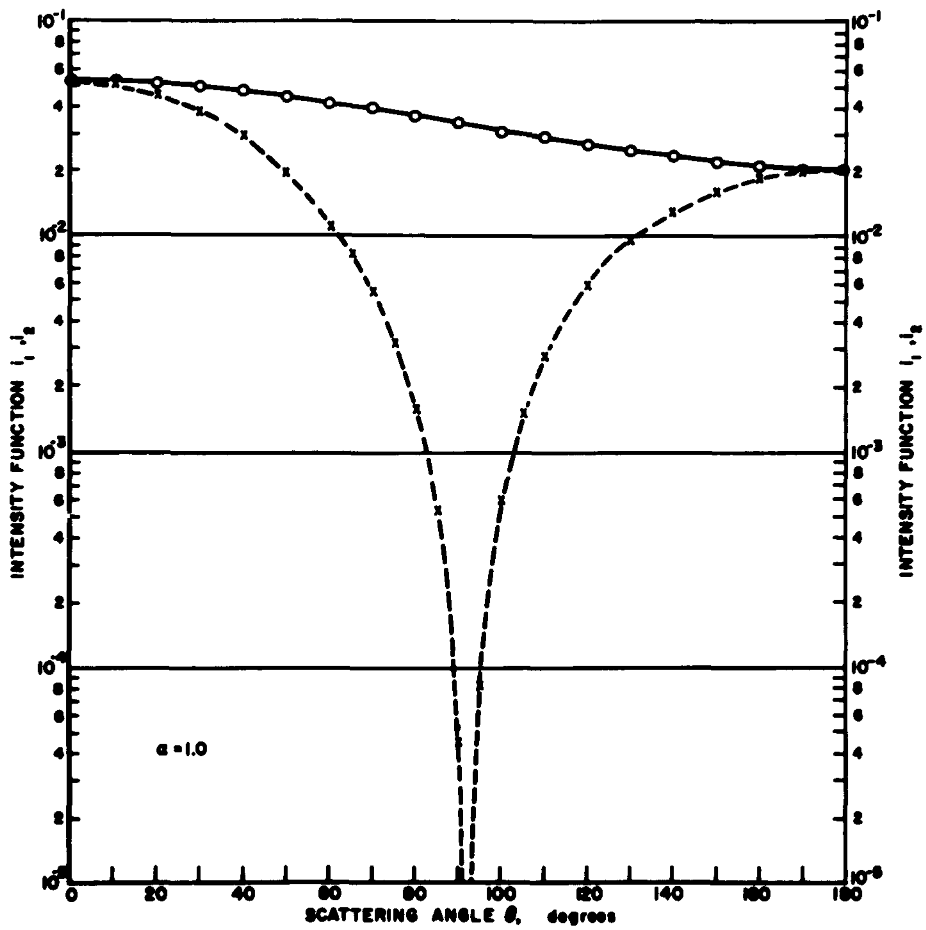
INTENSITY FUNCTION i_1, i_2
$\alpha = 6.8$
SCATTERING ANGLE θ, in degrees

6.23 Atlas of scattering diagrams

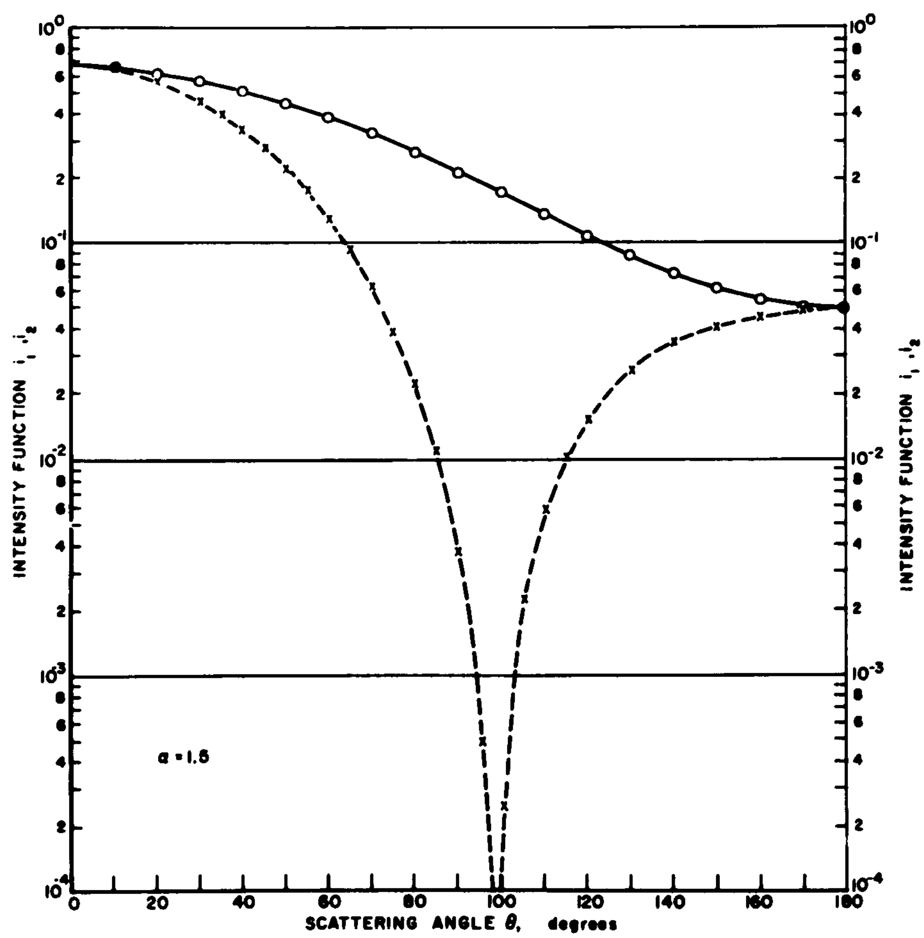
for $n = 1.33$

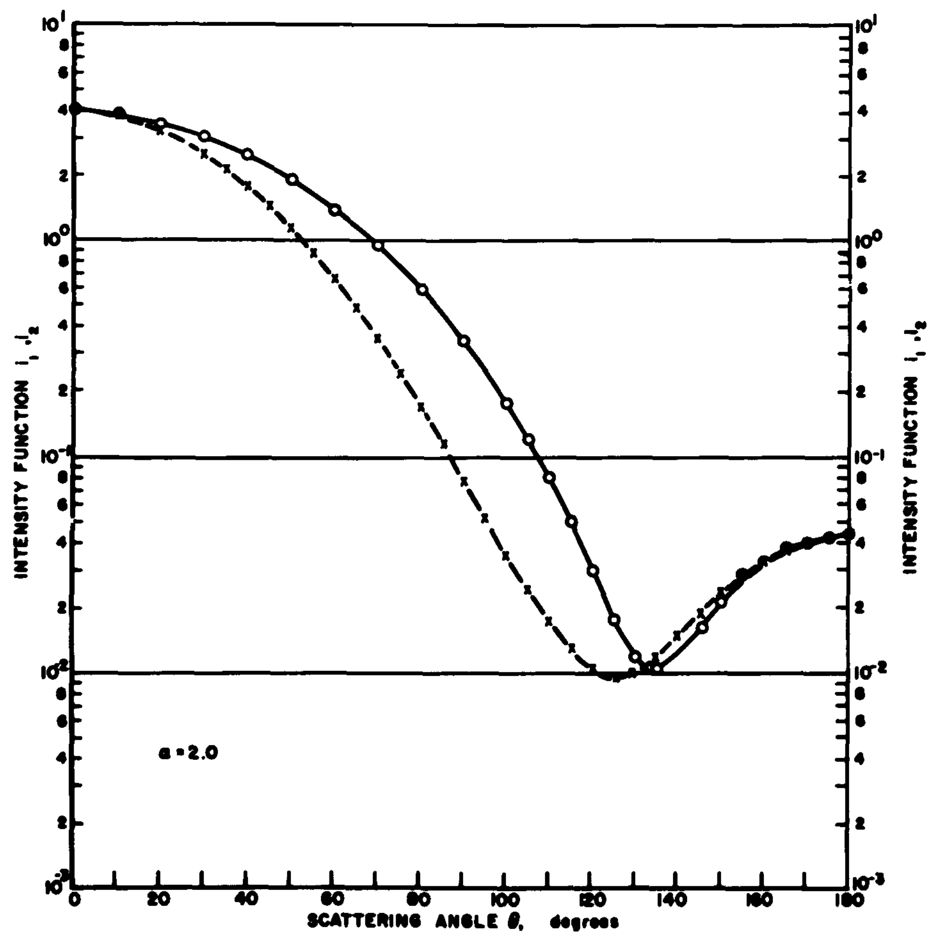


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 0.5$

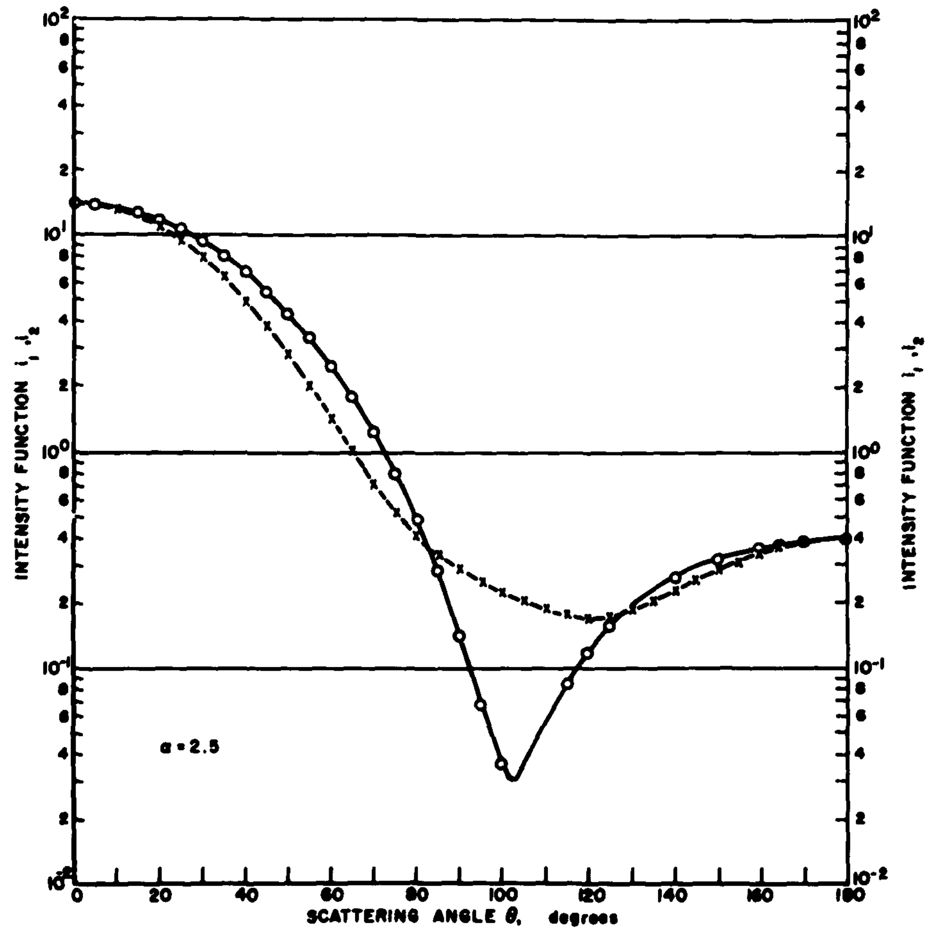


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 1.0$

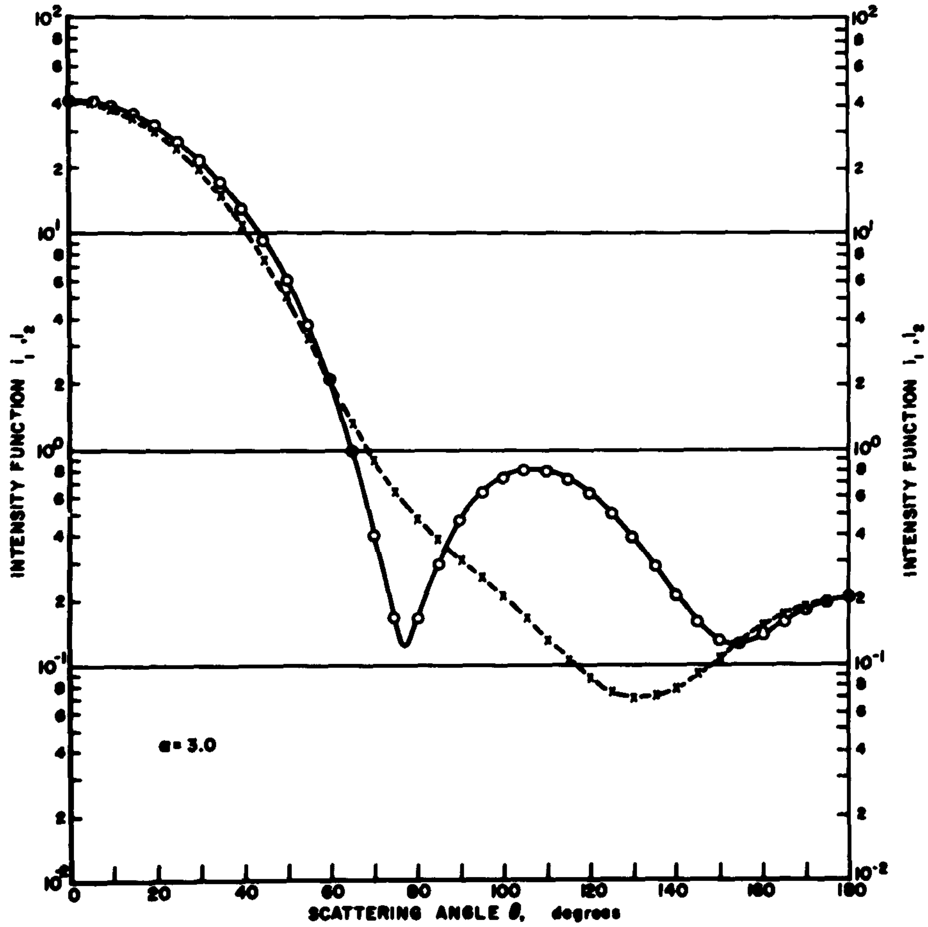

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 1.5$



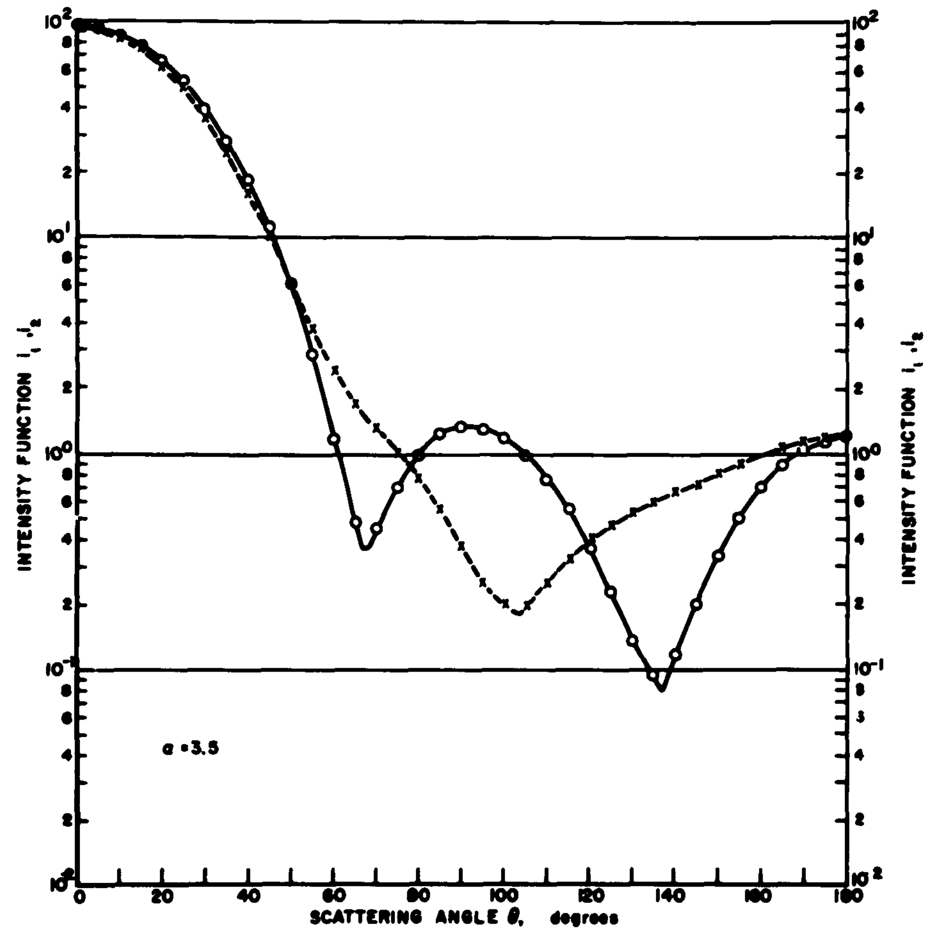
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α = 2.0



INTENSITY FUNCTION i_1, i_2
$\alpha = 2.5$
SCATTERING ANGLE θ, degrees



INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 3.0$



INTENSITY FUNCTION i_1, i_2
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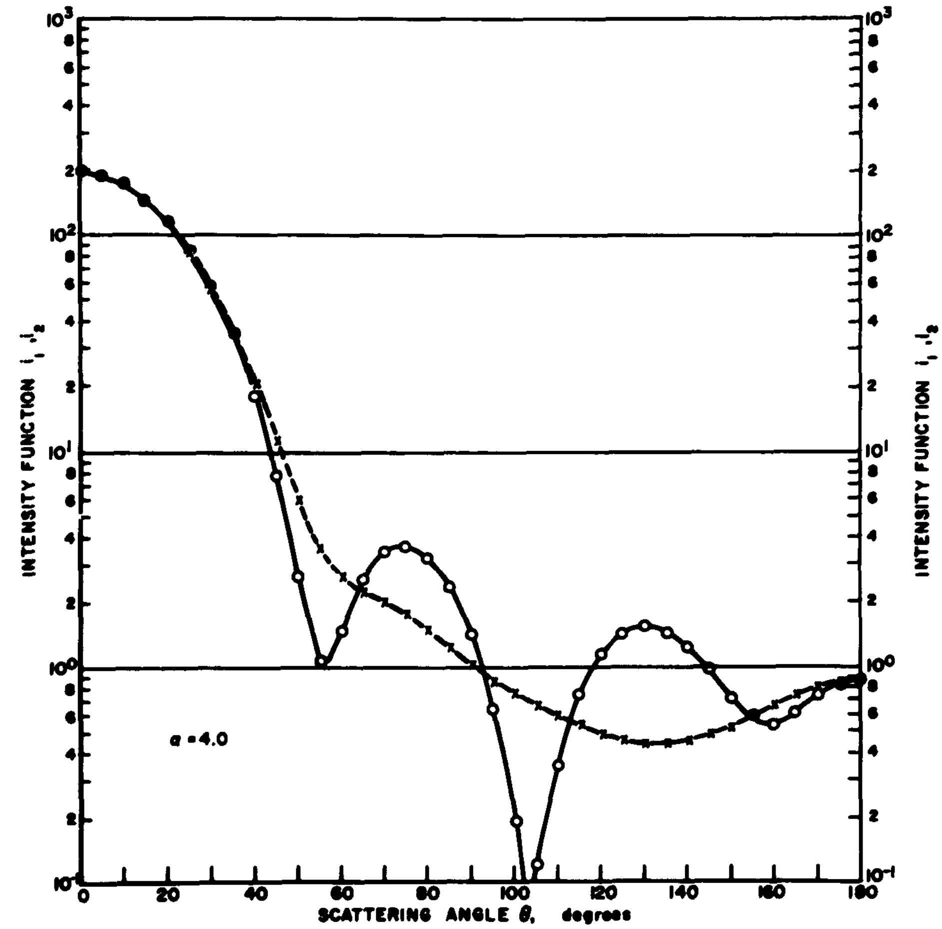
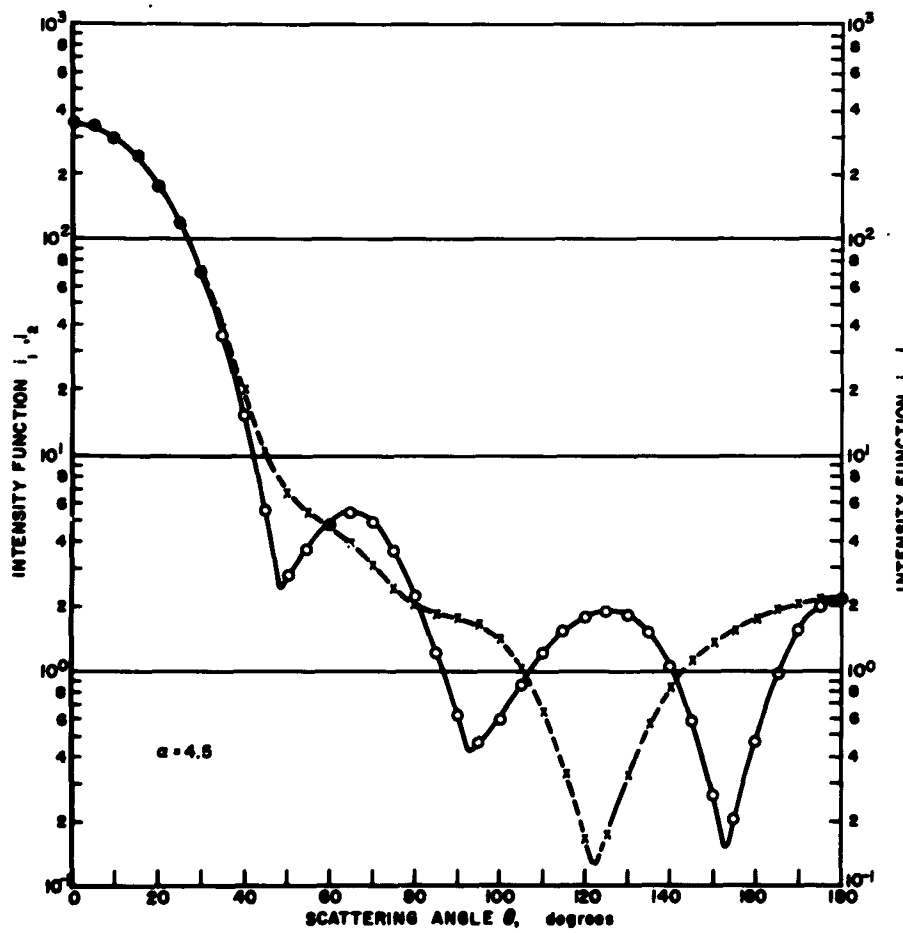
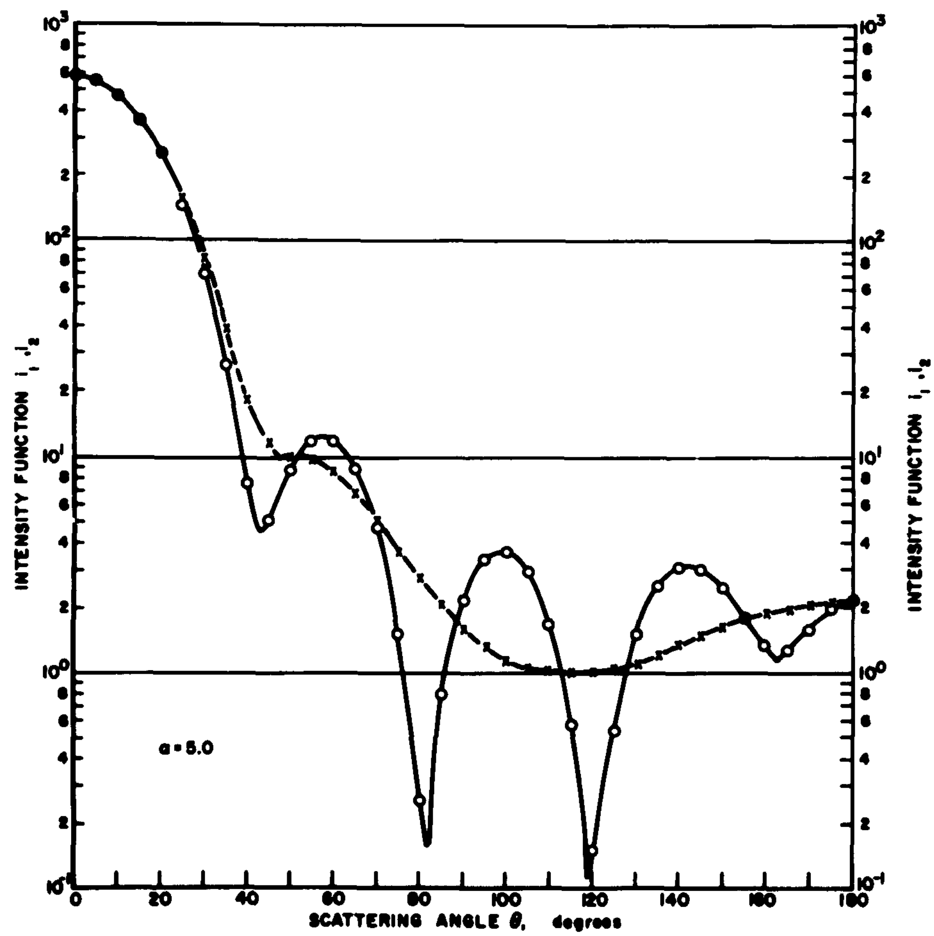
$\alpha = 3.5$



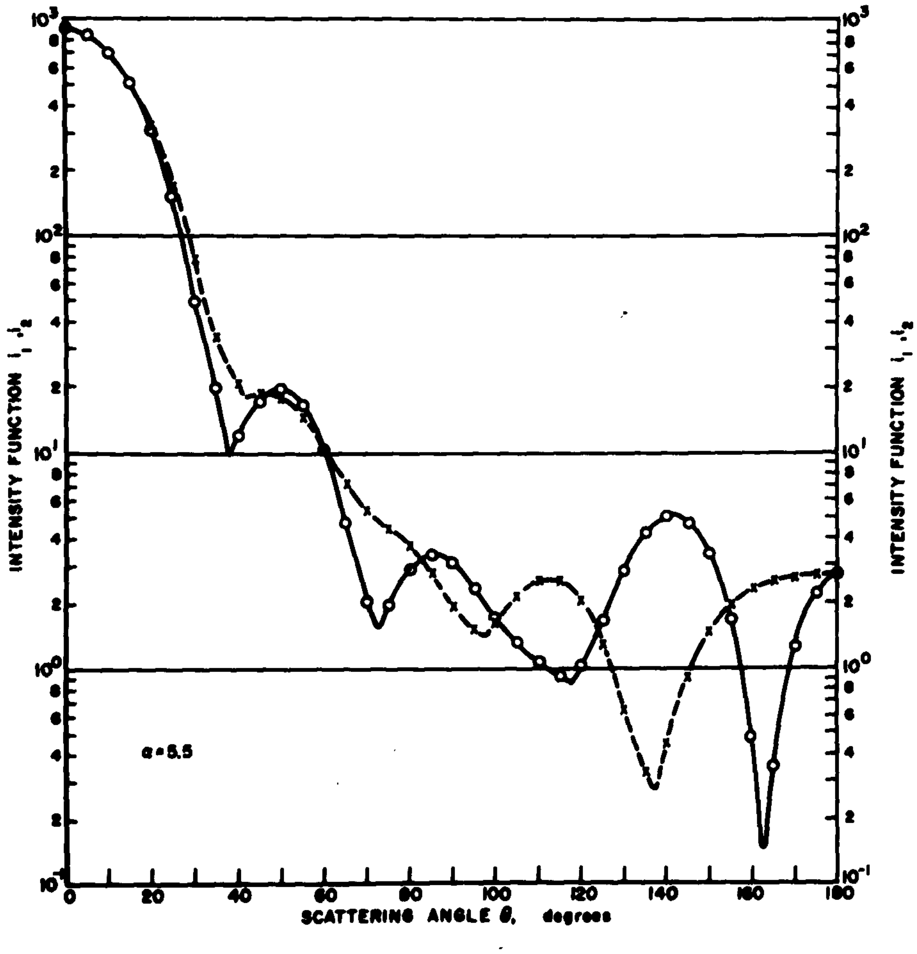
INTENSITY FUNCTION i_1, i_2
$\alpha = 4.0$
SCATTERING ANGLE θ, degrees



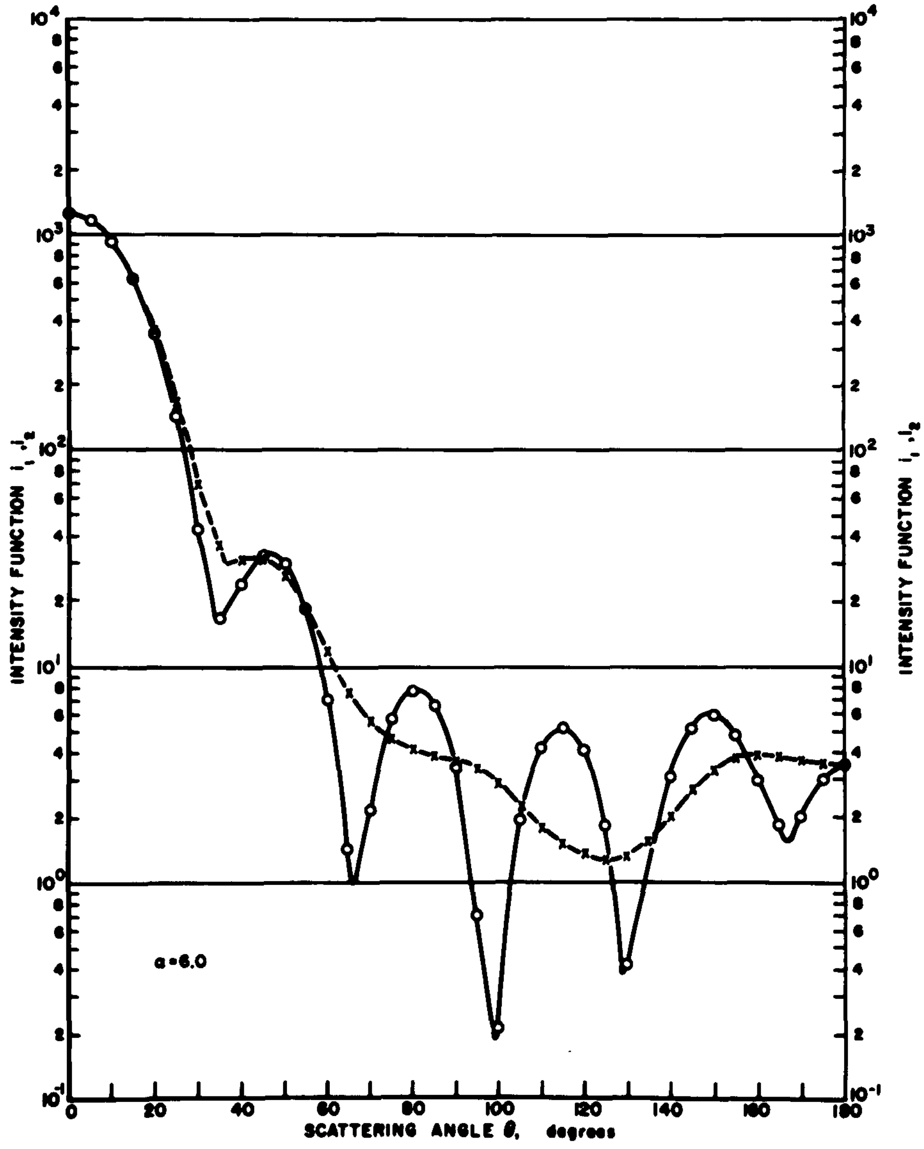
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α = 4.5

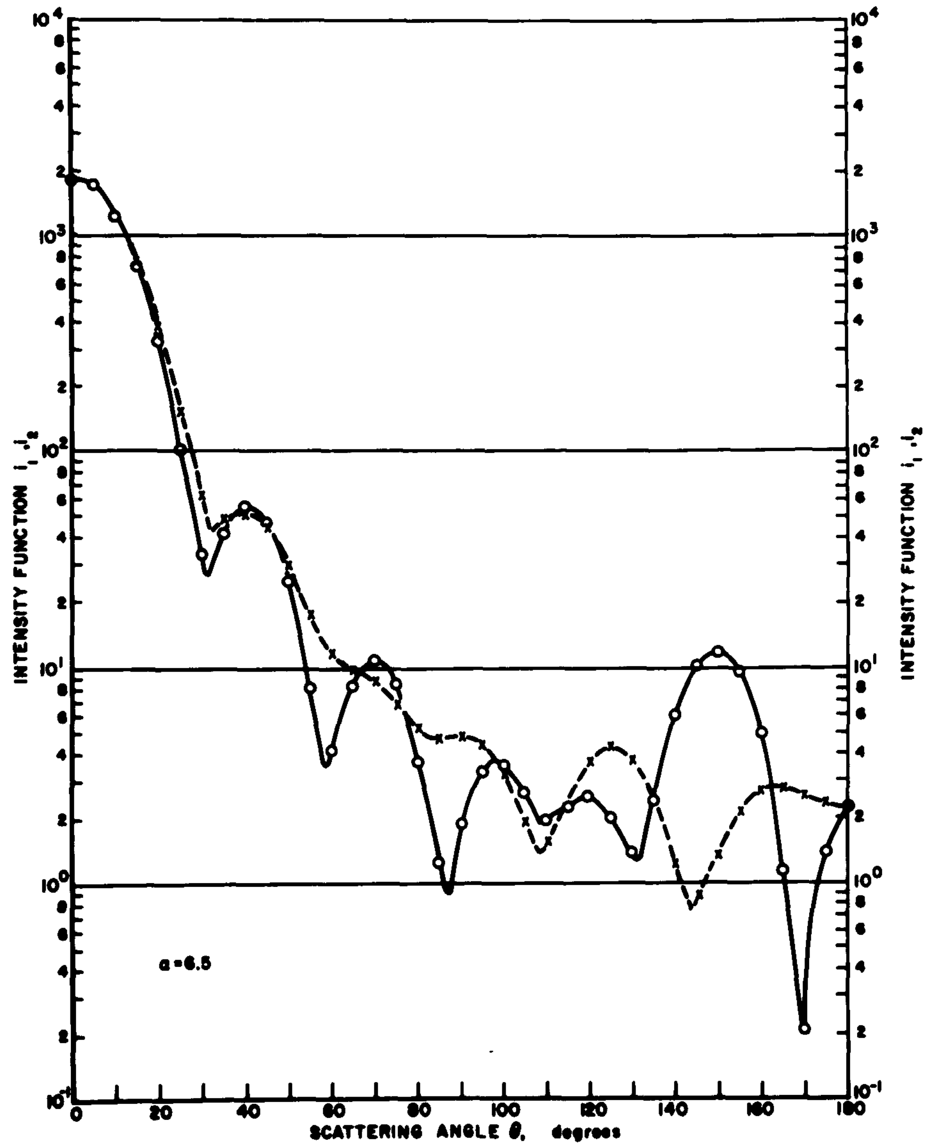


INTENSITY FUNCTION i_1, i_2
10^3
10^2
10^1
10^0
10^{-1}
$\alpha = 5.0$
0
20
40
60
80
100
120
140
160
180
SCATTERING ANGLE θ, degrees

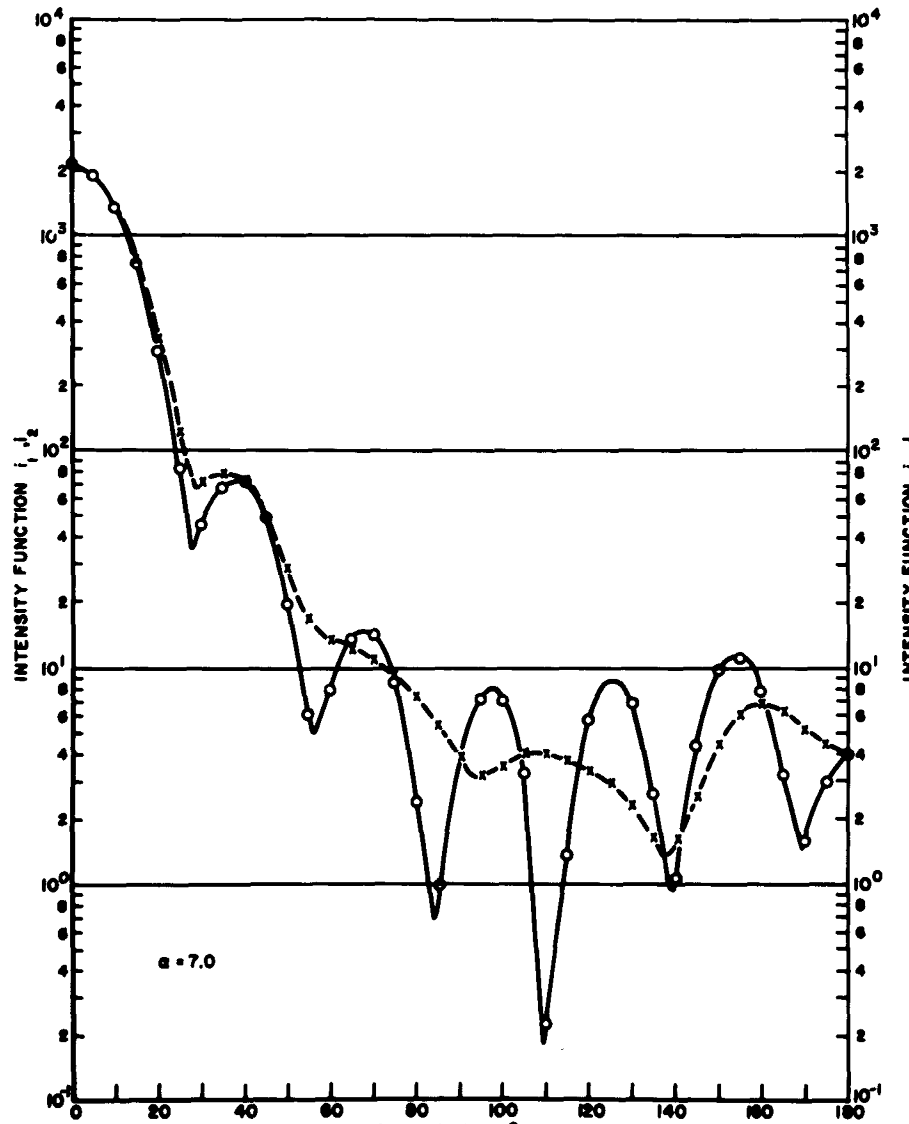


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α = 5.5

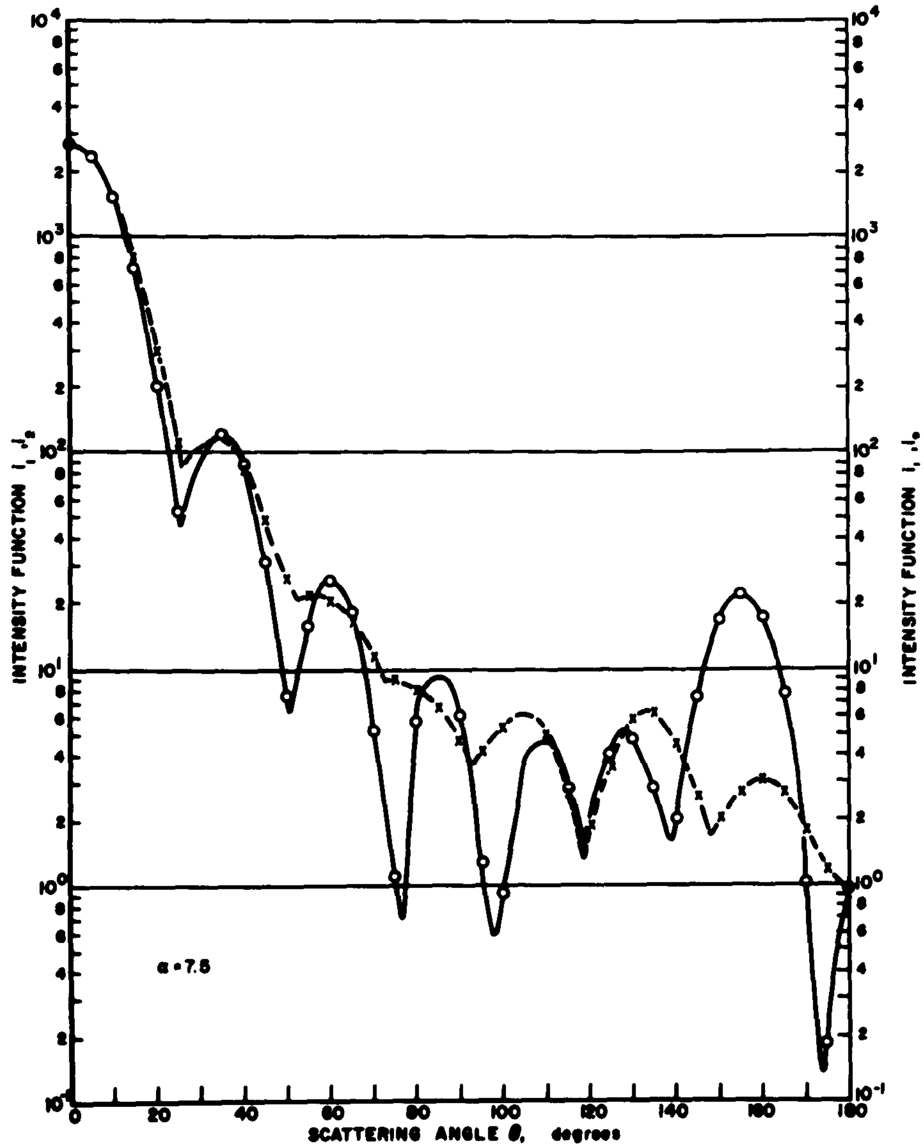

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 6.0$



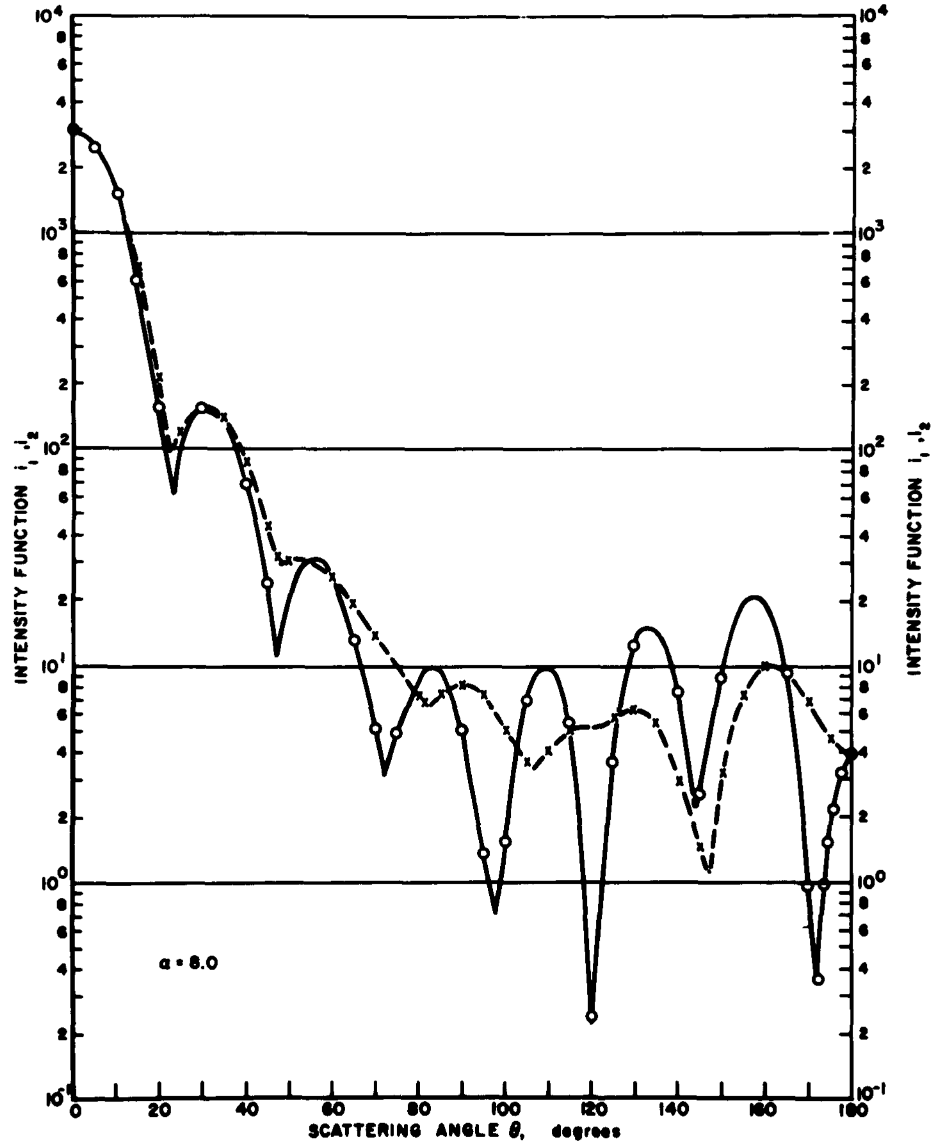
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α = 6.5

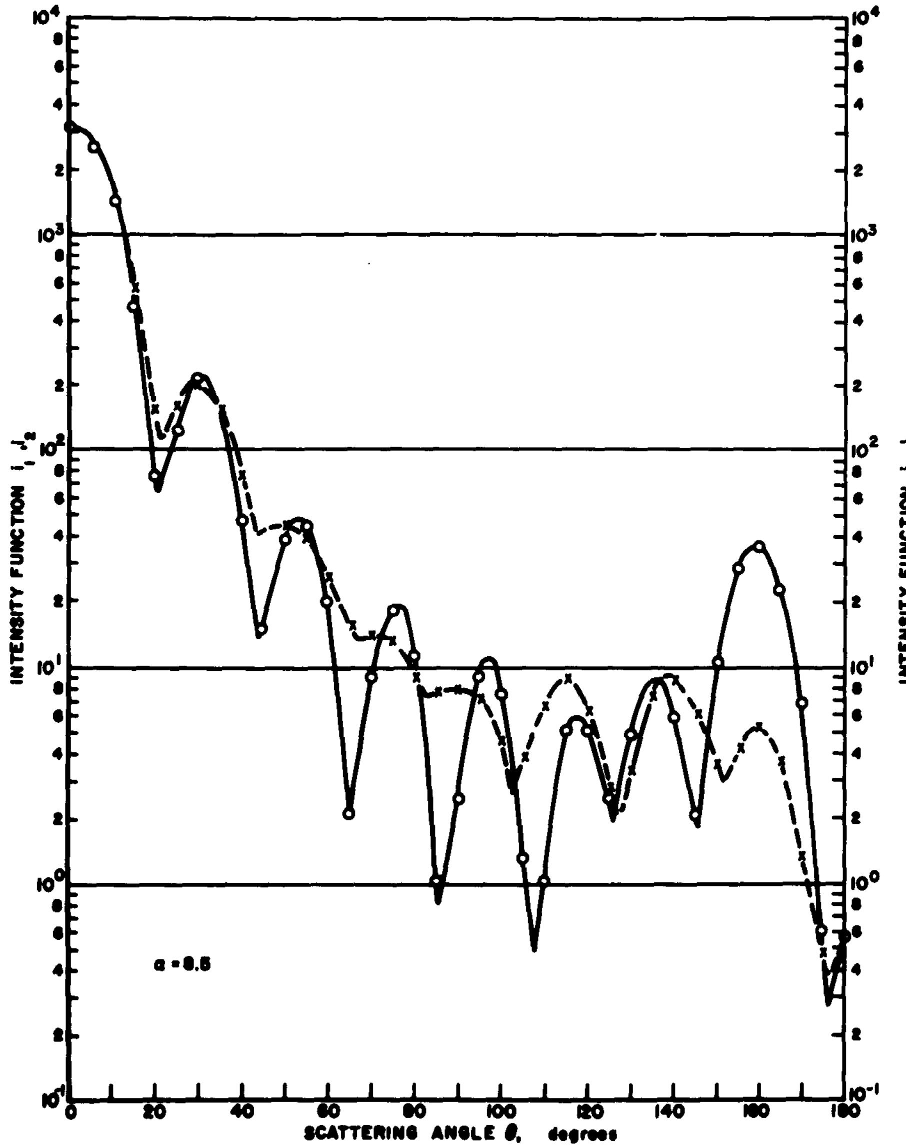


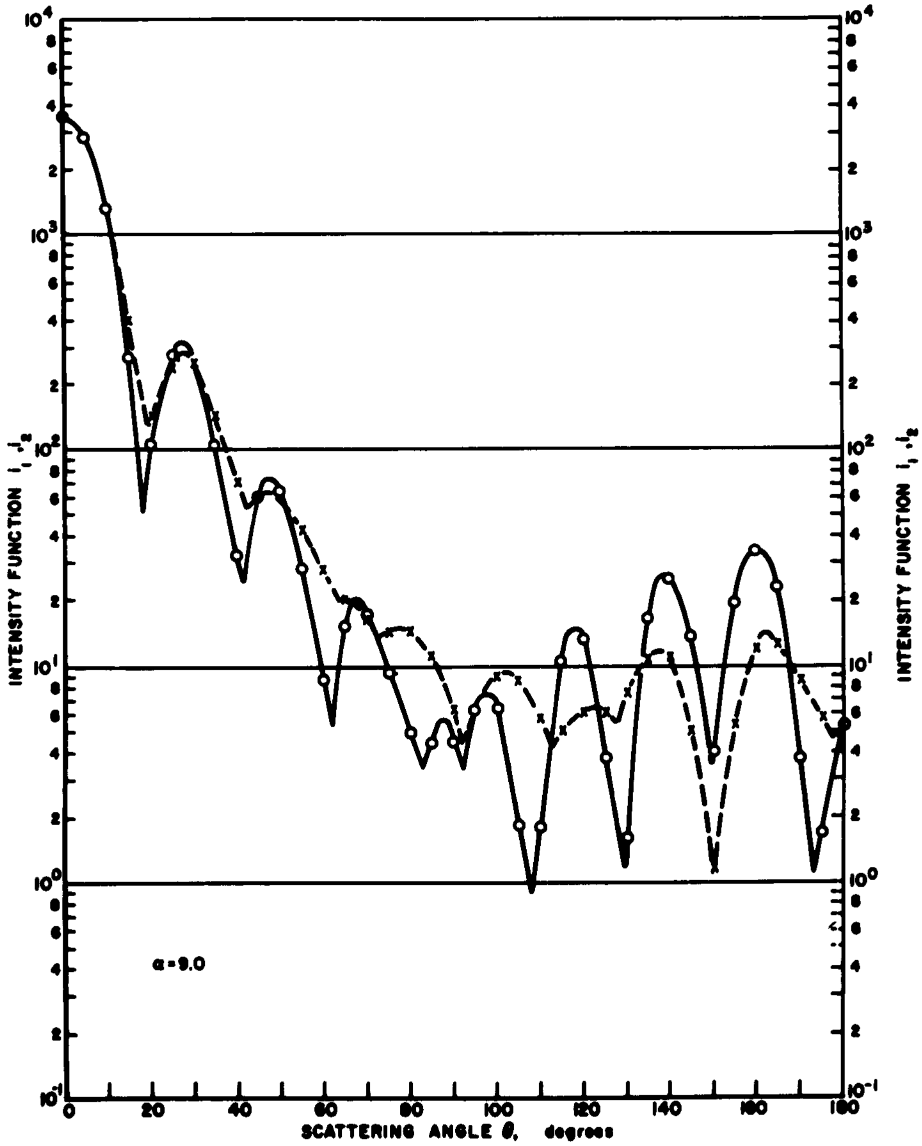
INTENSITY FUNCTION i_1, i_2
$\alpha = 7.0$
SCATTERING ANGLE θ, degrees

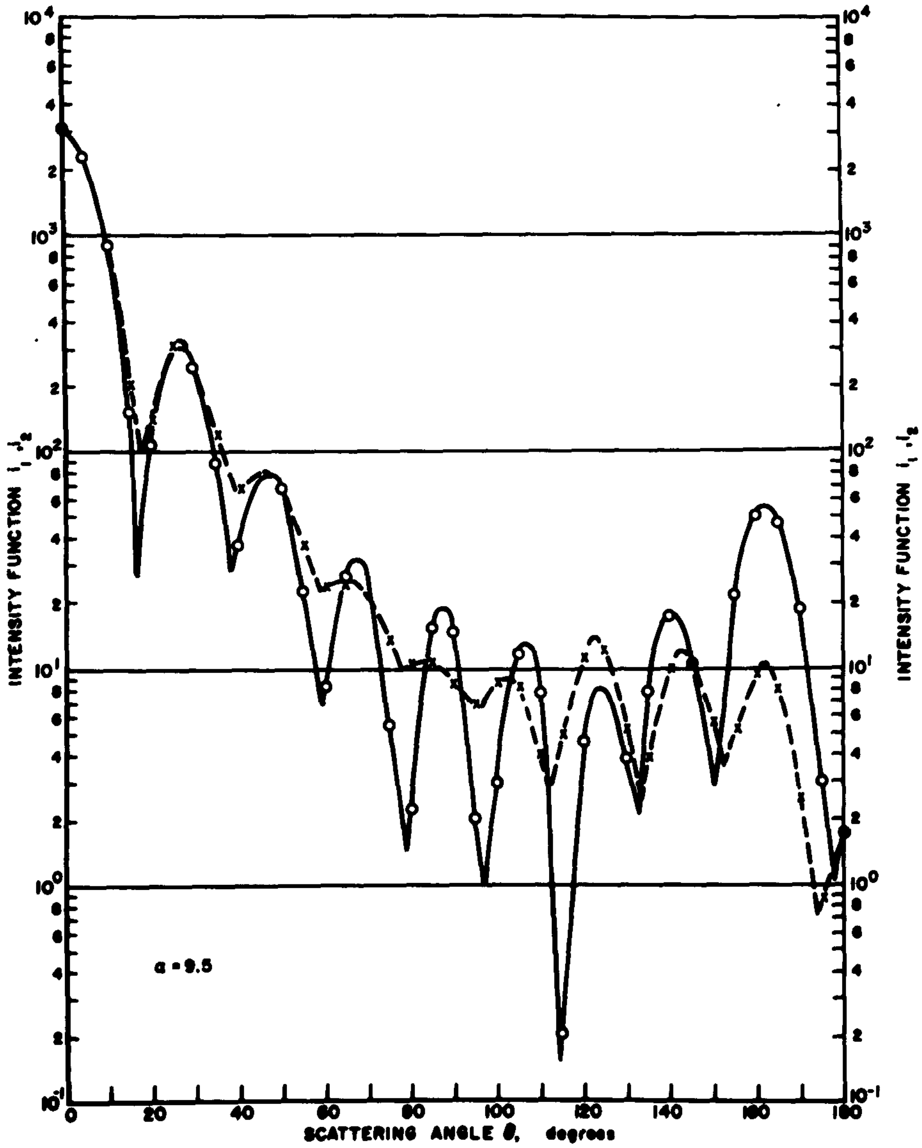


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 7.5$

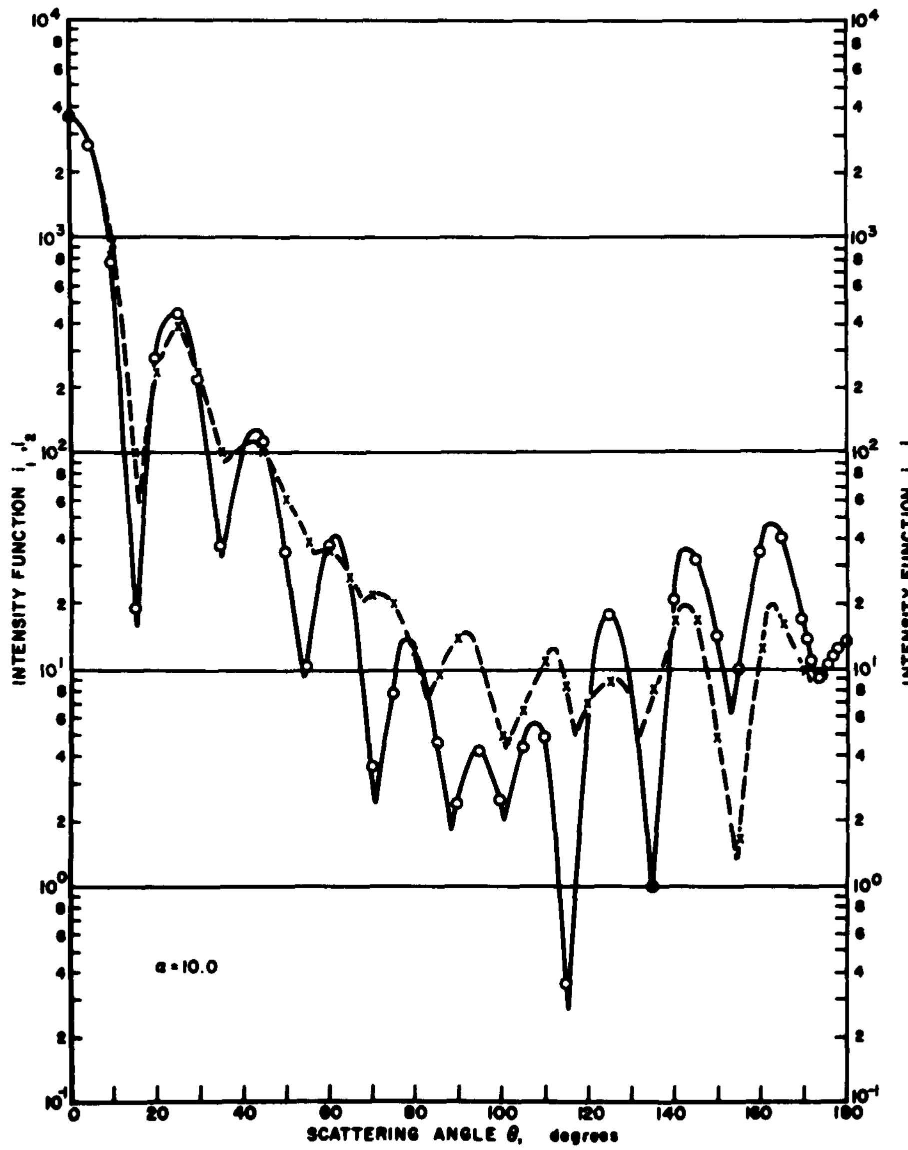

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 8.0$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 8.5$

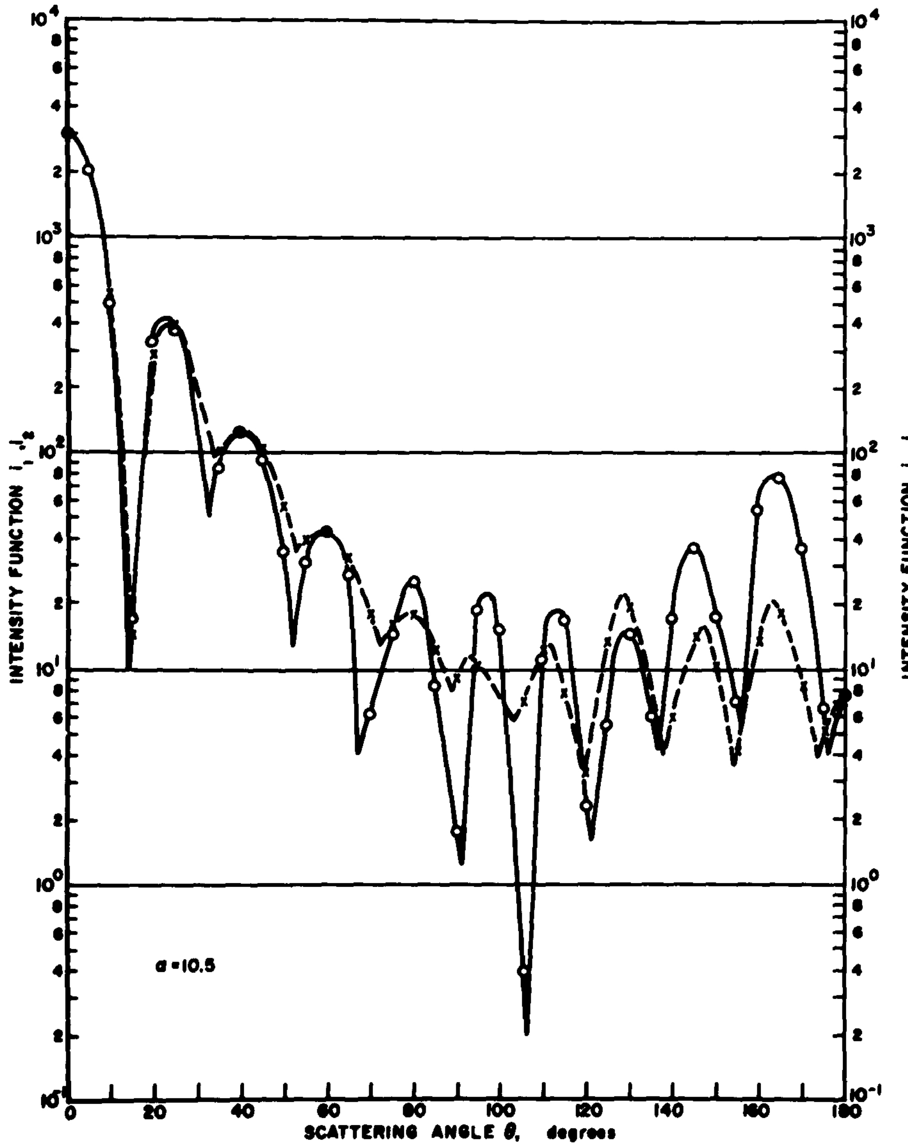

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 9.0$

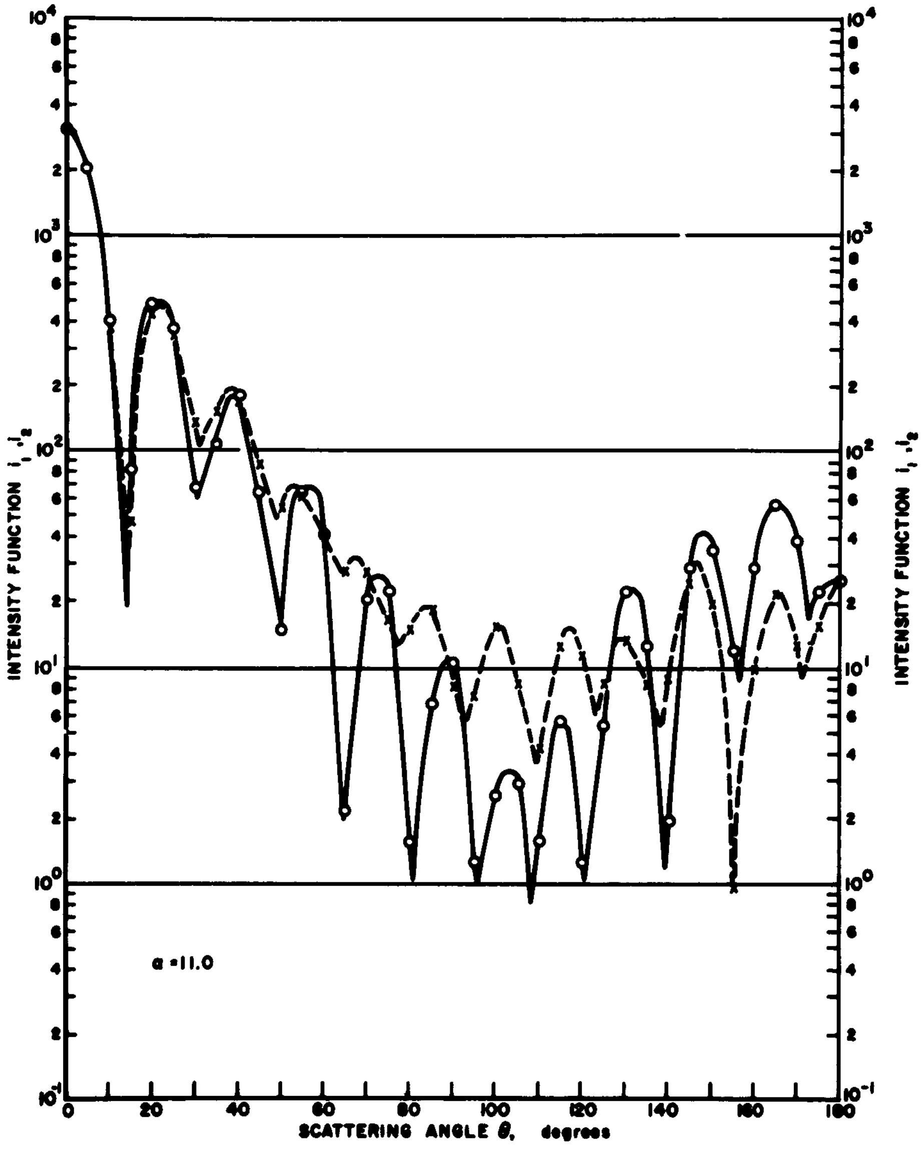


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α = 9.5

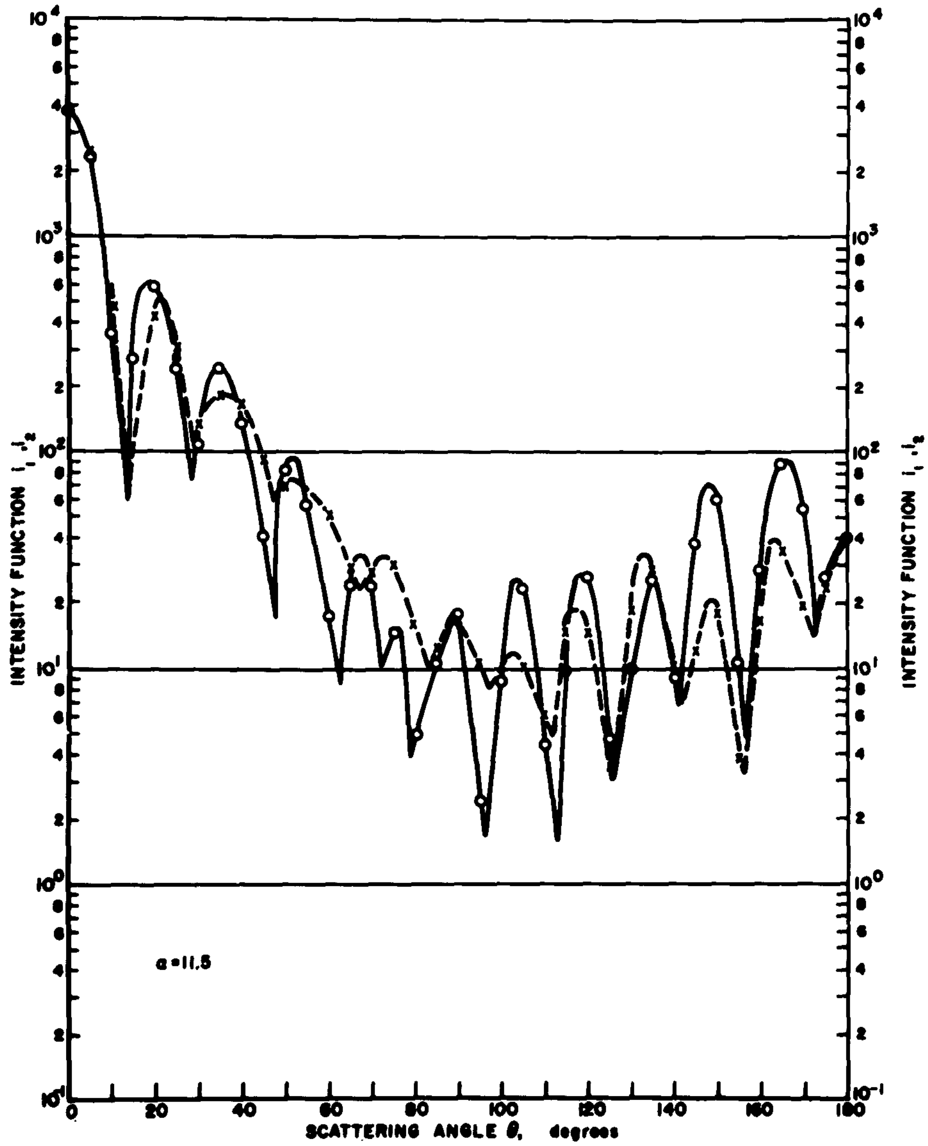


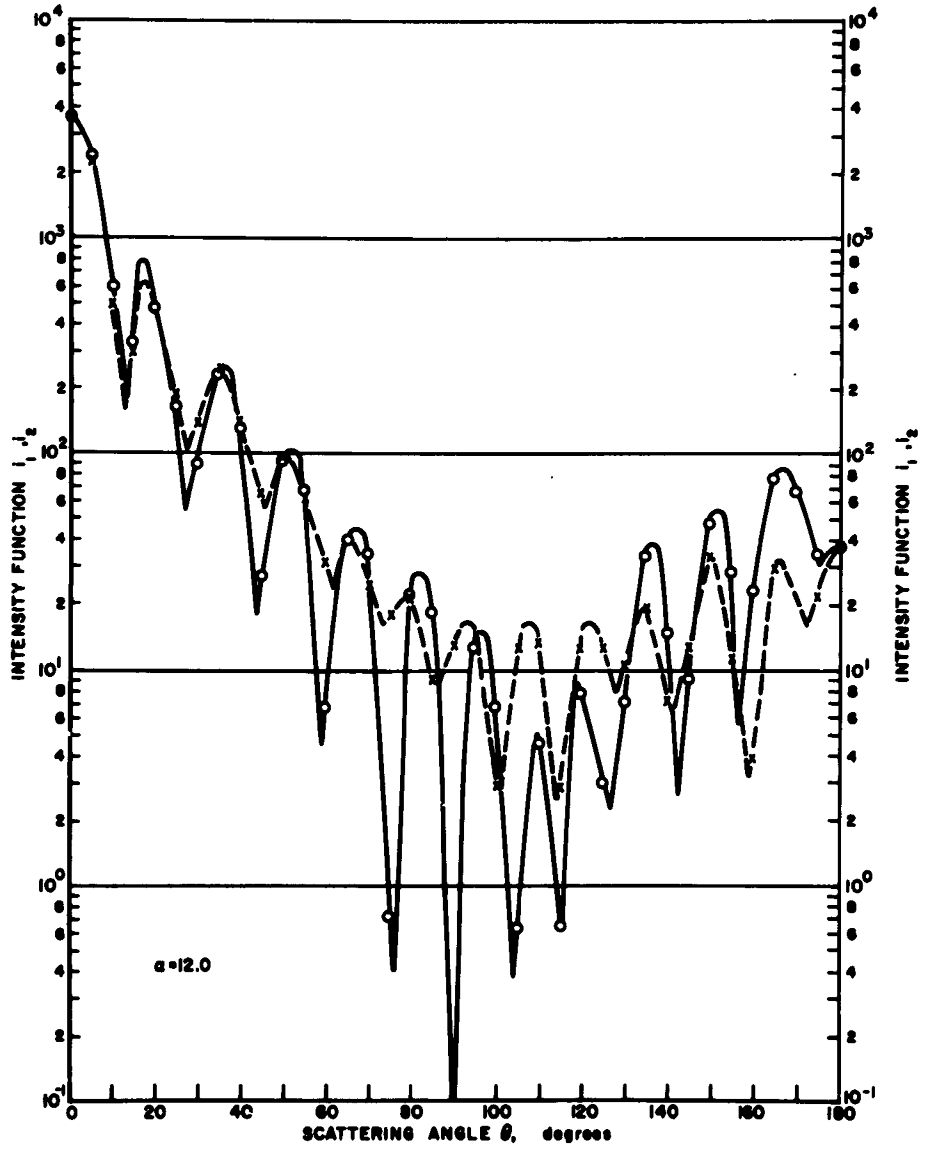
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 10.0$


INTENSITY FUNCTION $i_1 + i_2$
SCATTERING ANGLE θ, degrees
α = 10.5

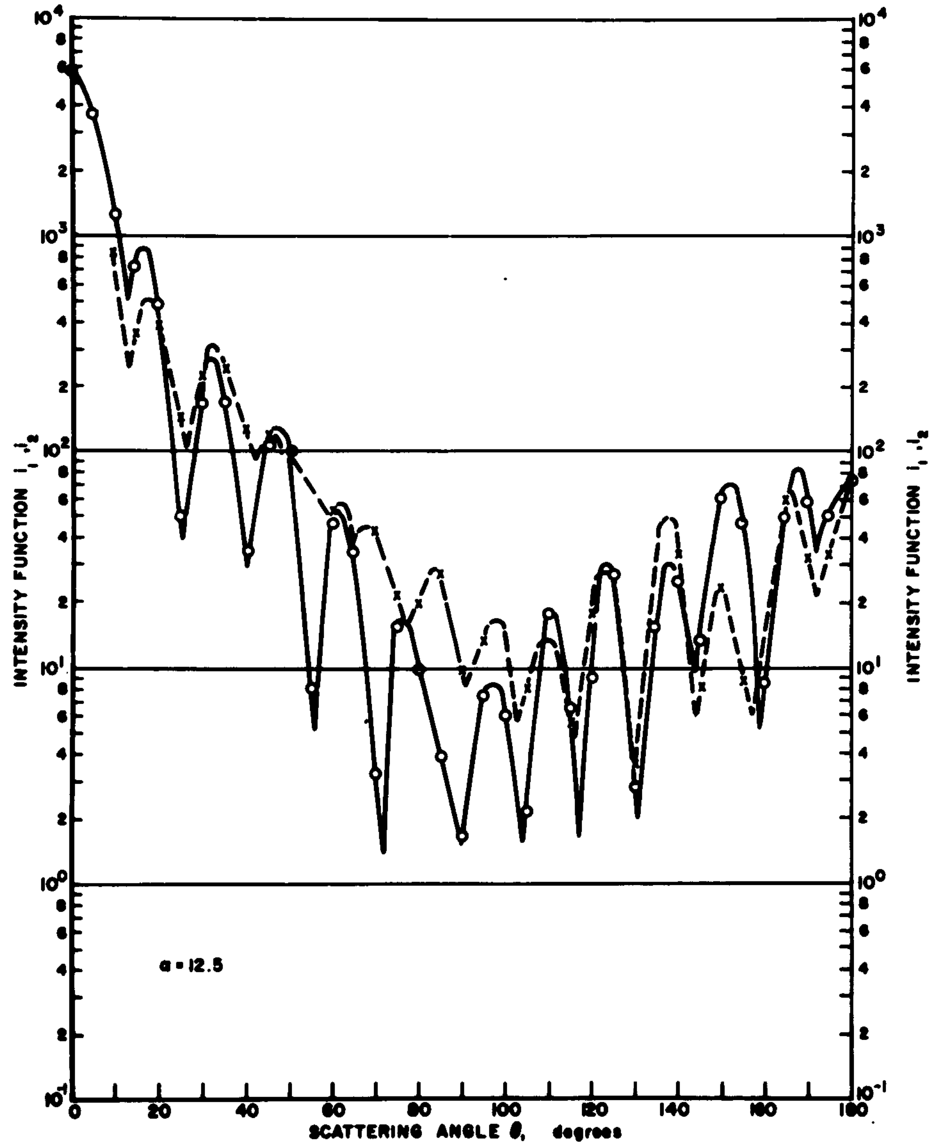


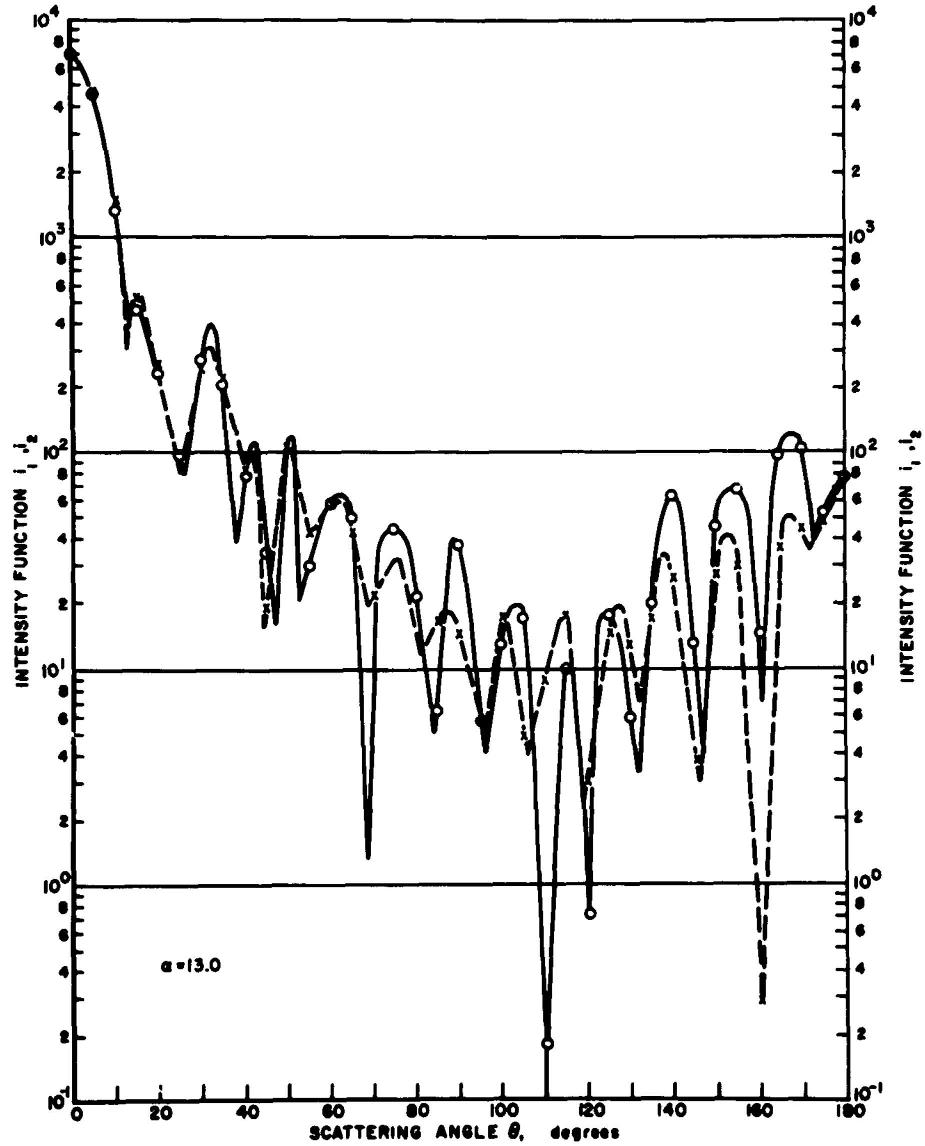
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 11.0$

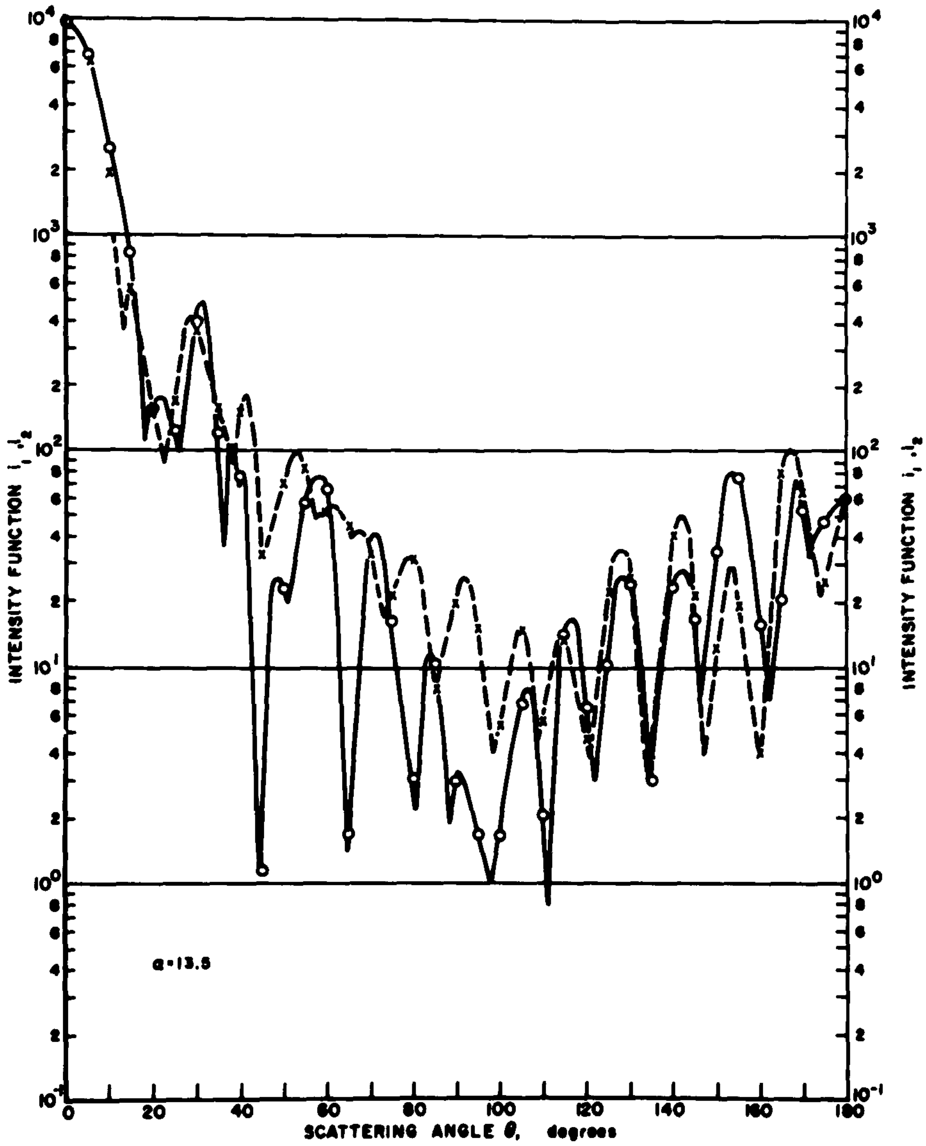

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 11.5$

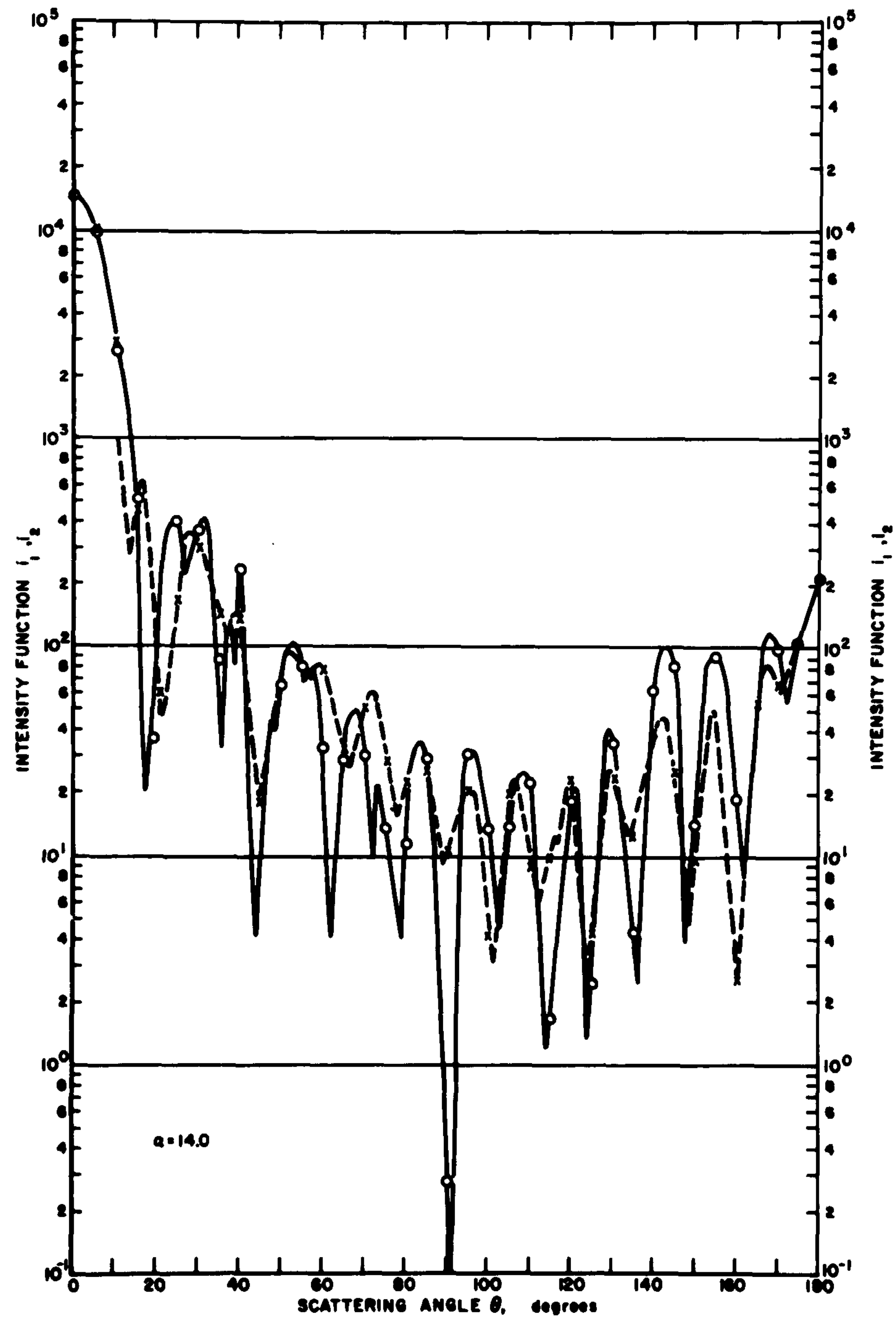


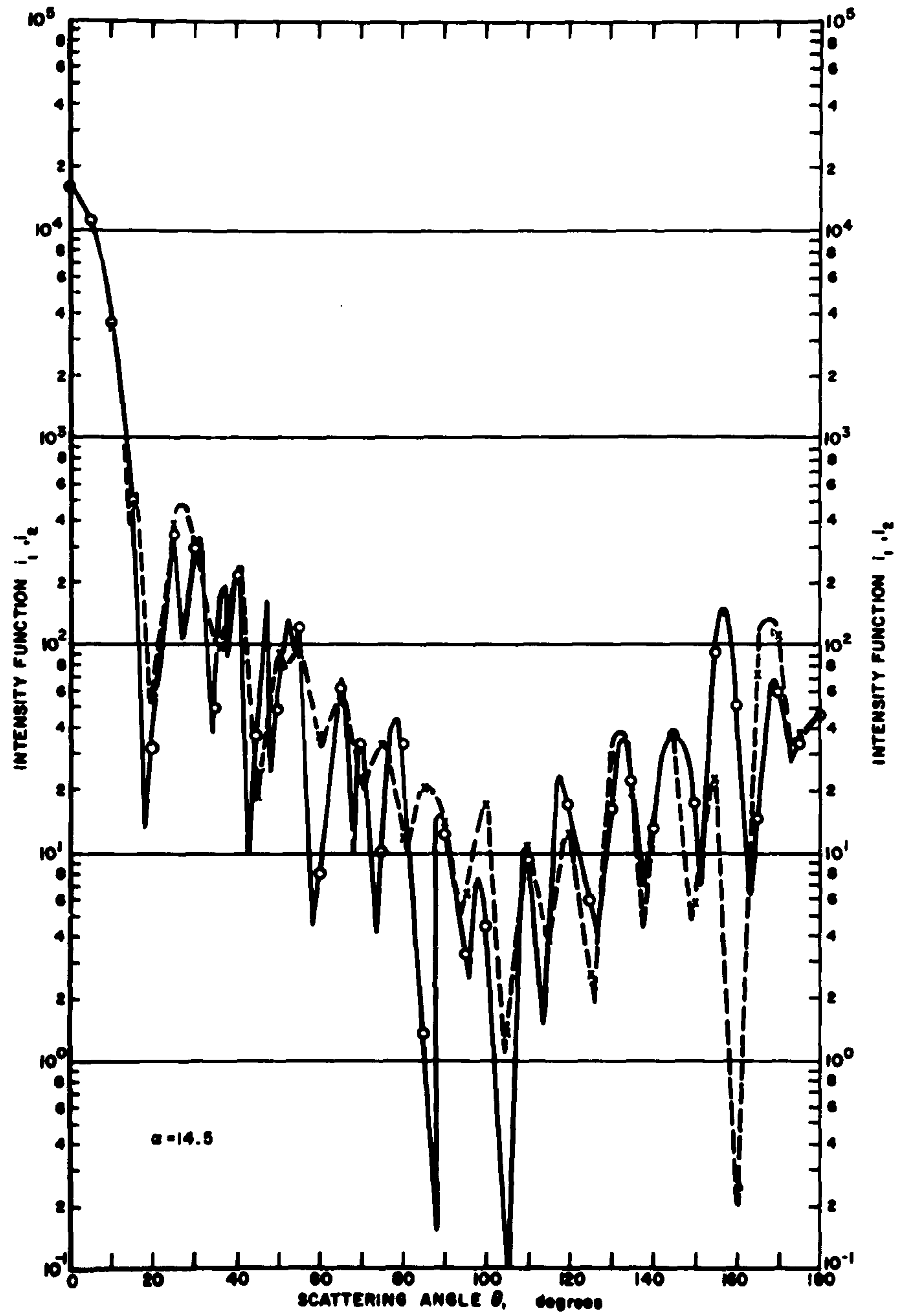
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 12.0$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 12.5$

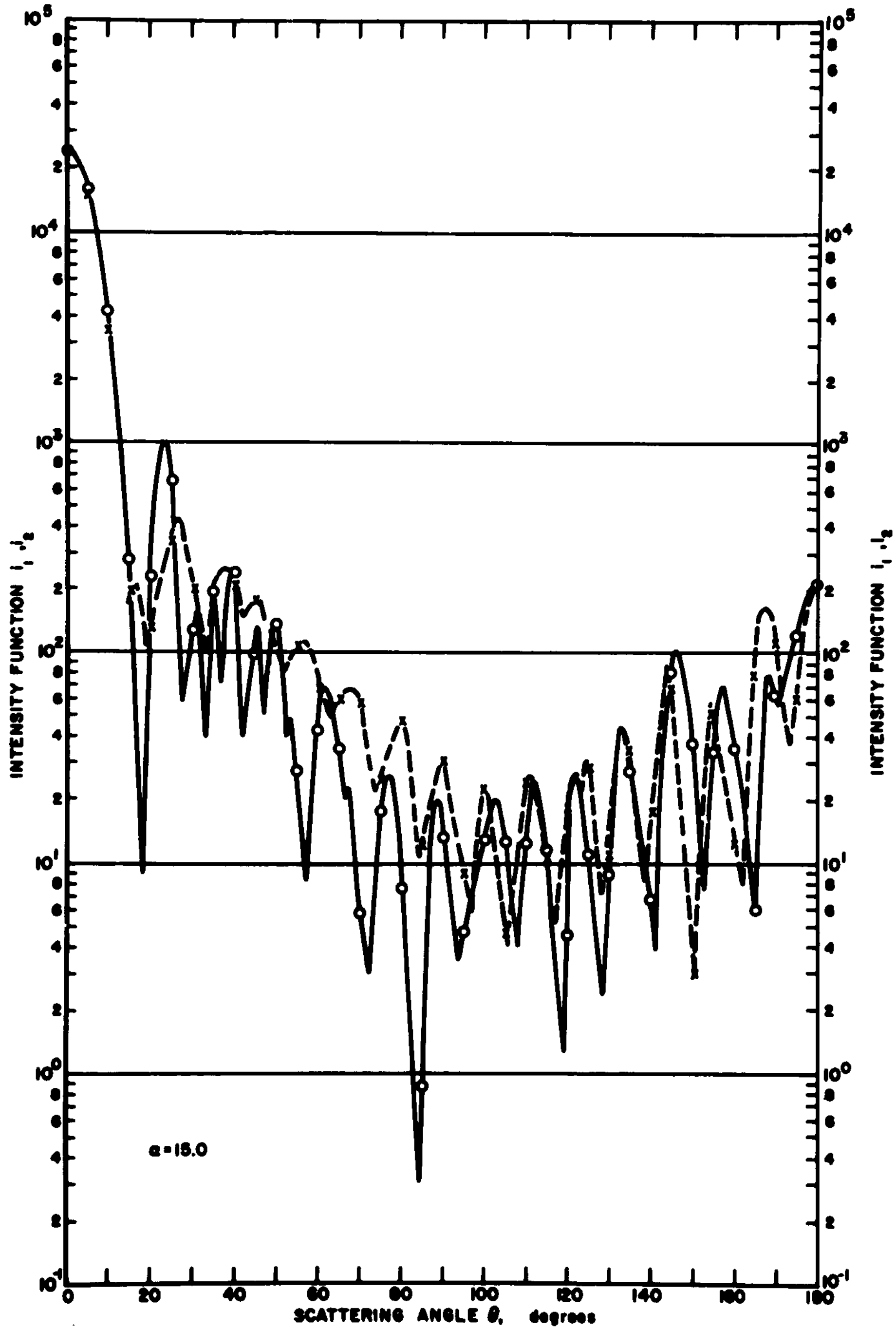

INTENSITY FUNCTION $i_1 + i_2$
SCATTERING ANGLE θ, degrees
α = 13.0


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α=13.5


INTENSITY FUNCTION $i_1 \cdot i_2$
SCATTERING ANGLE θ, degrees
$\alpha = 14.0$

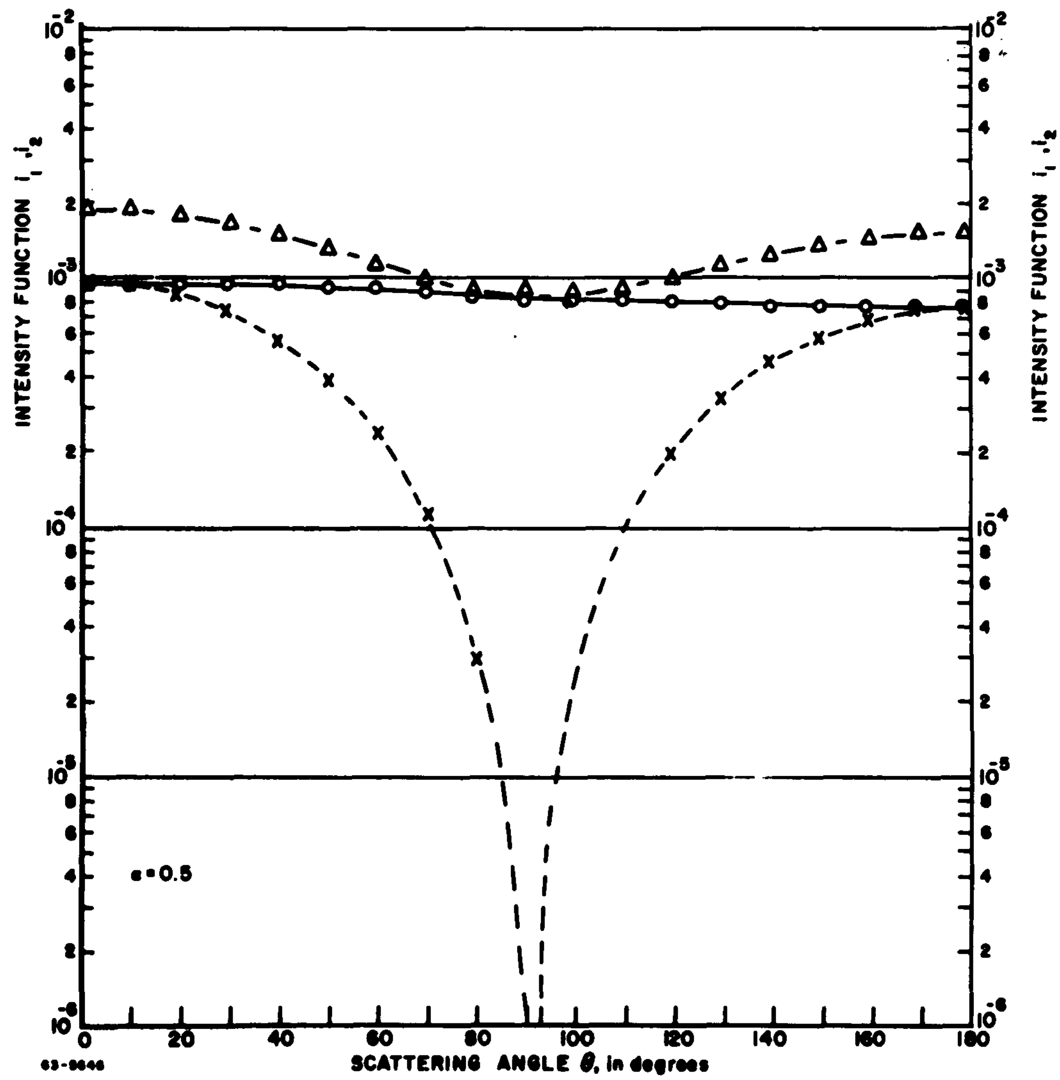


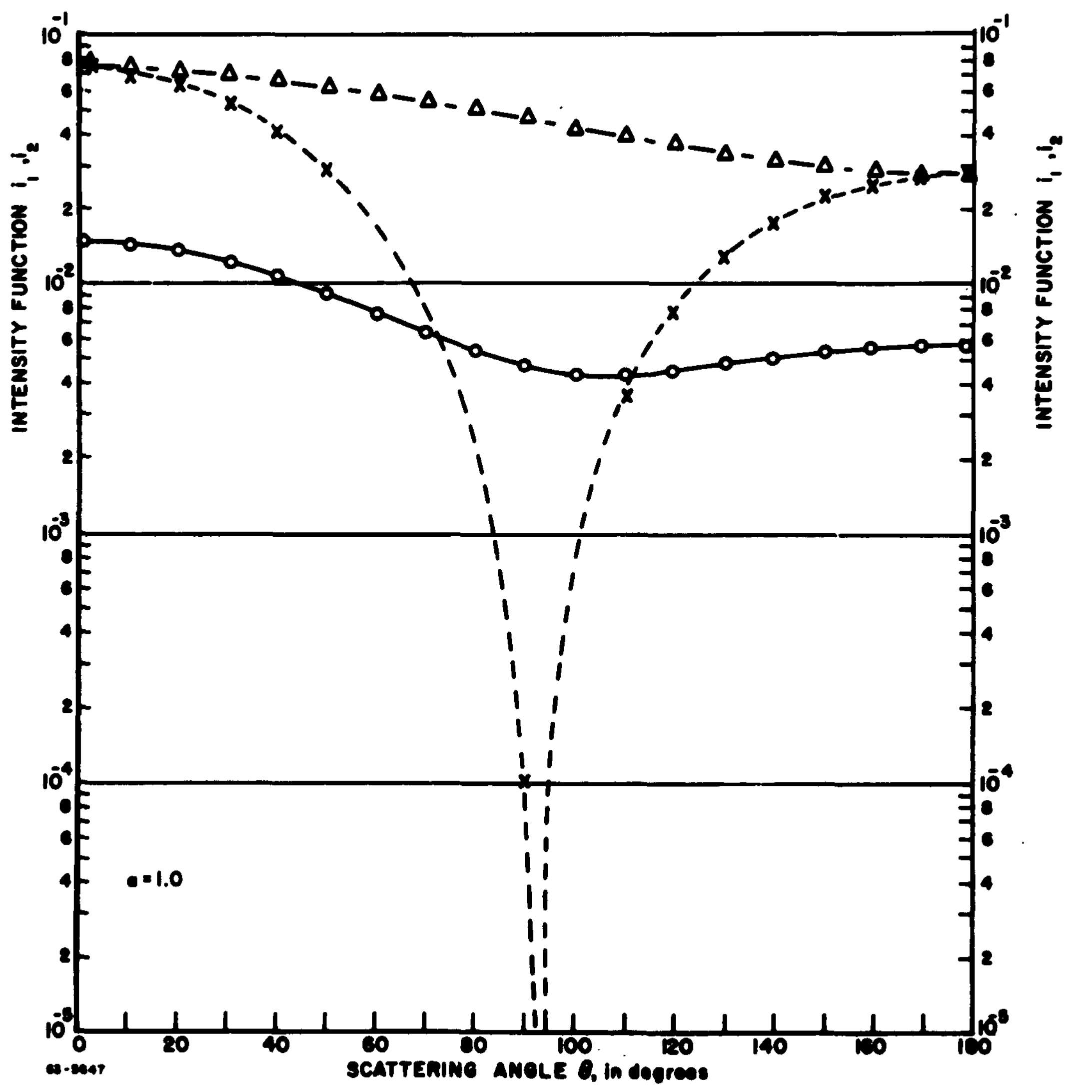
INTENSITY FUNCTION i_1+i_2
SCATTERING ANGLE θ, degrees
α = 14.5

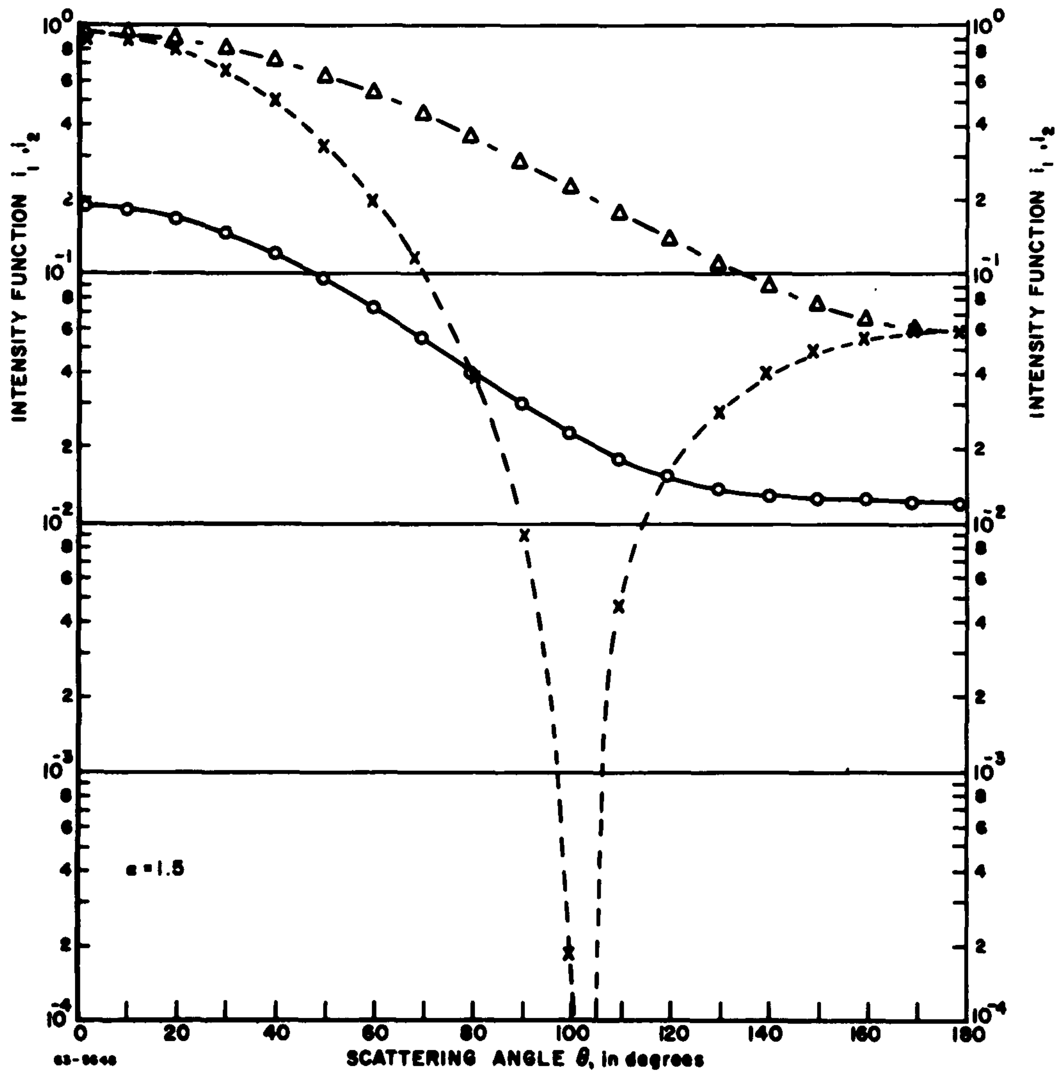

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 15.0$

6.24 Atlas of scattering diagrams

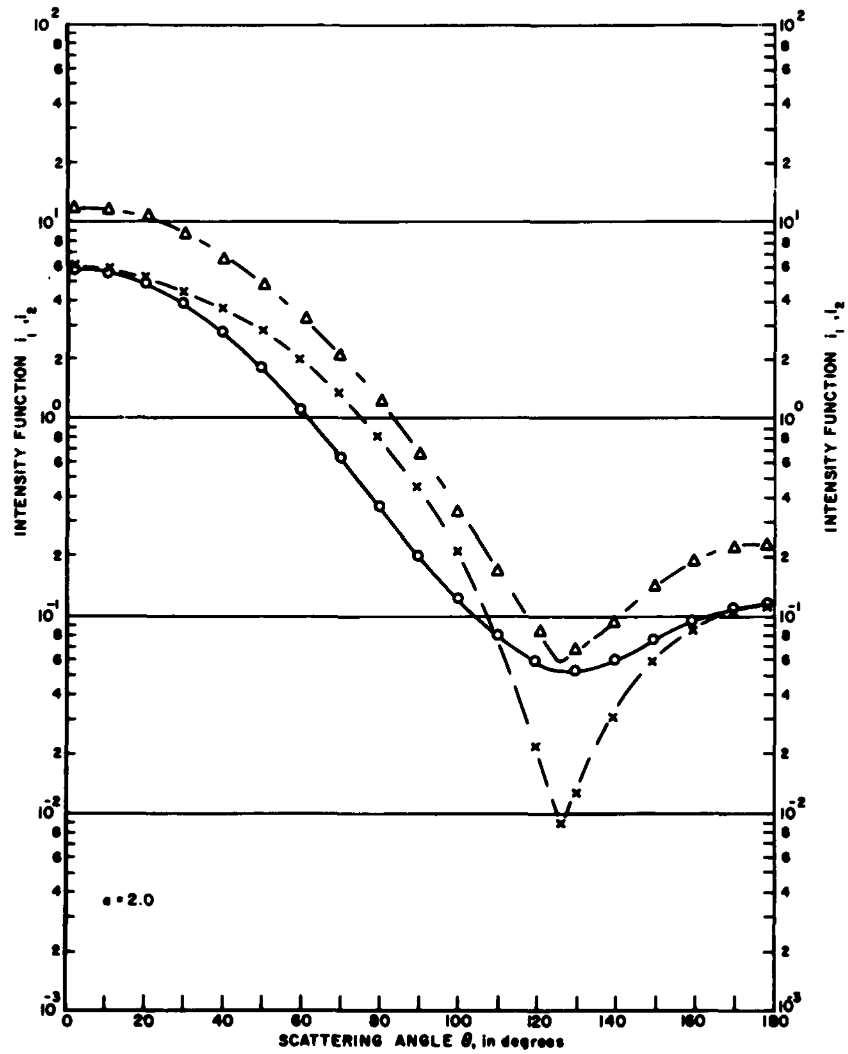
for $n = 1.4$


INTENSITY FUNCTION $i_1 \cdot i_2$
SCATTERING ANGLE θ, in degrees
α = 0.5
63-9646

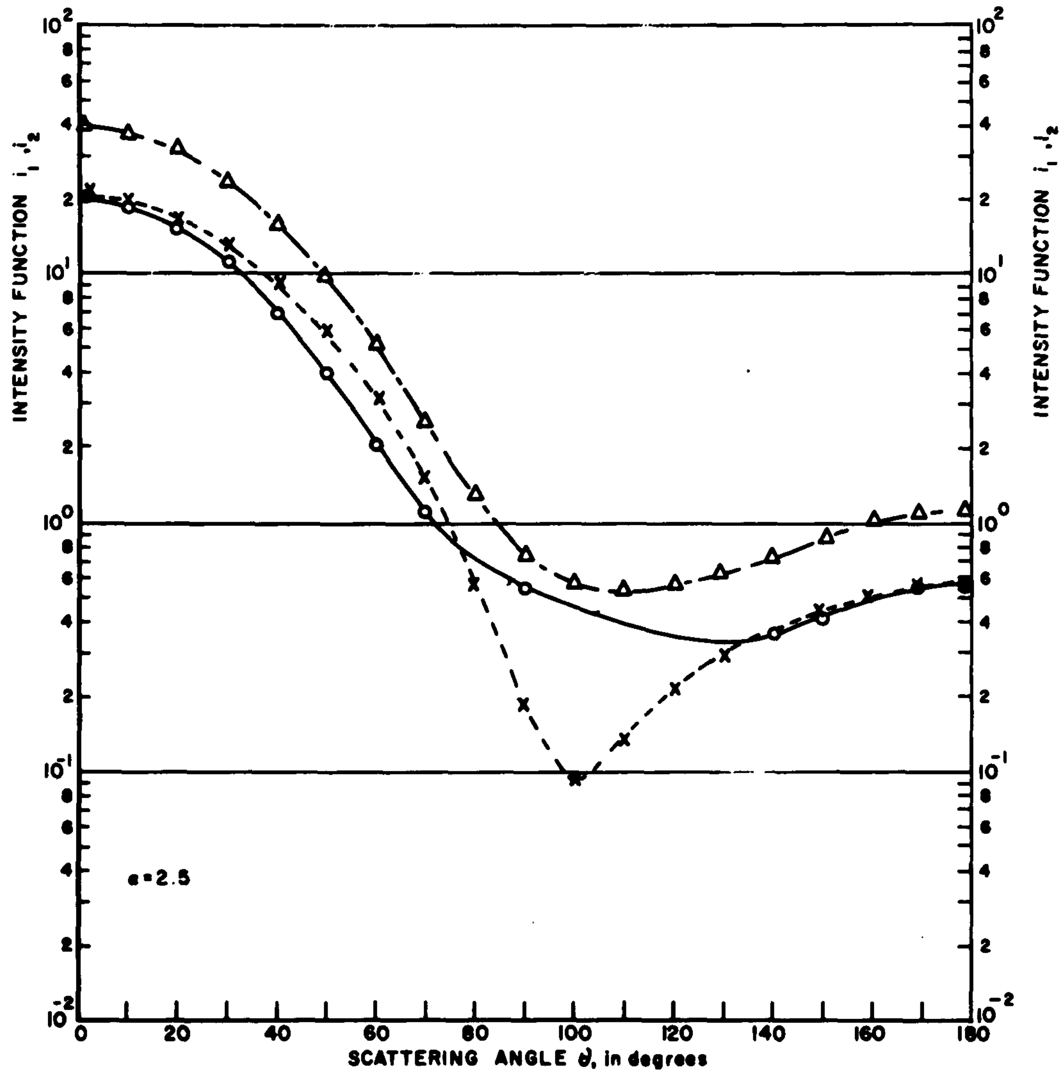

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
α = 1.0

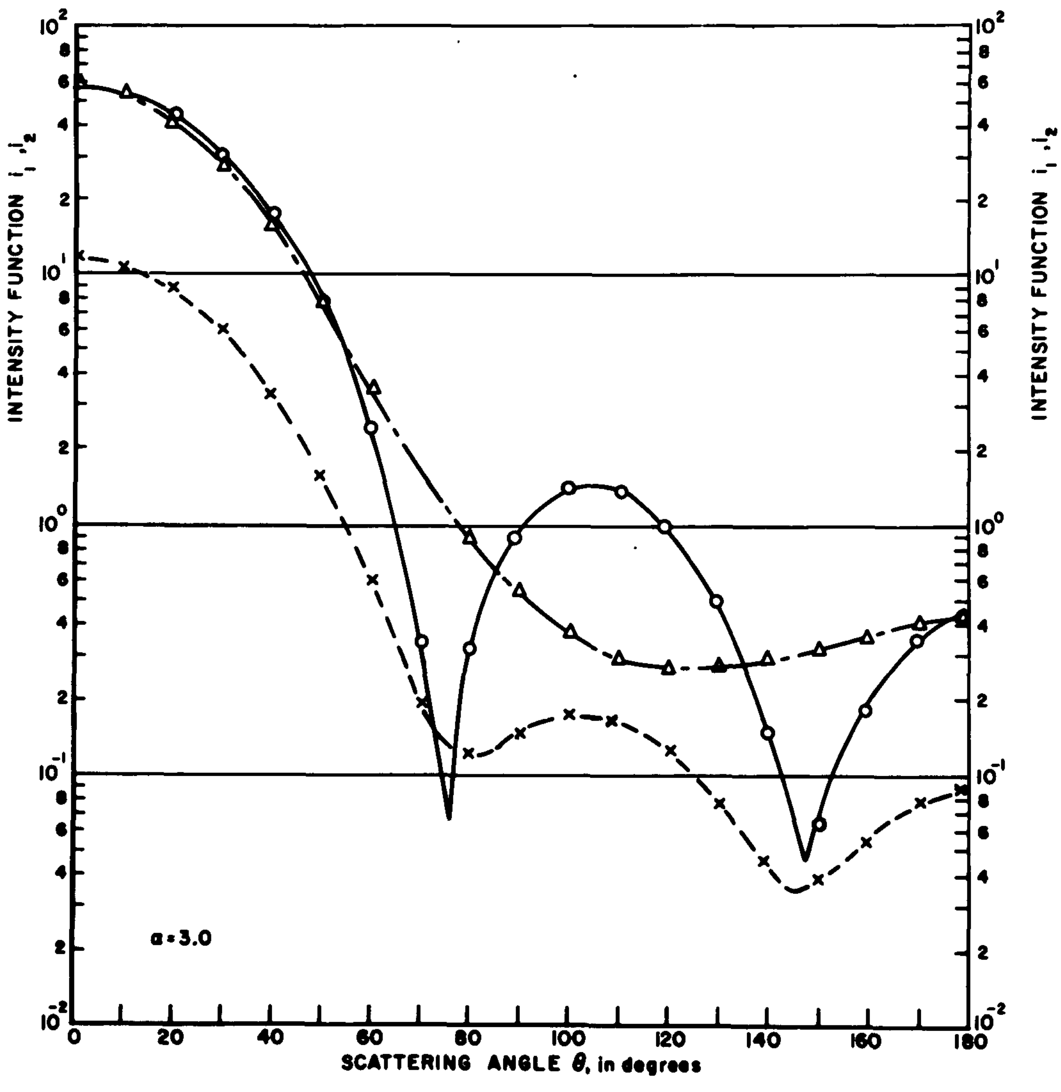


INTENSITY FUNCTION i_1, i_2
10^0
8
6
4
2
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\alpha = 1.5$
0
20
40
60
80
100
120
140
160
180
SCATTERING ANGLE θ, in degrees

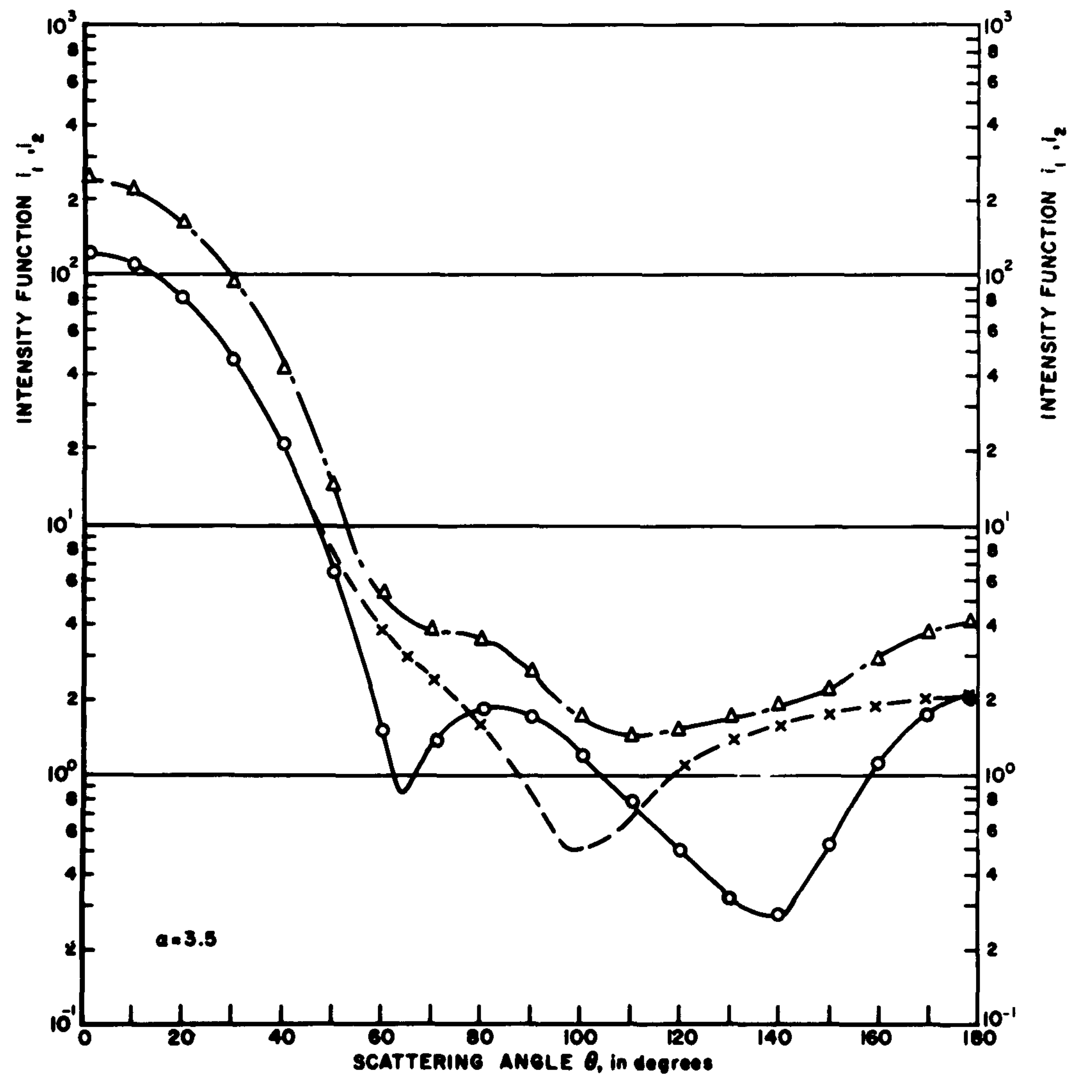


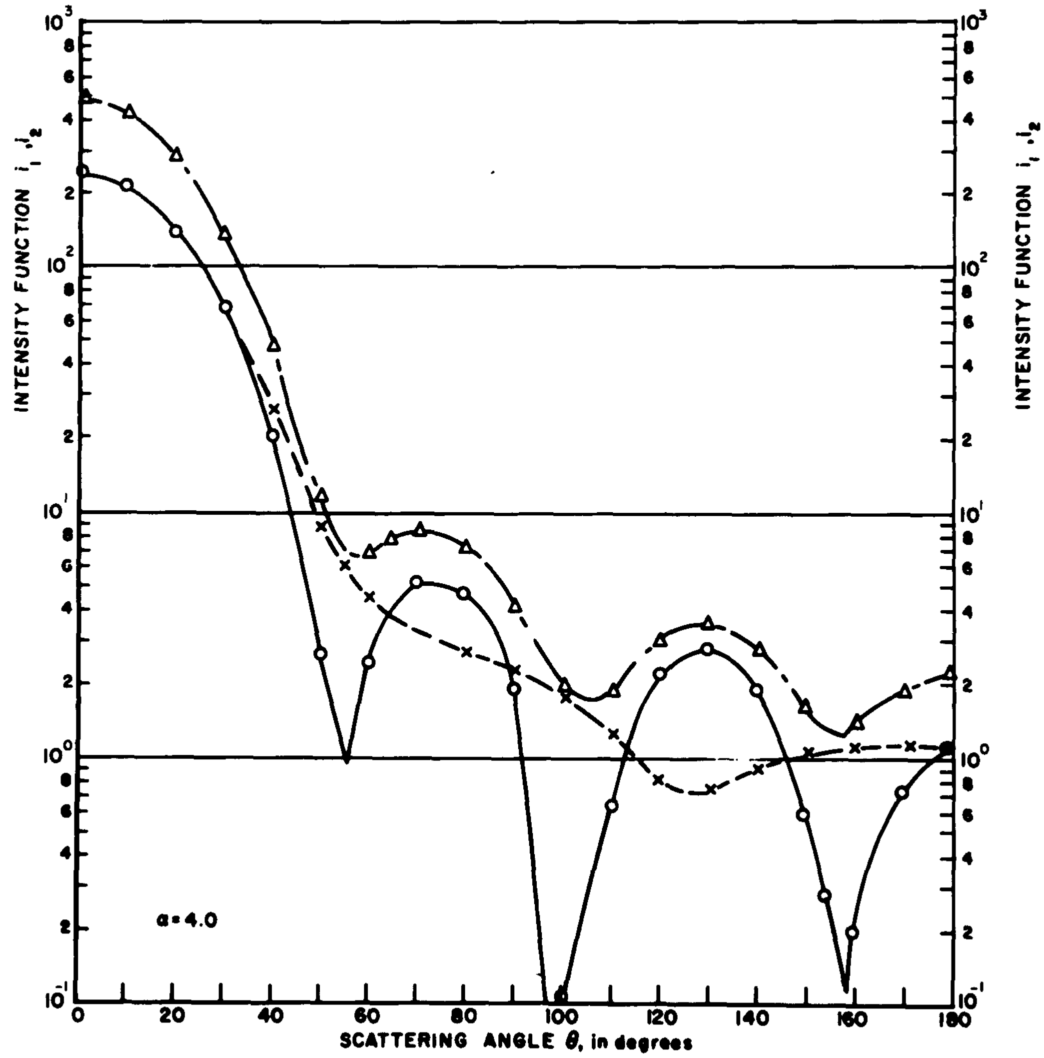
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 2.0$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE ϑ, in degrees
$\alpha = 2.5$

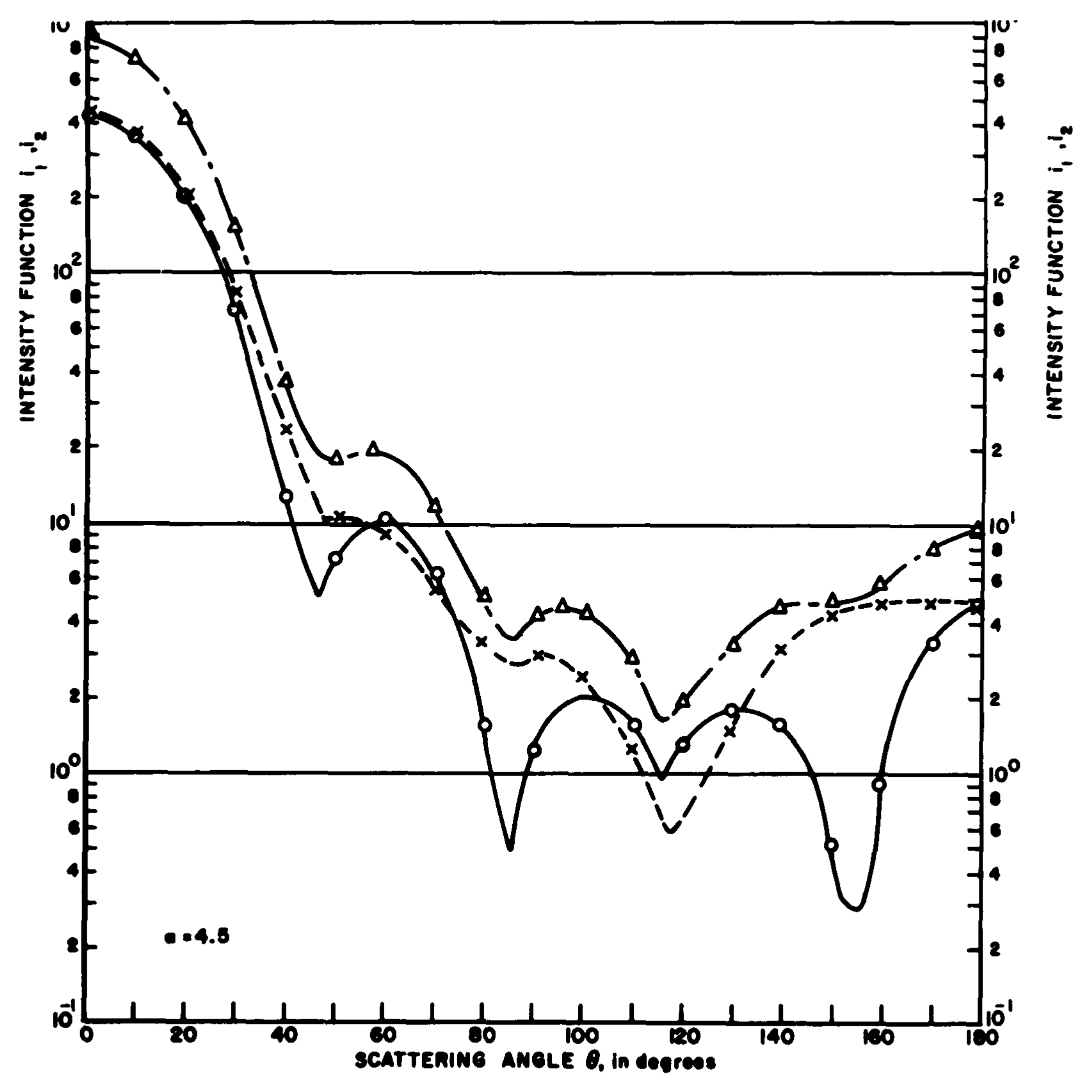


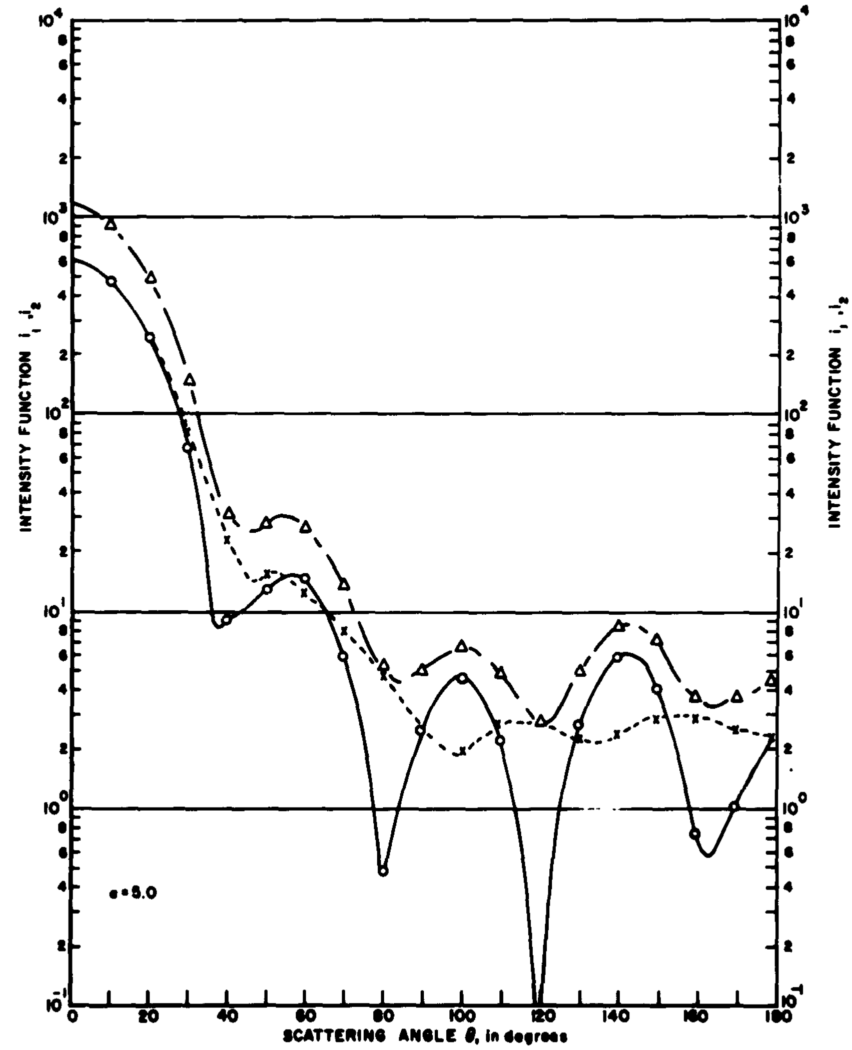
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 3.0$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 3.5$
10^3
10^2
10^1
10^0
10^{-1}
0
20
40
60
80
100
120
140
160
180

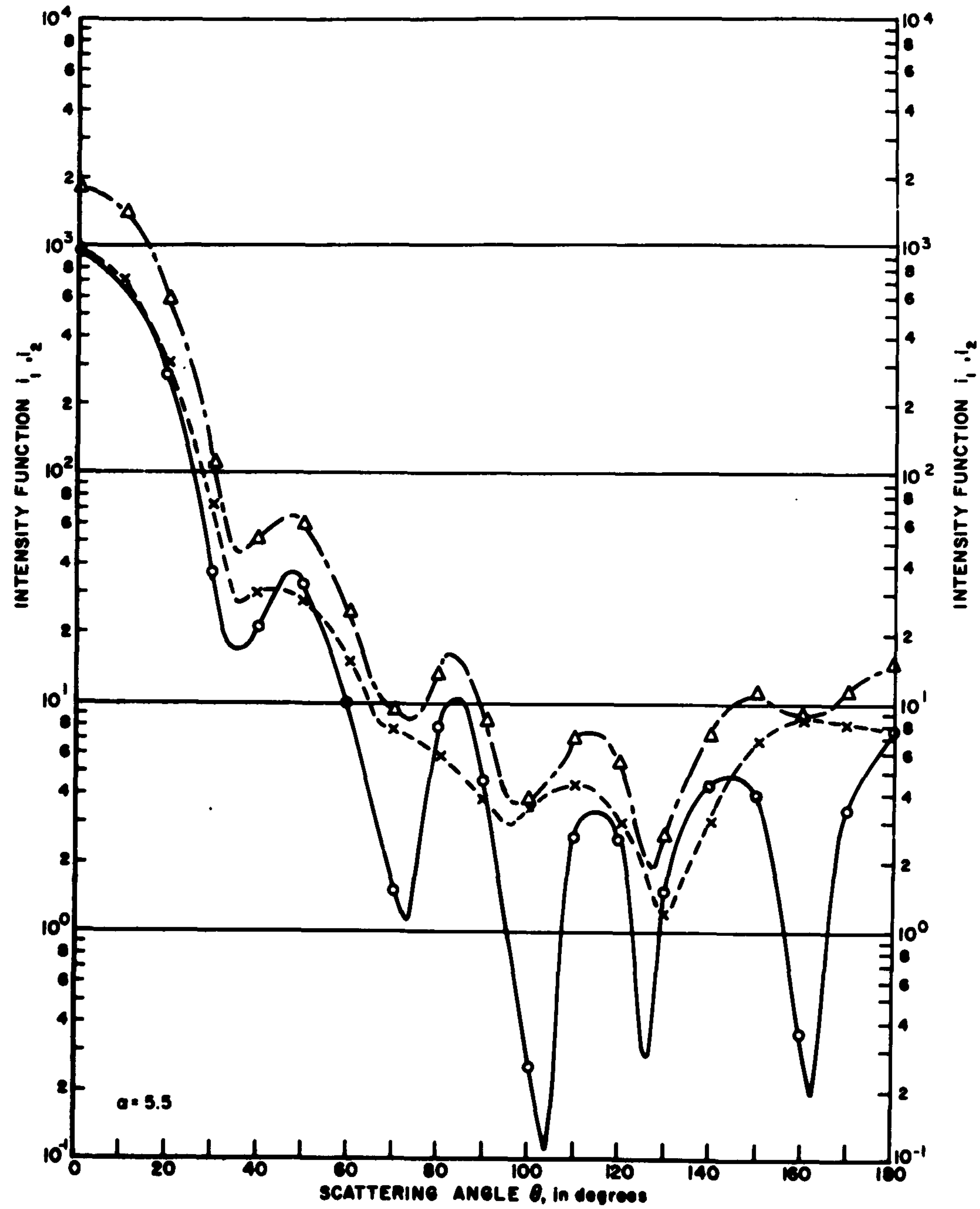


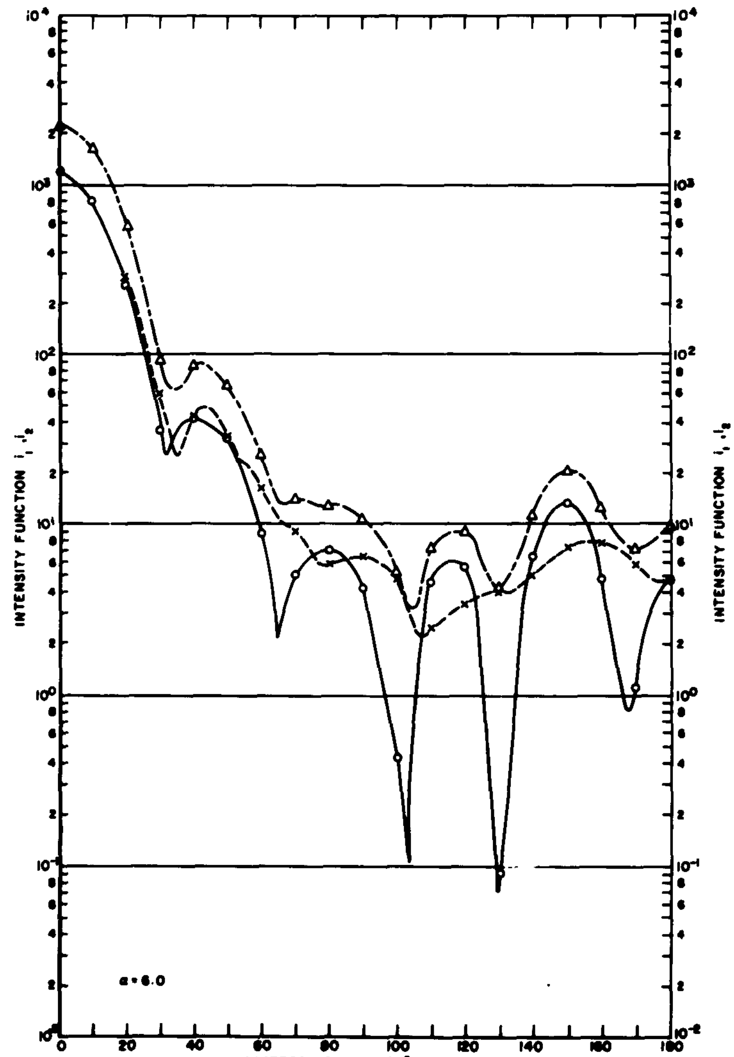
INTENSITY FUNCTION i_1, i_2
$\alpha = 4.0$
SCATTERING ANGLE θ, in degrees

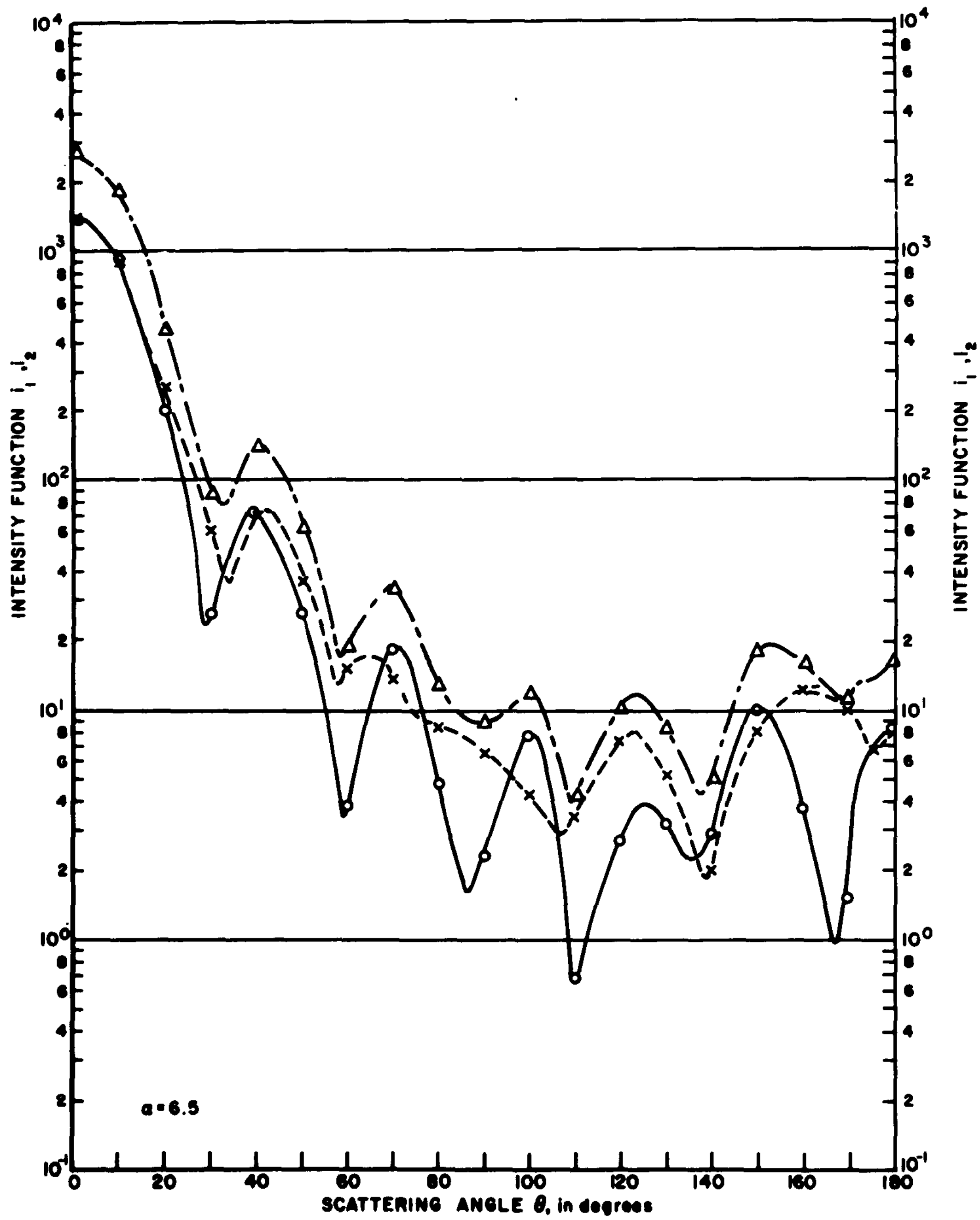

INTENSITY FUNCTION i_1, i_2
10^2
10^1
10^0
10^{-1}
α =4.5
SCATTERING ANGLE θ, in degrees



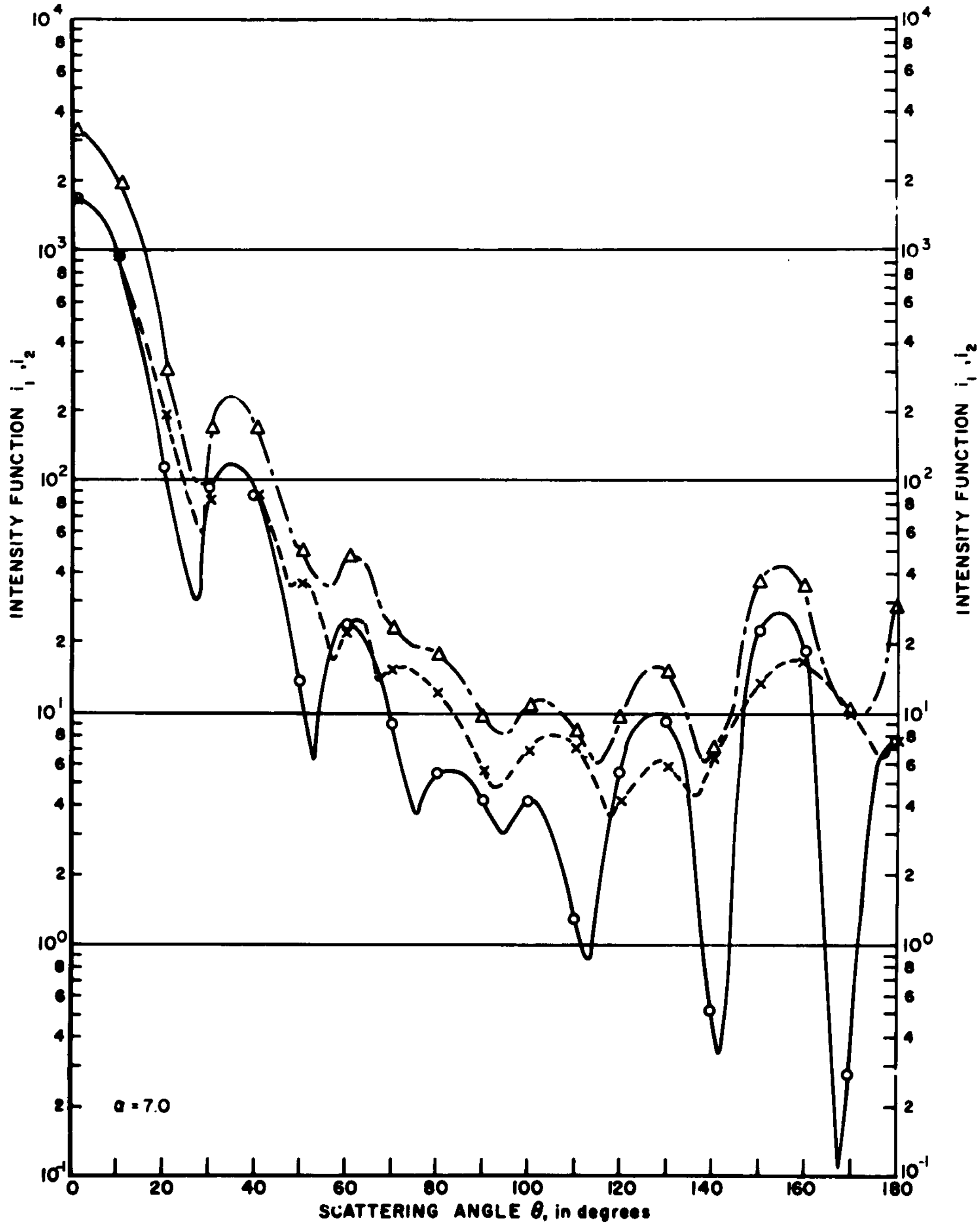
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
α = 5.0


INTENSITY FUNCTION $i_1 \cdot i_2$
SCATTERING ANGLE θ, in degrees
$\alpha = 5.5$

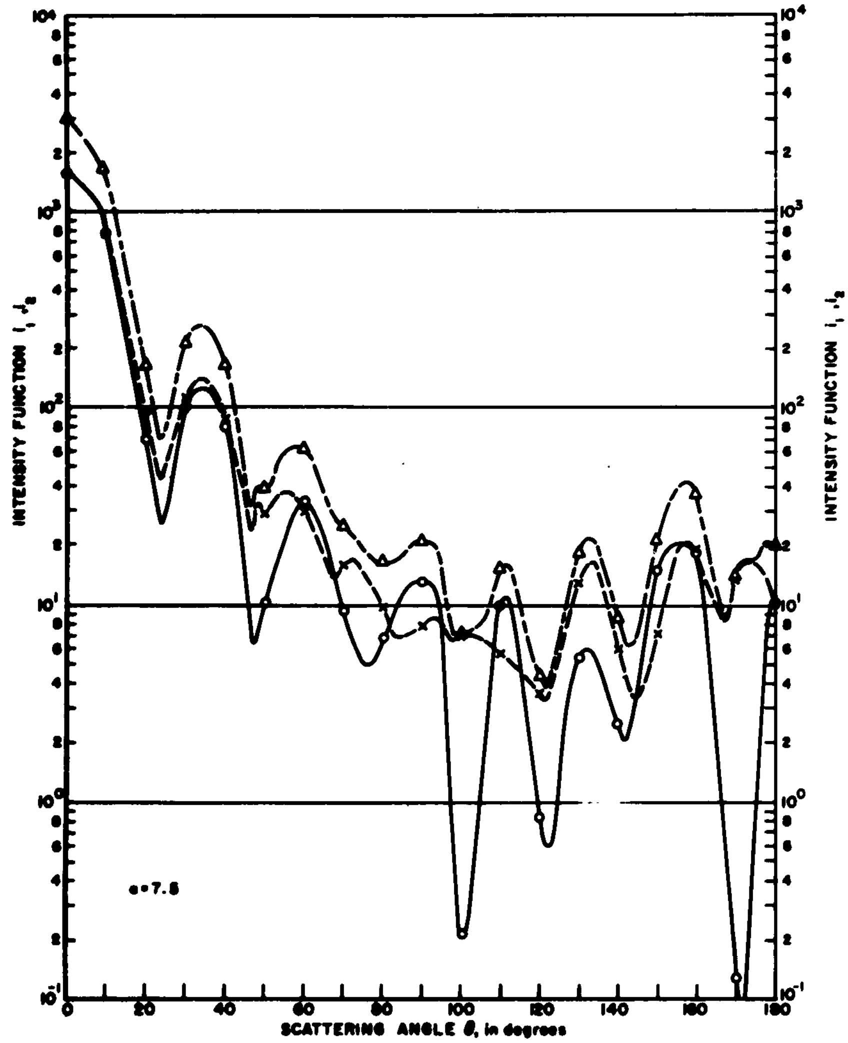

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 6.0$

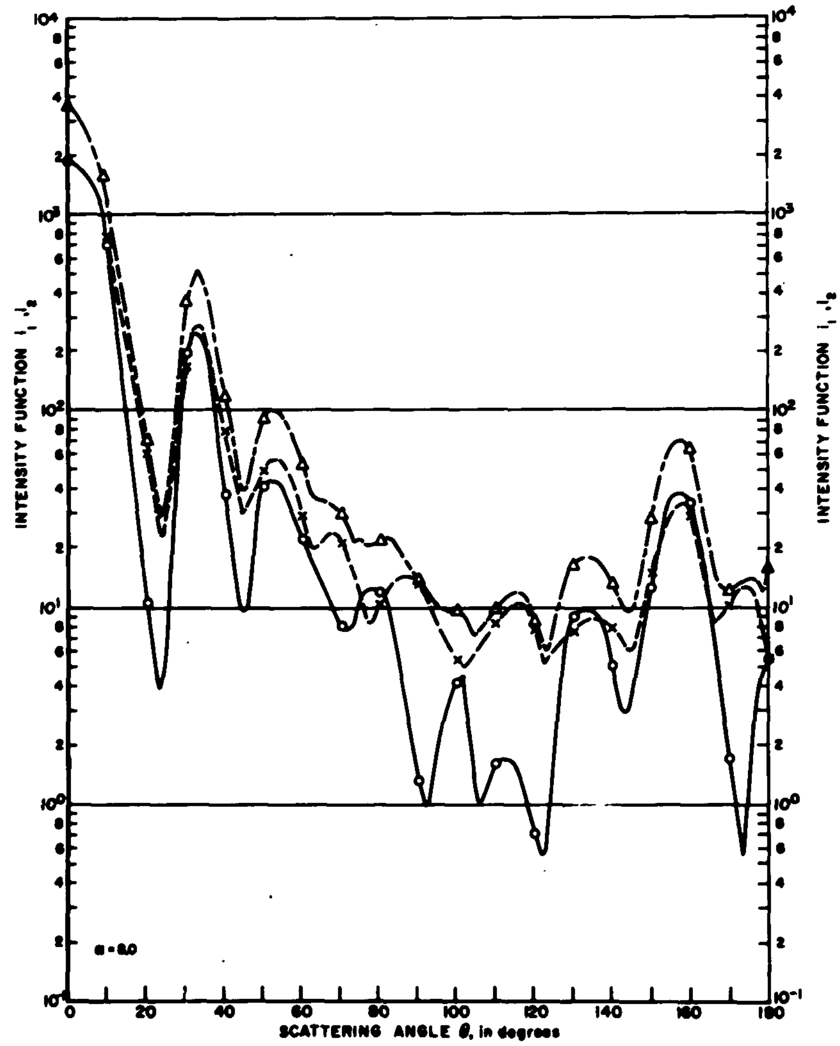


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 6.5$

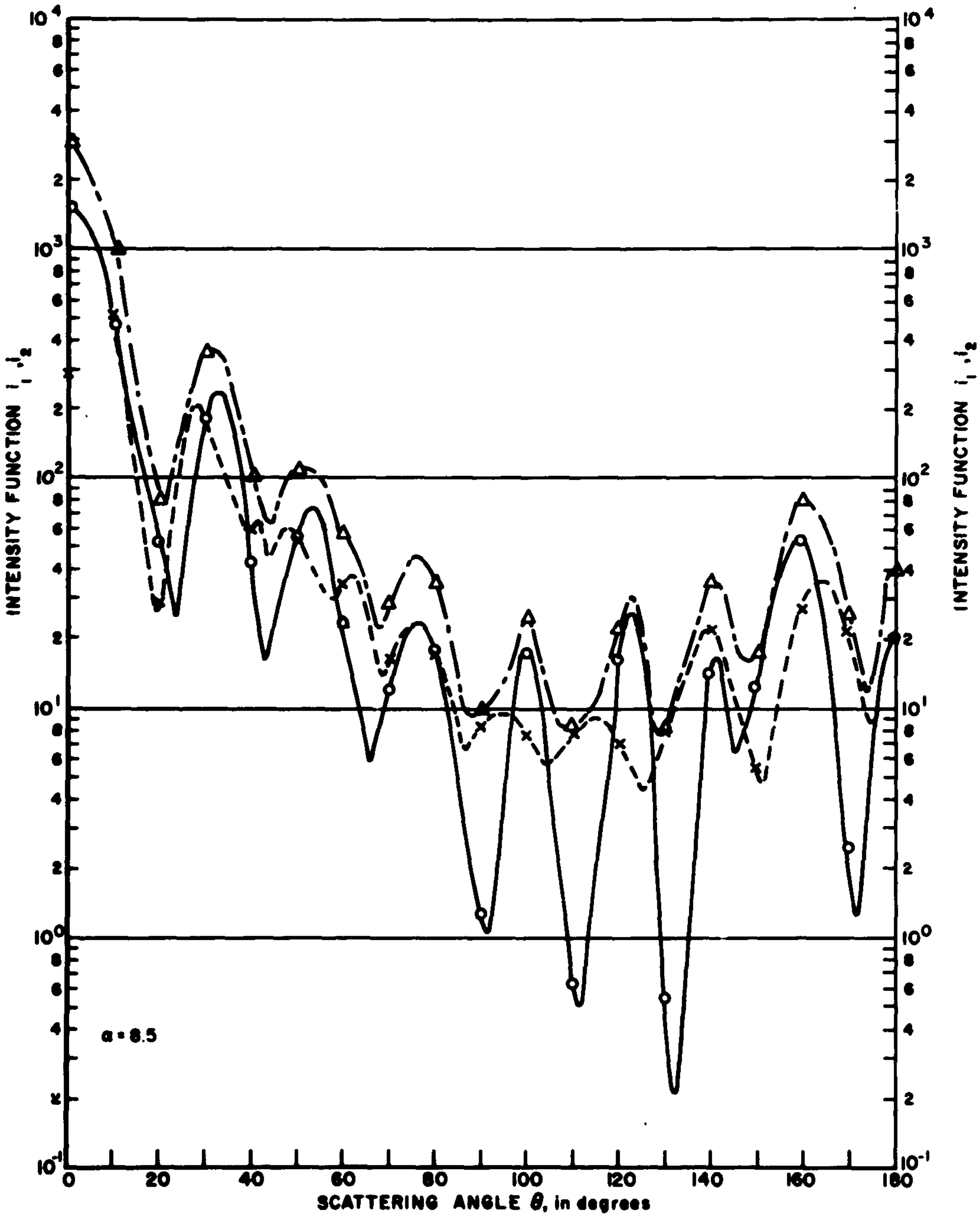


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 7.0$

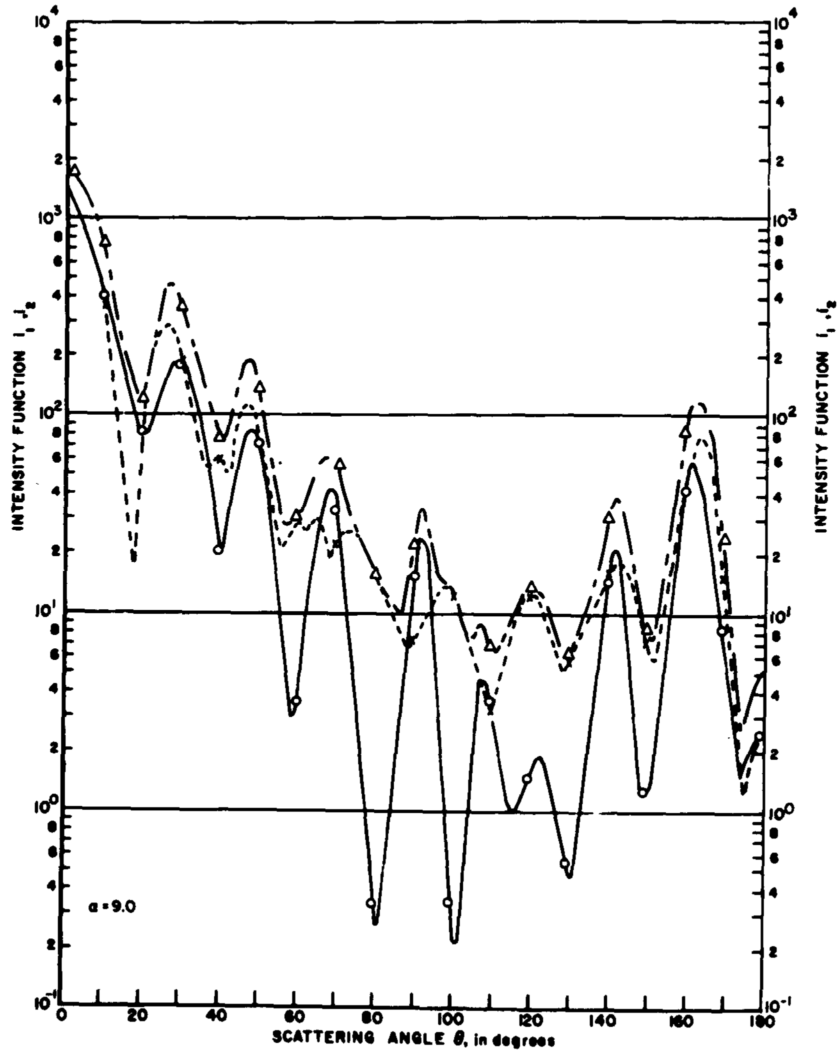

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 7.5$

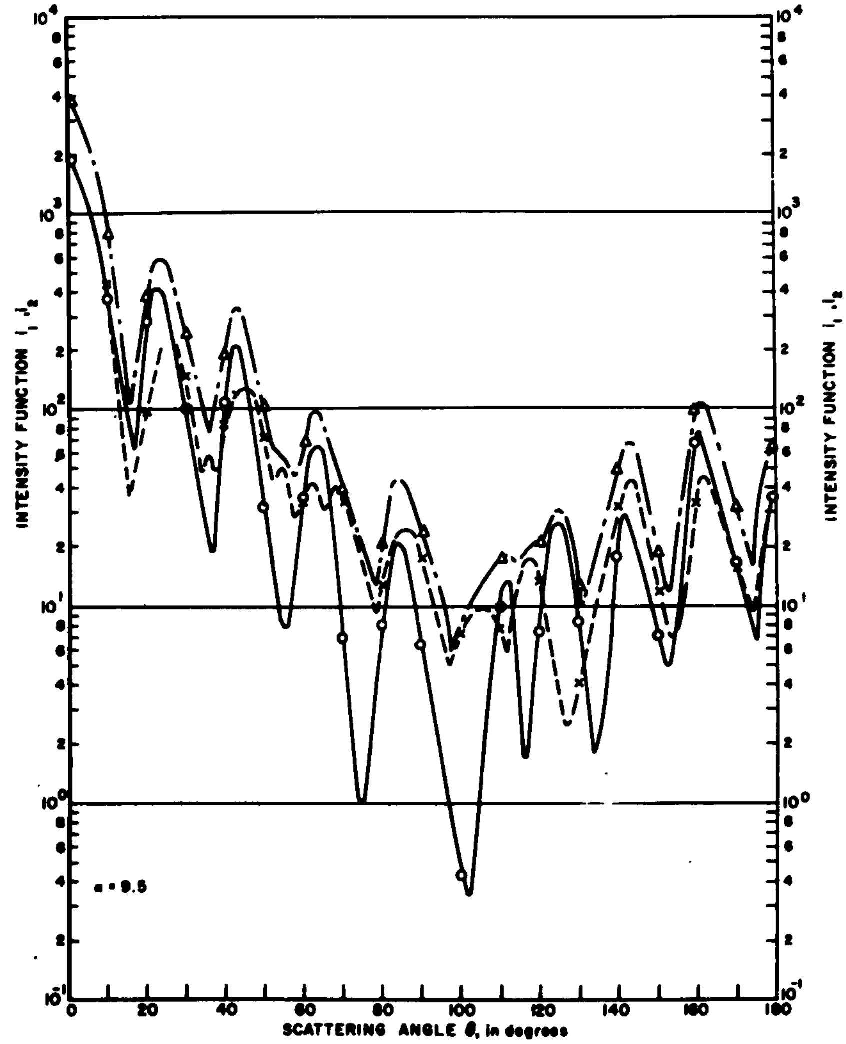


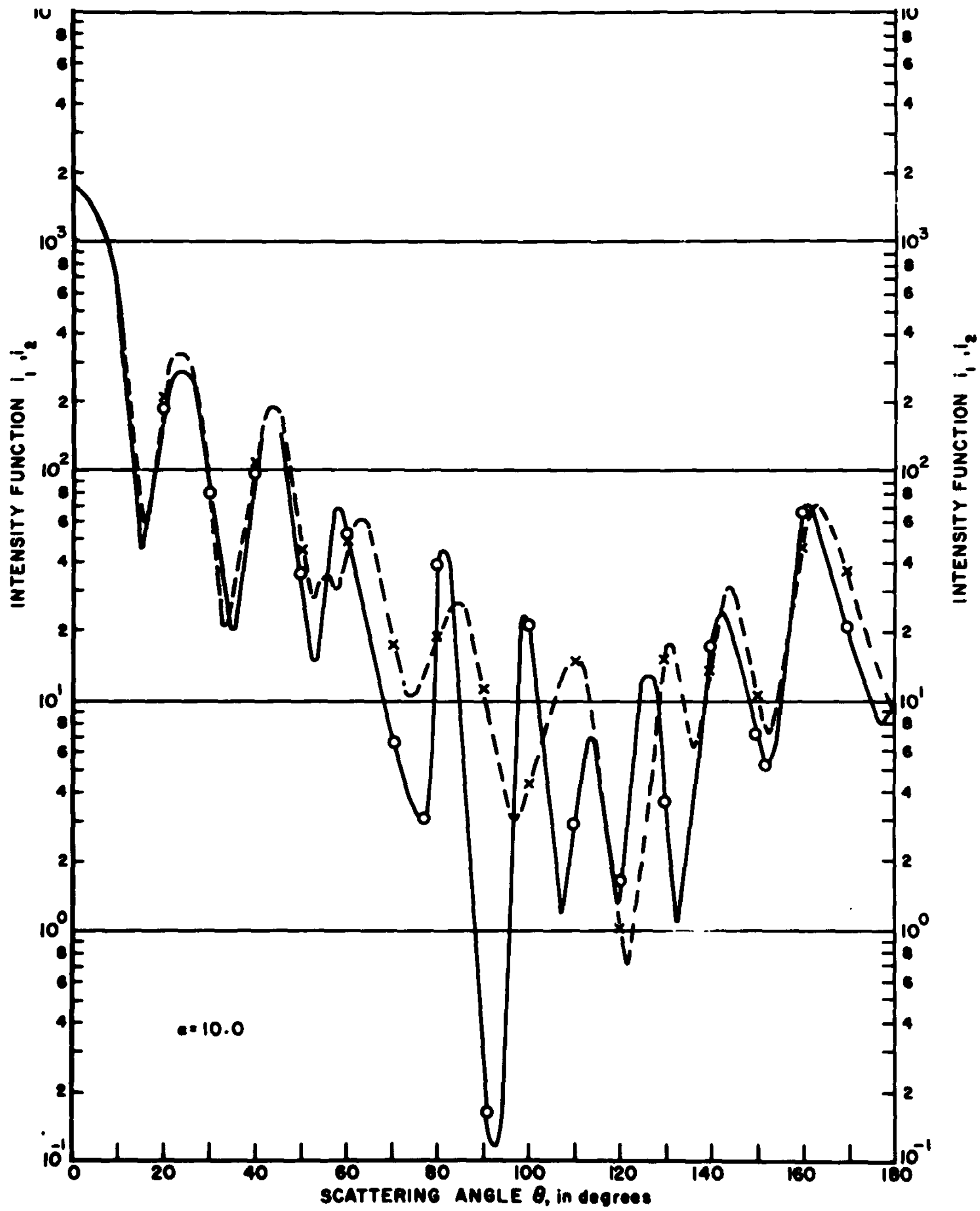
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees



INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
α = 8.5

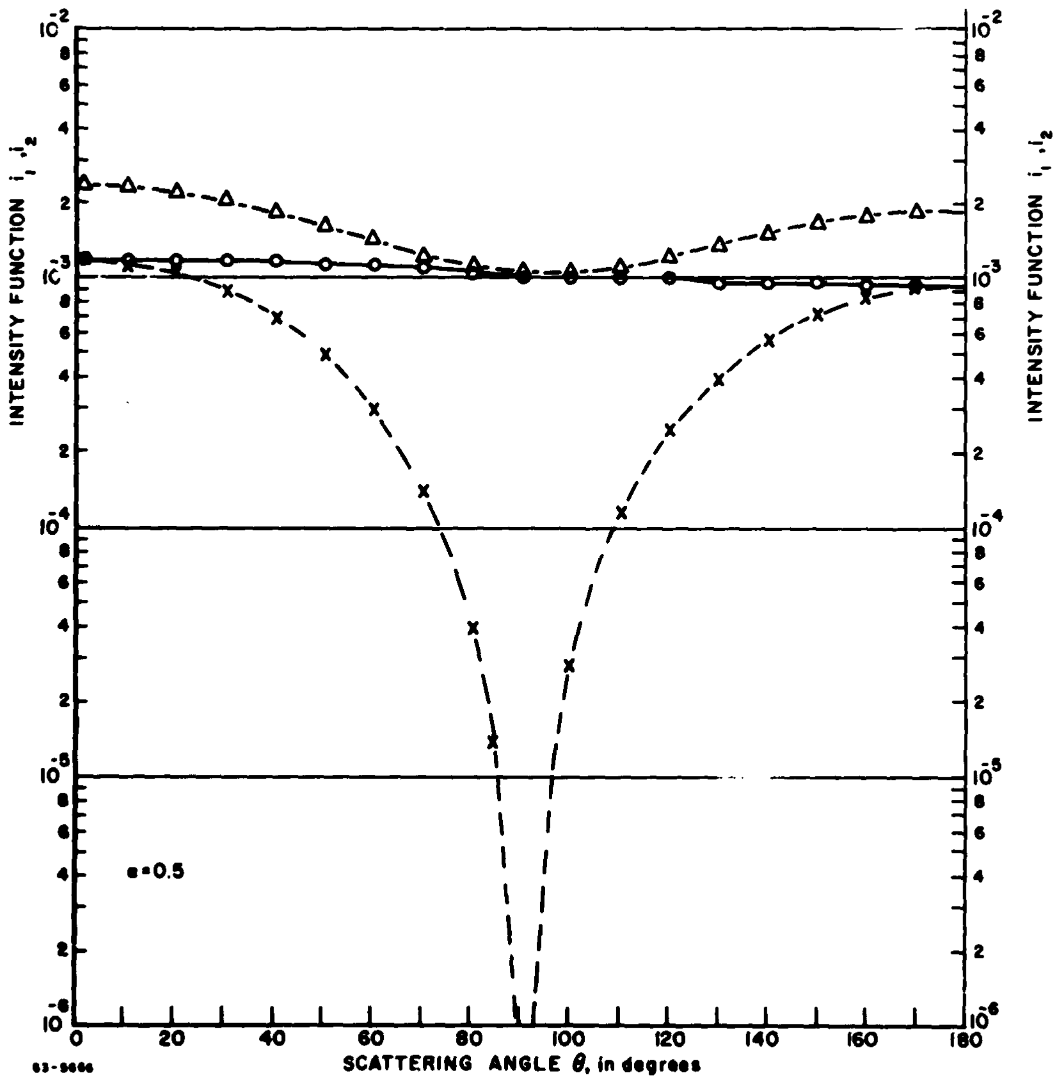

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 9.0$

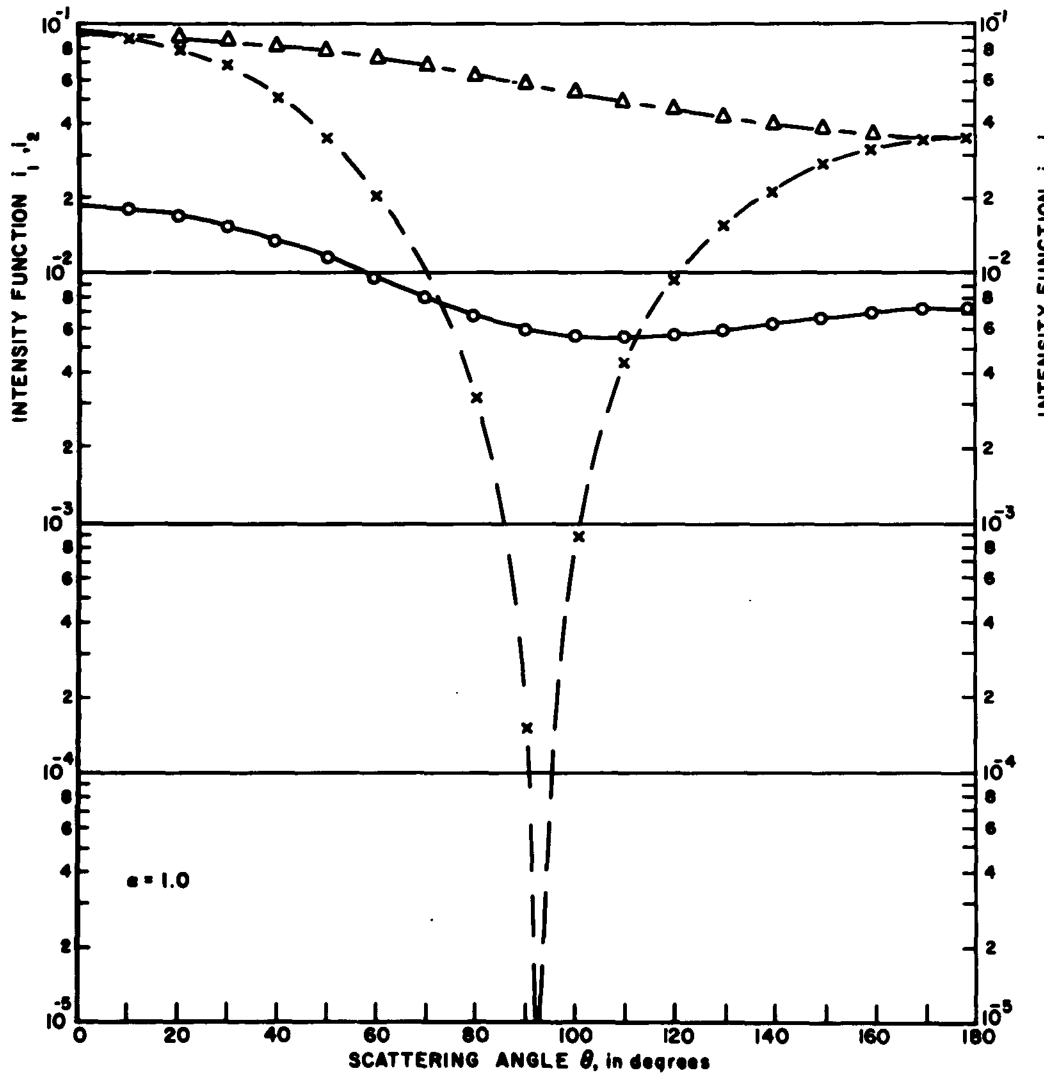

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
α = 9.5


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 10.0$

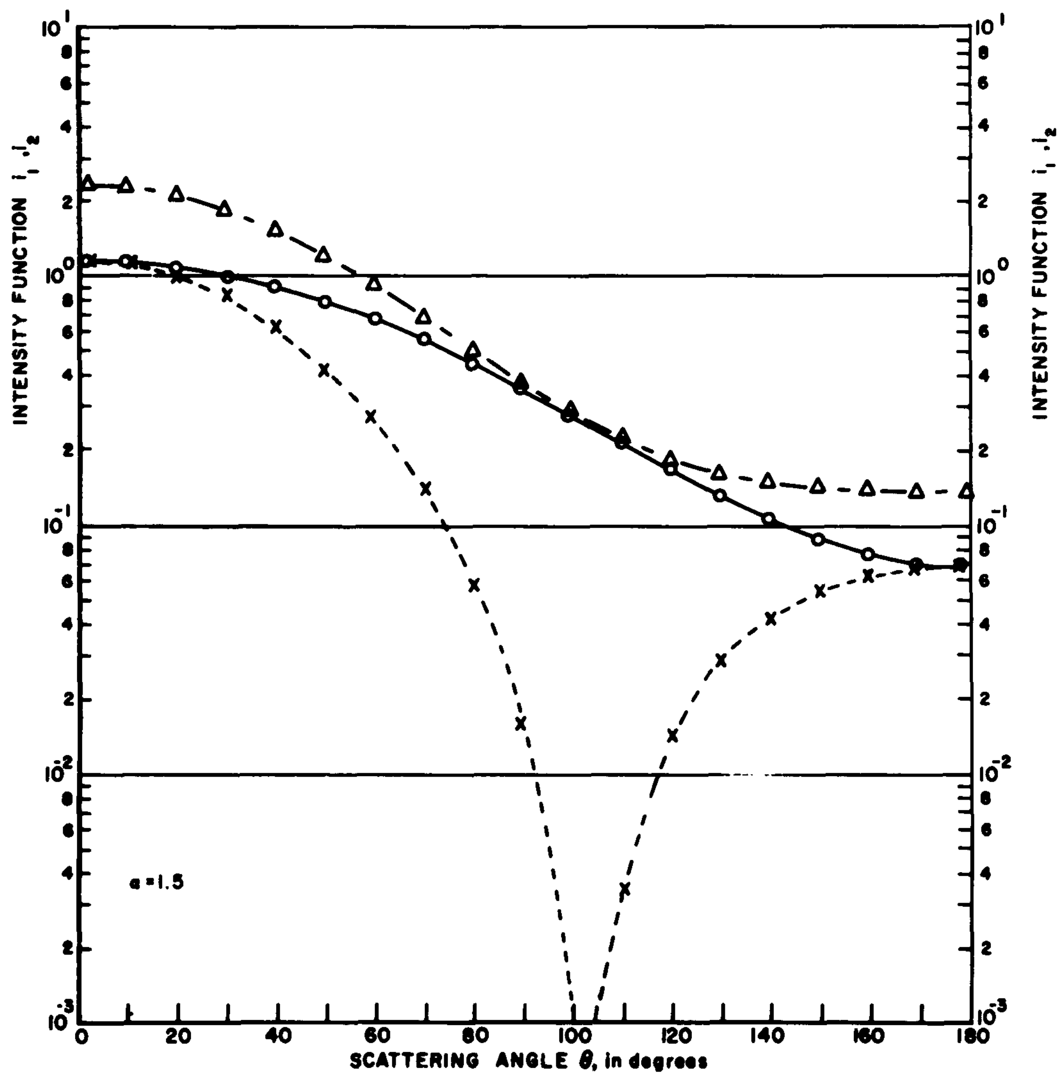
6.25 Atlas of scattering diagrams

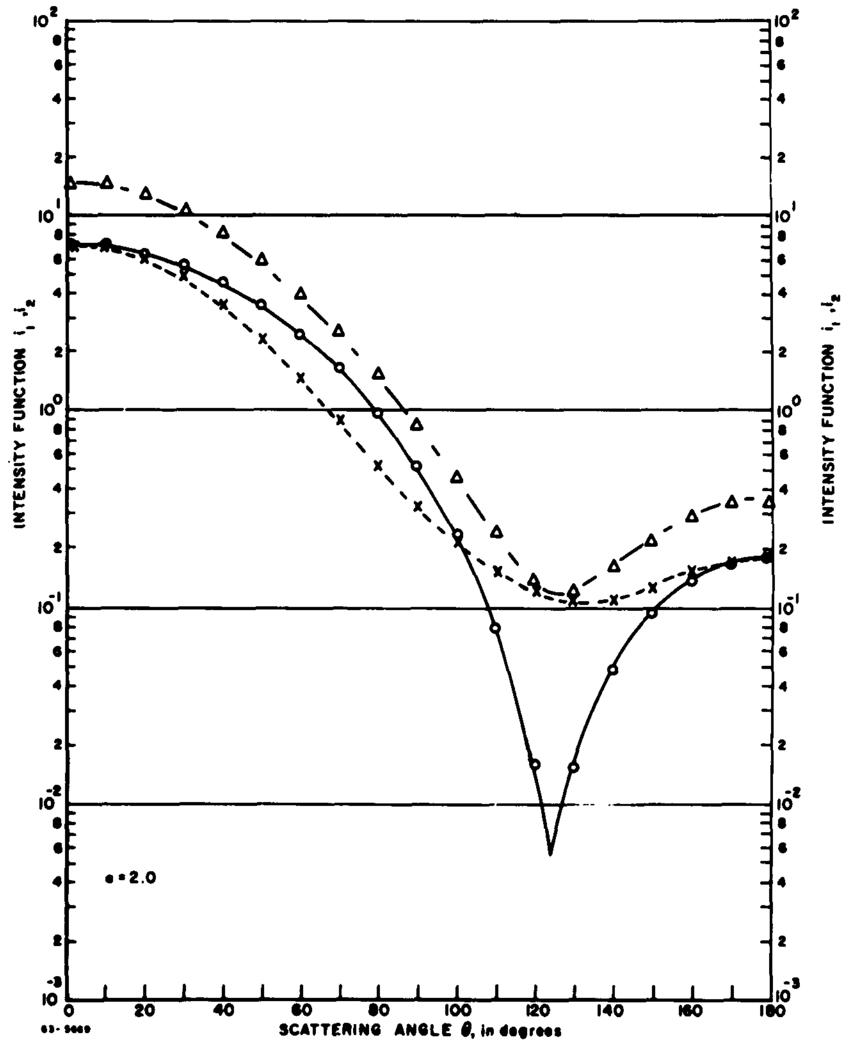
for n = 1.44

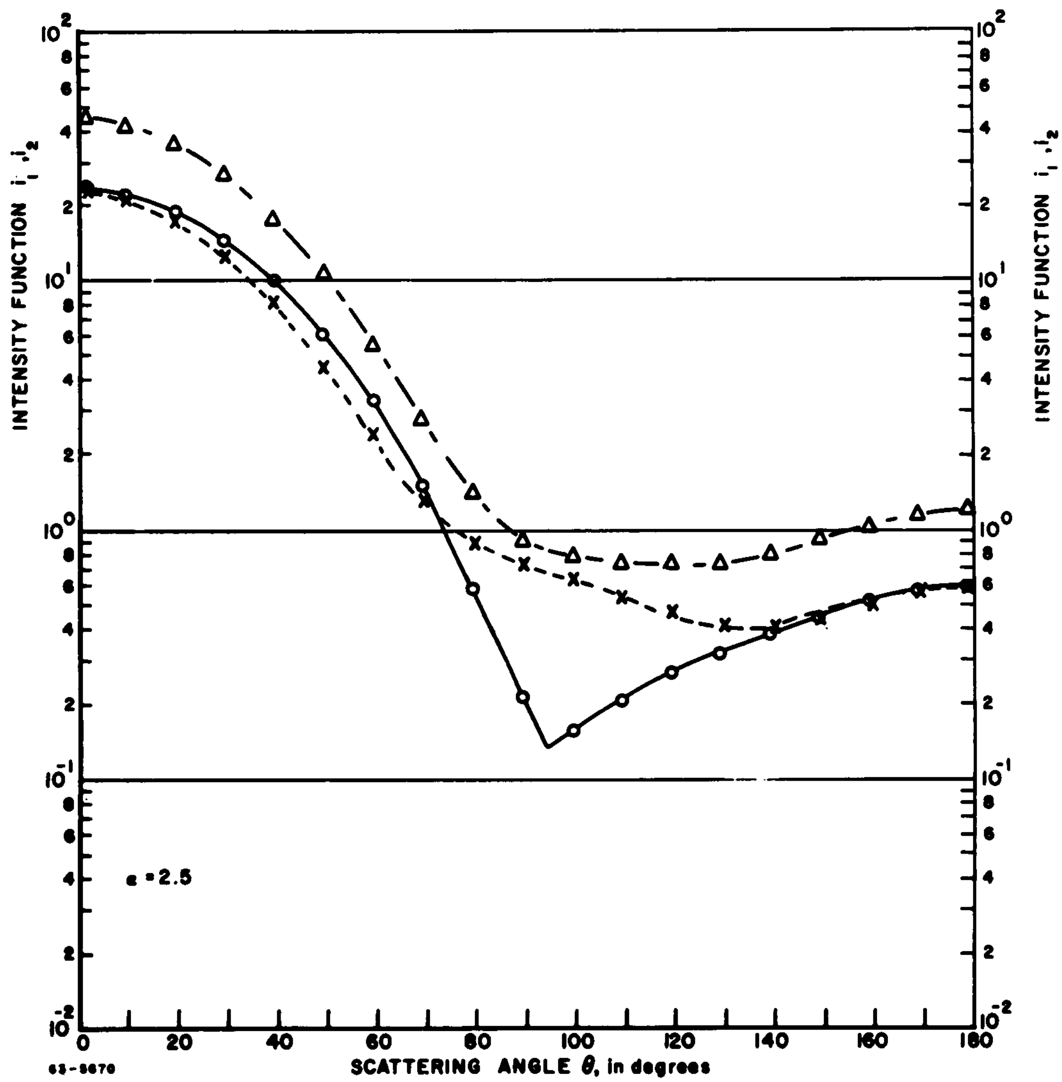

INTENSITY FUNCTION i_1, i_2
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
a = 0.5
SCATTERING ANGLE θ, in degrees
0 20 40 60 80 100 120 140 160 180



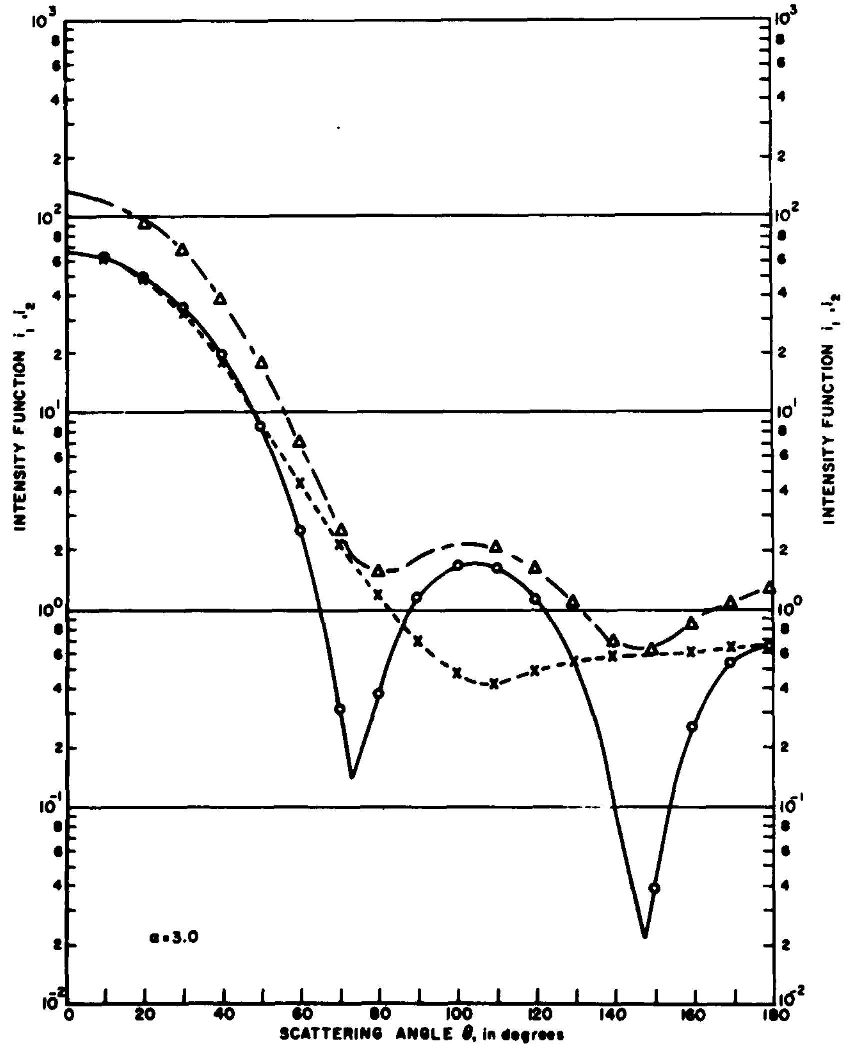
INTENSITY FUNCTION i_1, i_2
$\alpha = 1.0$
SCATTERING ANGLE θ, in degrees

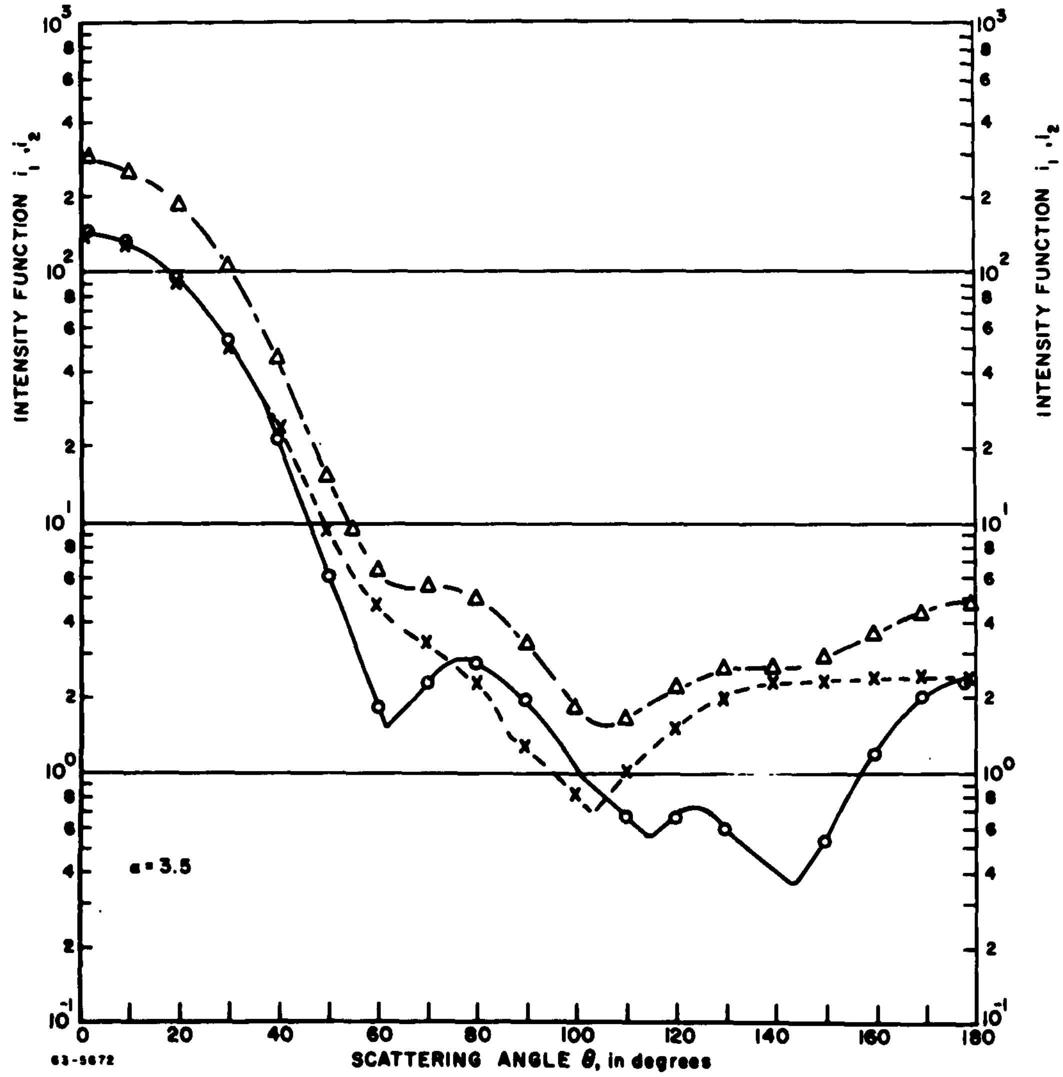

INTENSITY FUNCTION i_1, i_2
$\alpha = 1.5$
SCATTERING ANGLE θ, in degrees

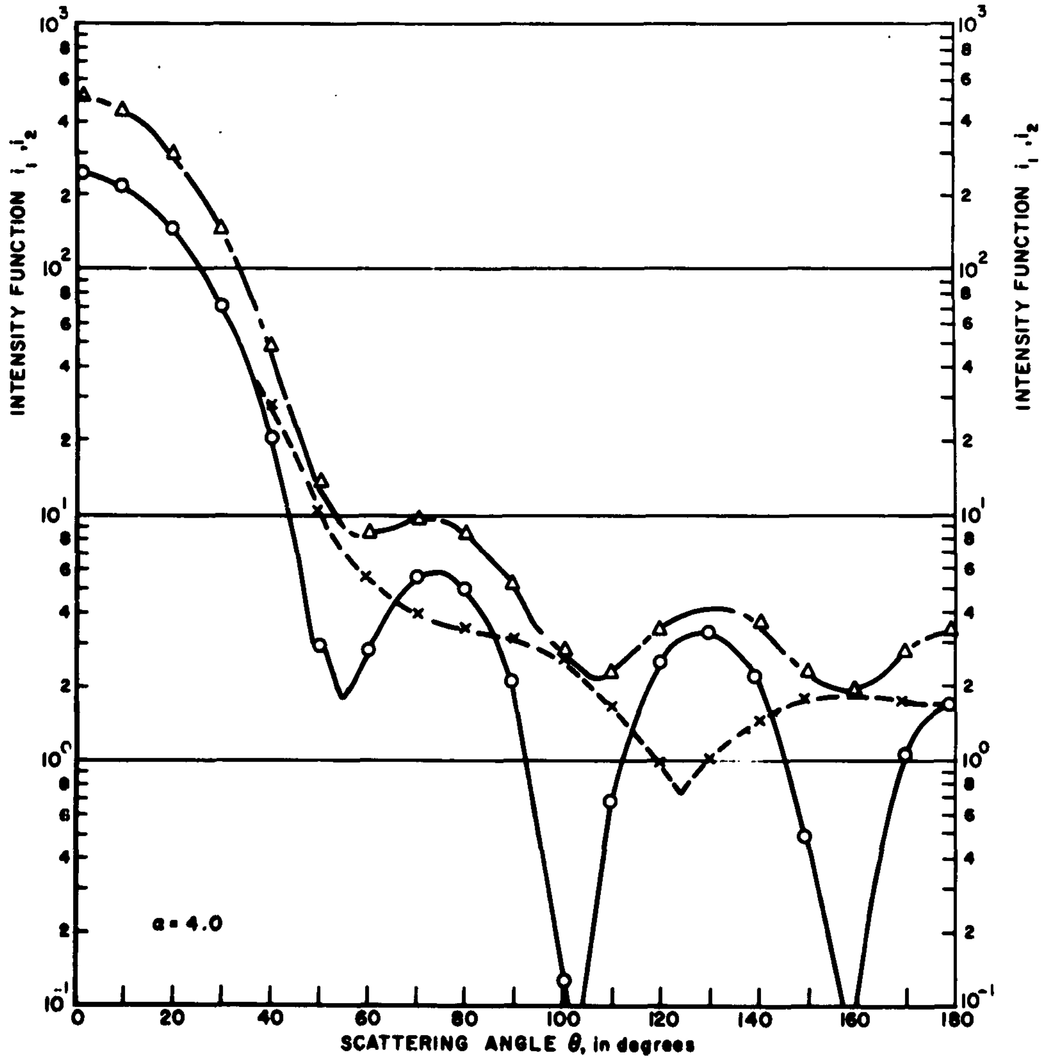

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 2.0$



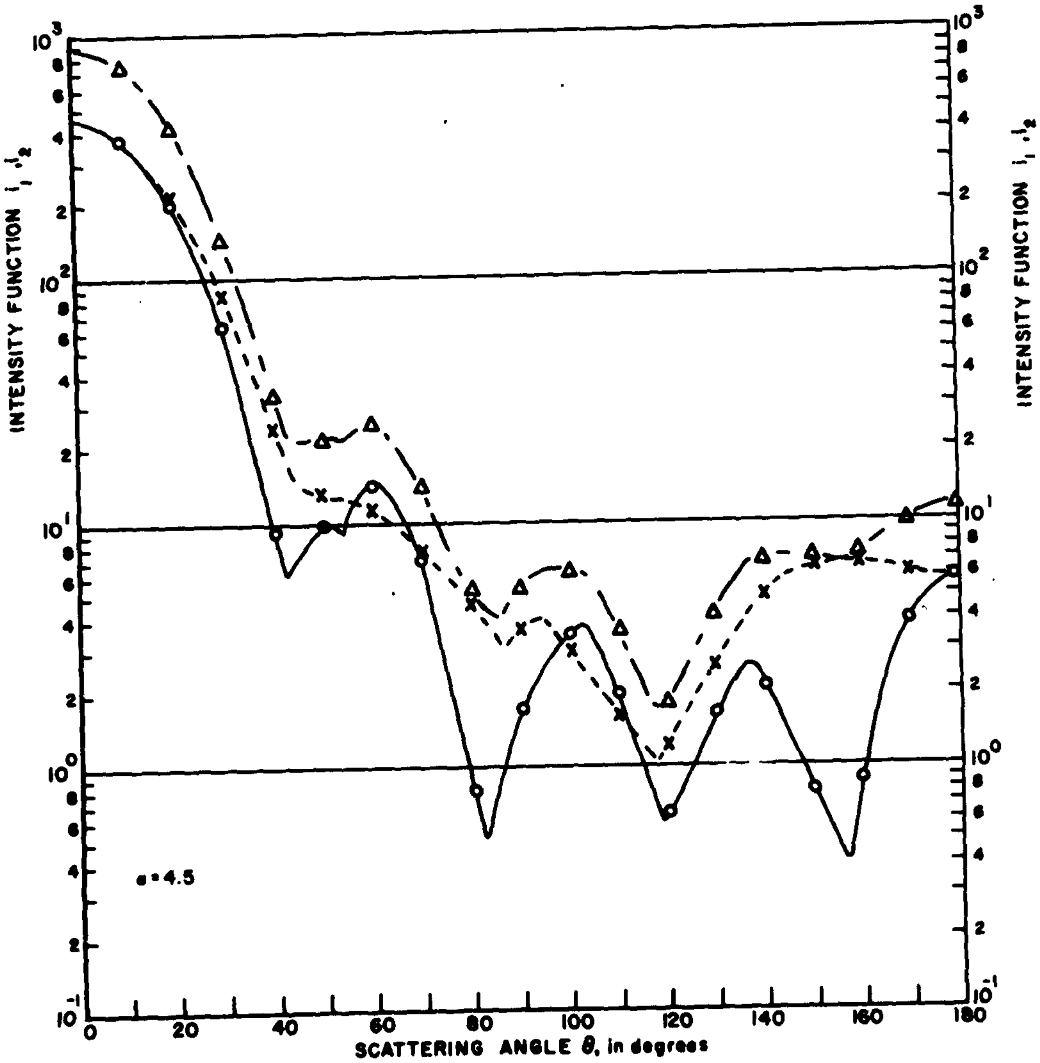
INTENSITY FUNCTION i_1, i_2
10^2
10^1
10^0
10^{-1}
10^{-2}
$\alpha = 2.5$
SCATTERING ANGLE θ, in degrees
63-5670


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 3.0$

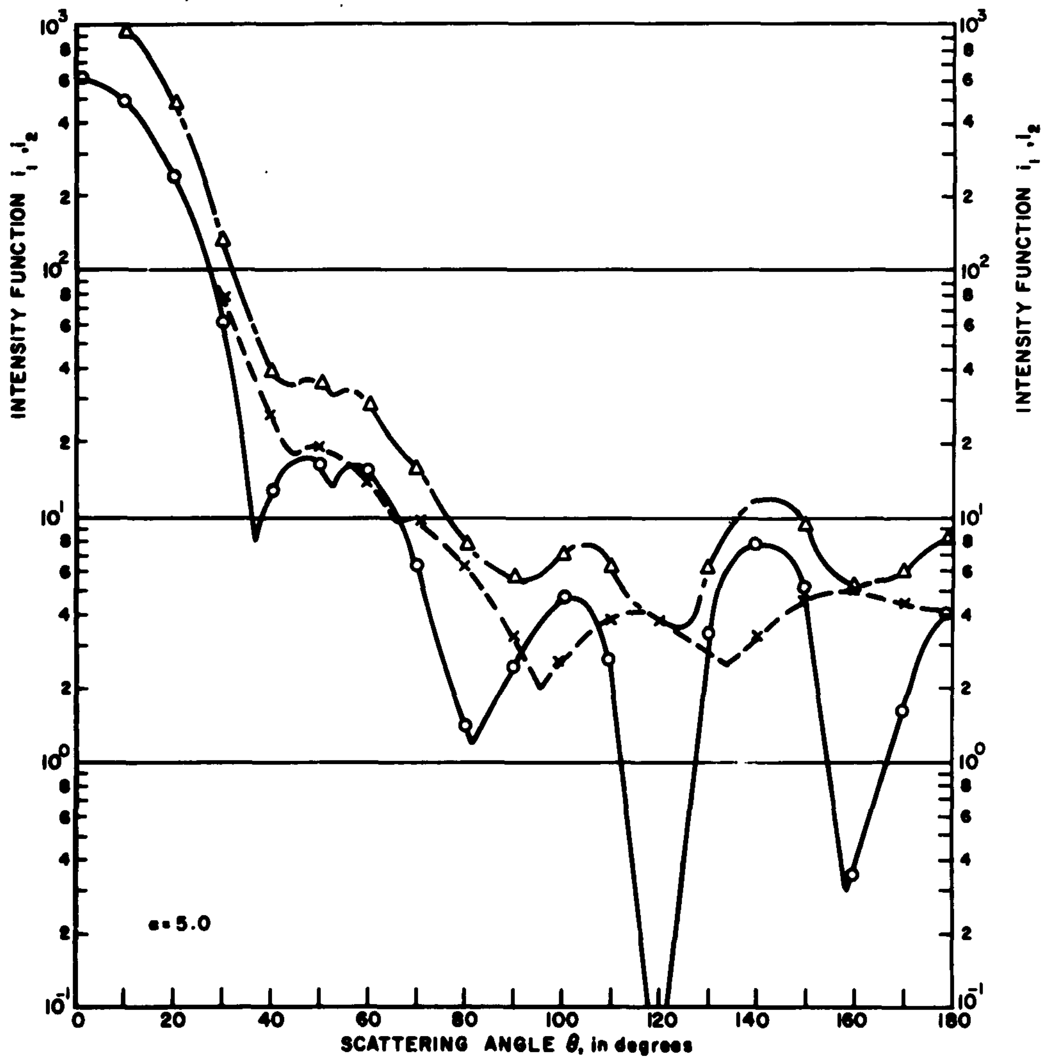

INTENSITY FUNCTION i_1, i_2
α = 3.5
63-5672
SCATTERING ANGLE θ, in degrees



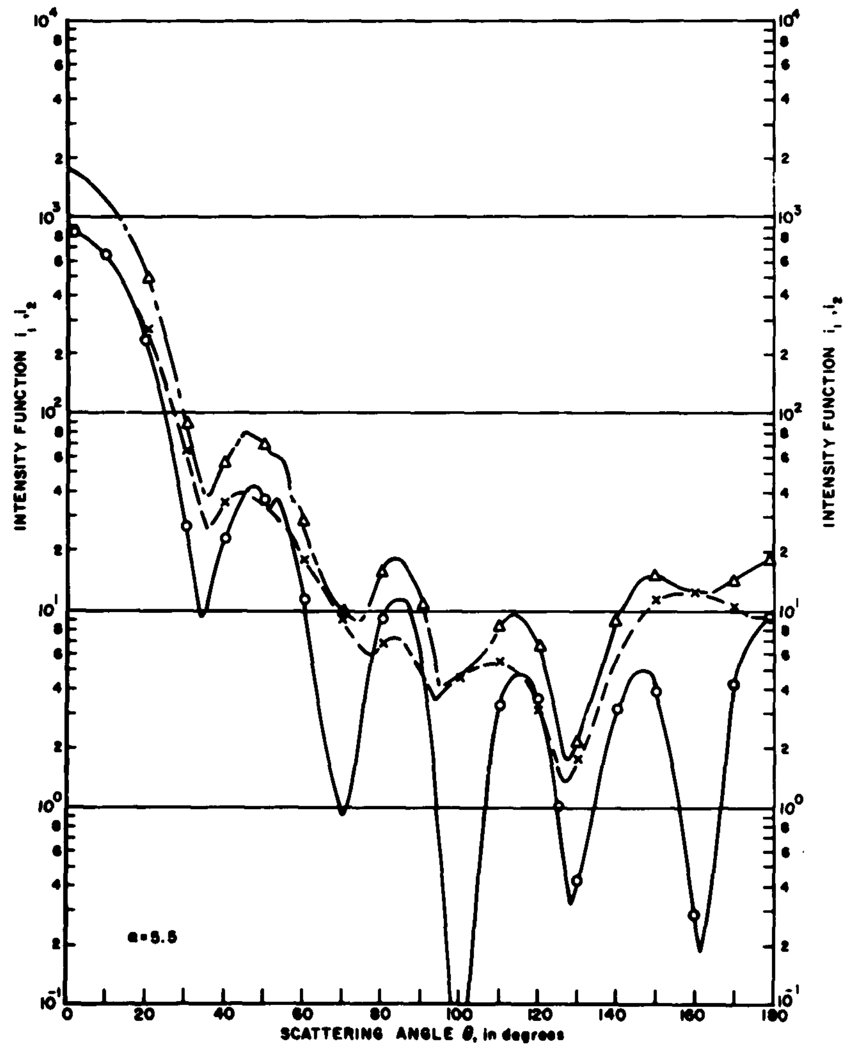
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 4.0$

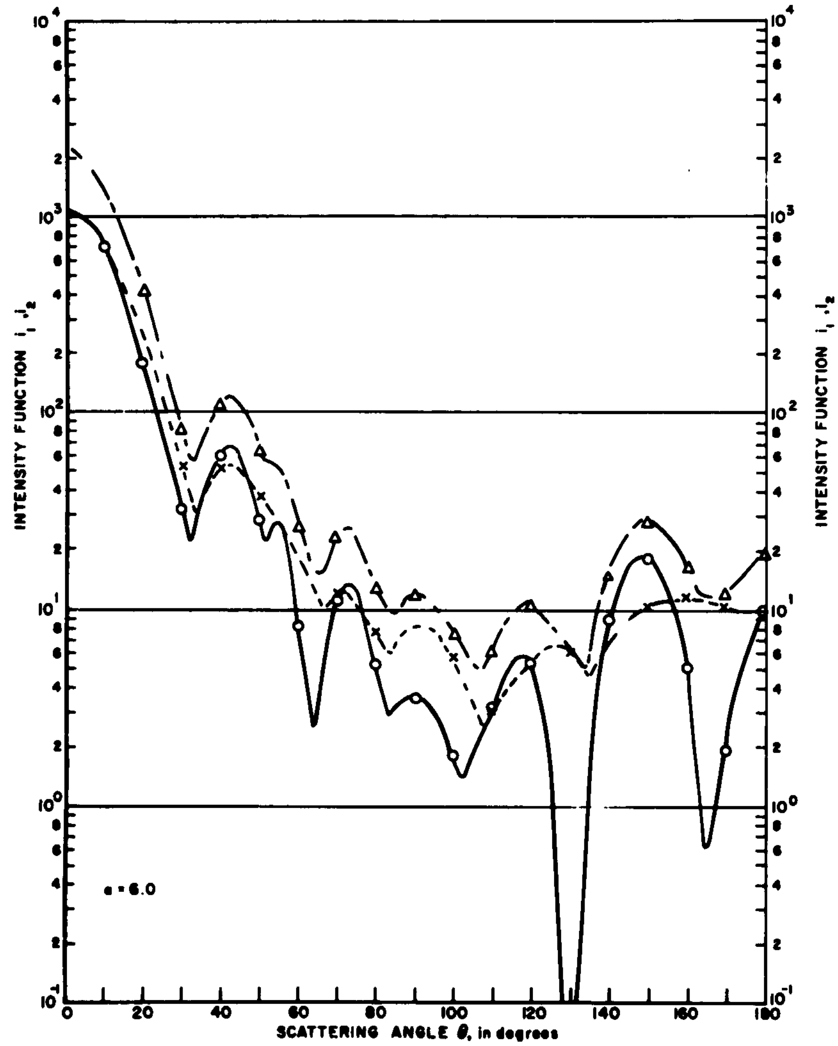


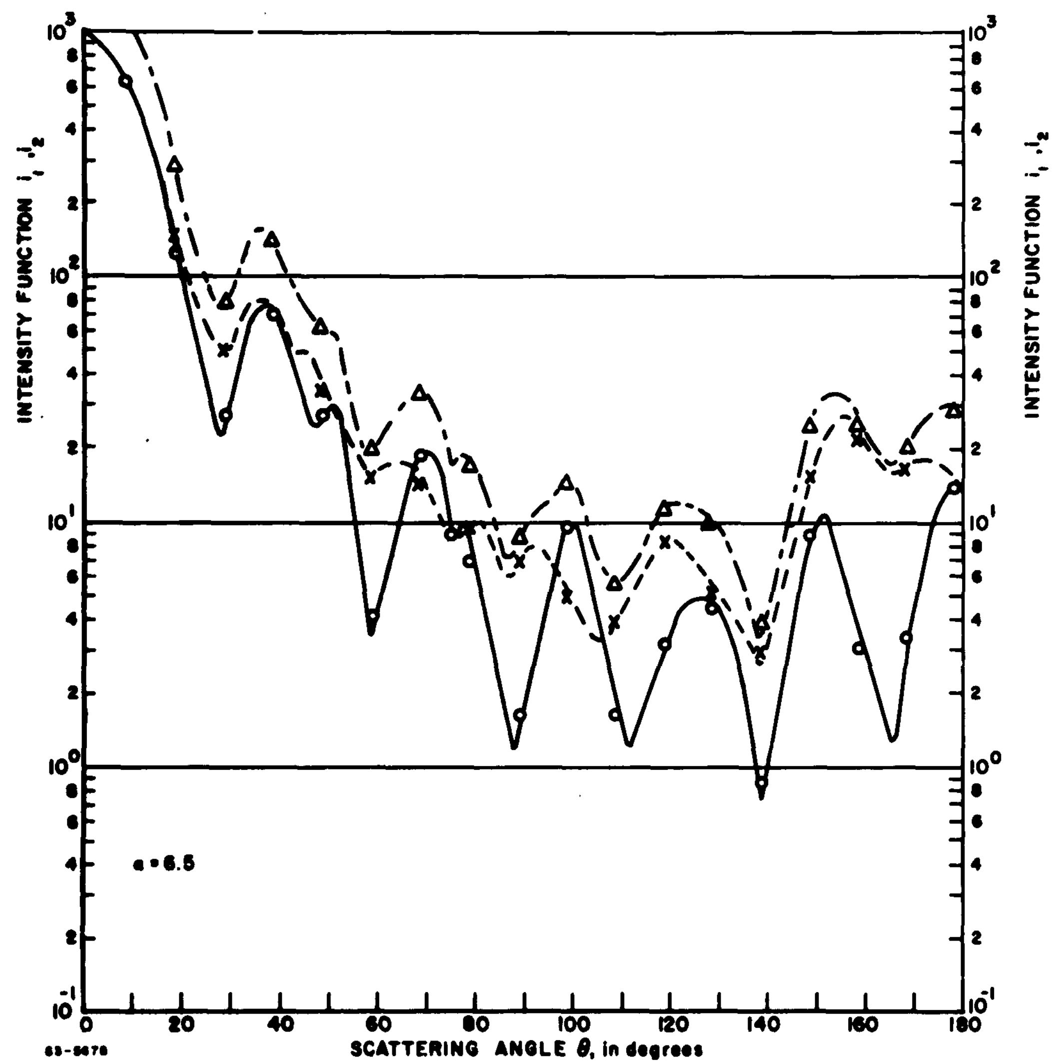
INTENSITY FUNCTION i_1, i_2
$\alpha = 4.5$
SCATTERING ANGLE θ, in degrees

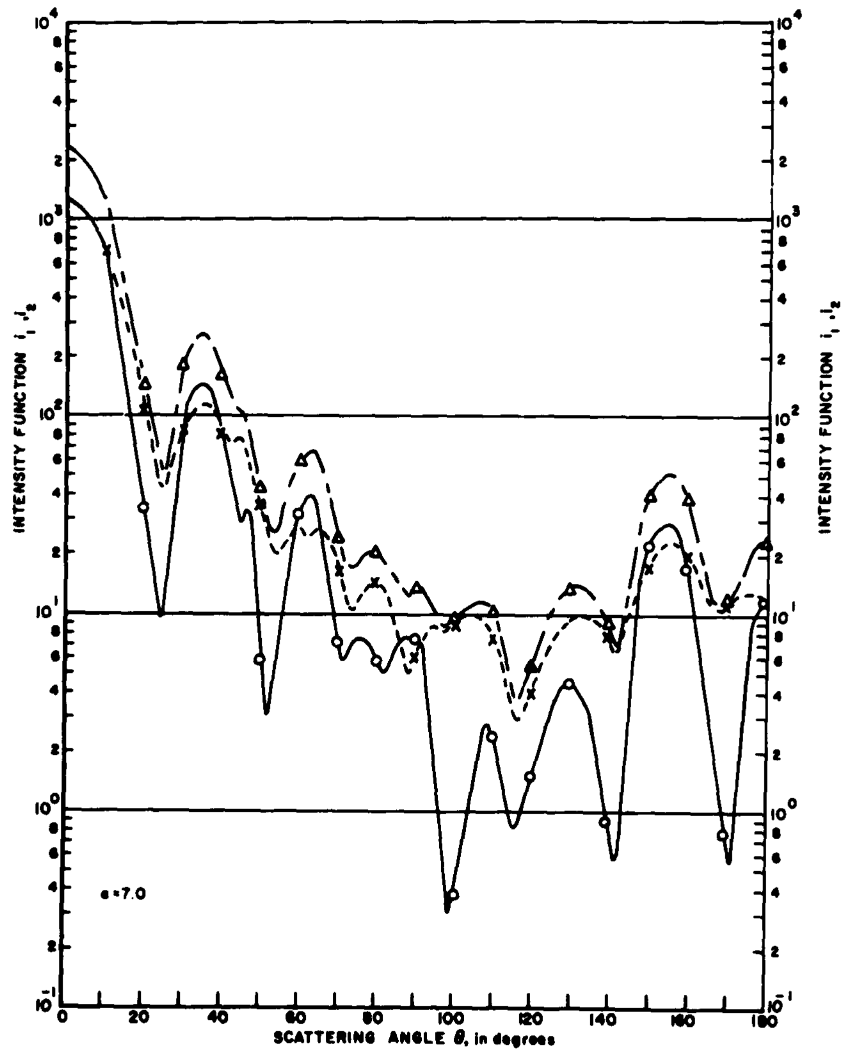


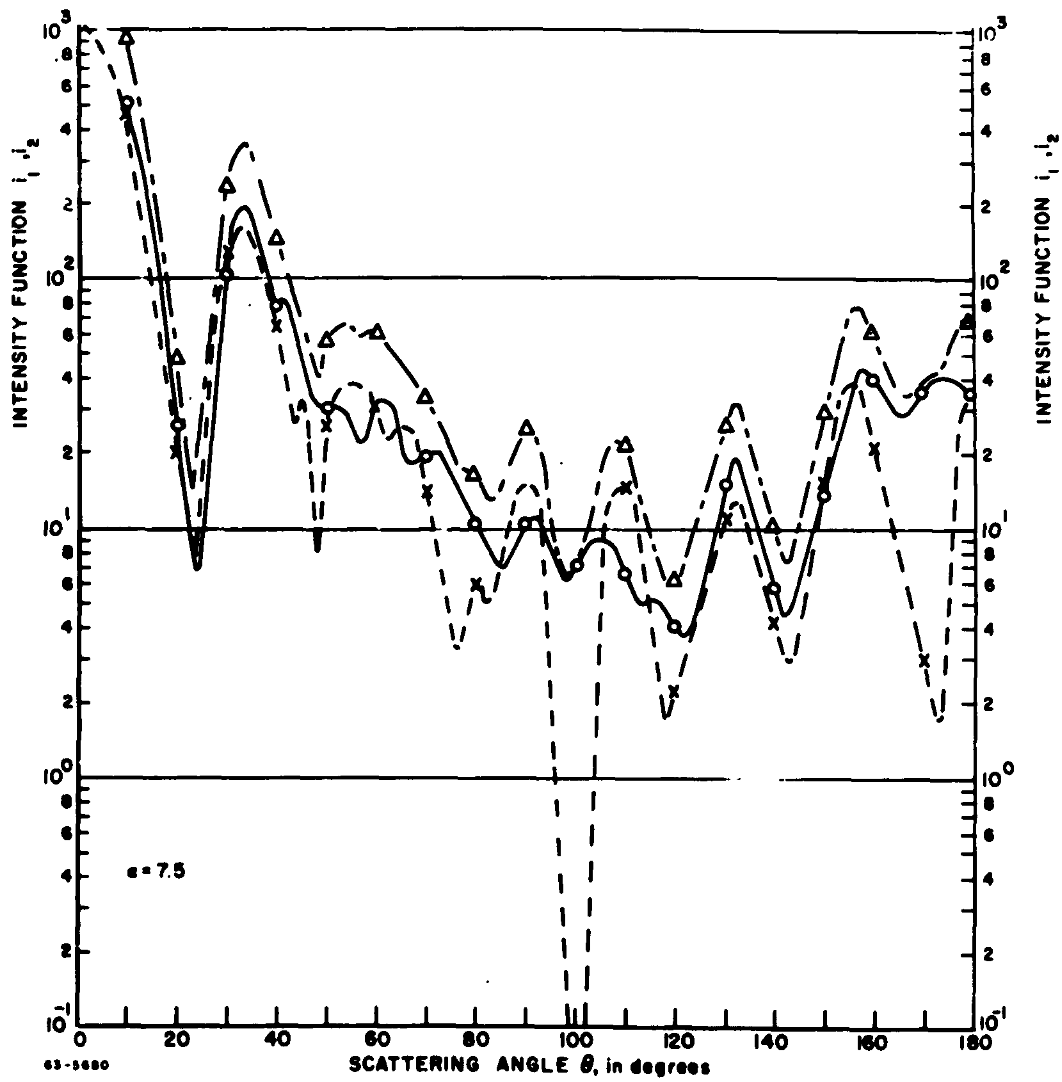
10^3
8
6
4
2
10^2
8
6
4
2
10^1
8
6
4
2
10^0
8
6
4
2
10^{-1}
INTENSITY FUNCTION $i_1 \cdot i_2$
$\alpha = 5.0$
0
20
40
60
80
100
120
140
160
180
SCATTERING ANGLE θ, in degrees


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
α=5.5

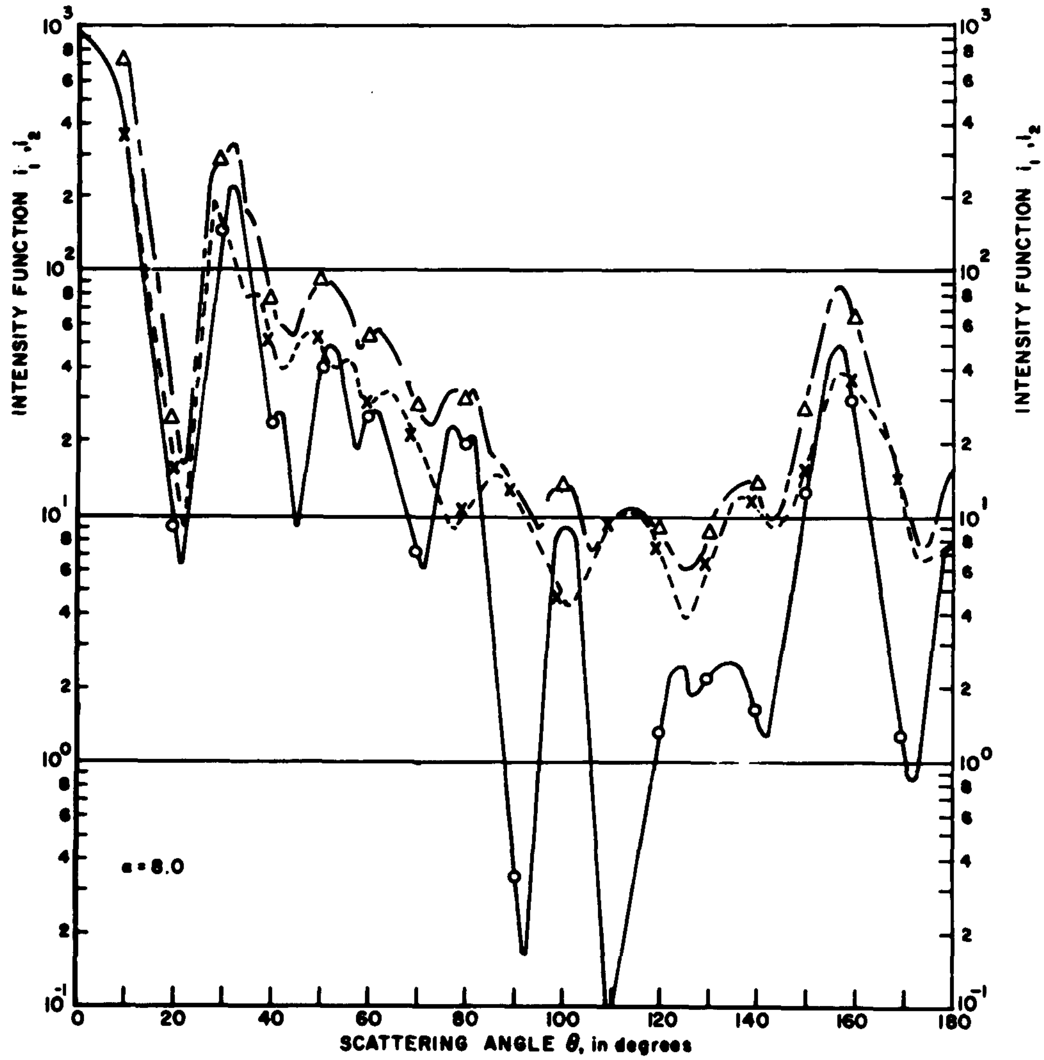

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
α = 6.0


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 6.5$
63-5678

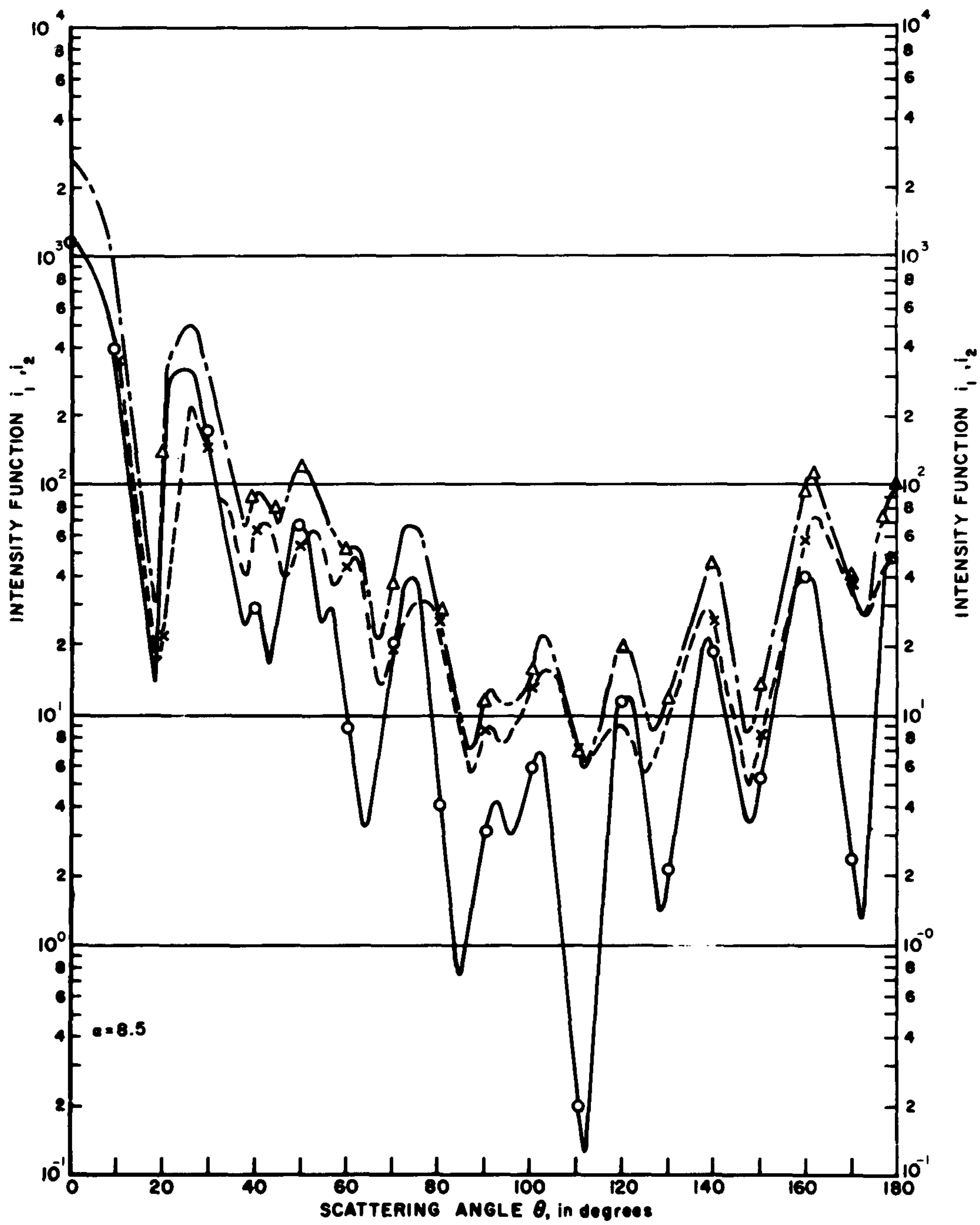

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 7.0$

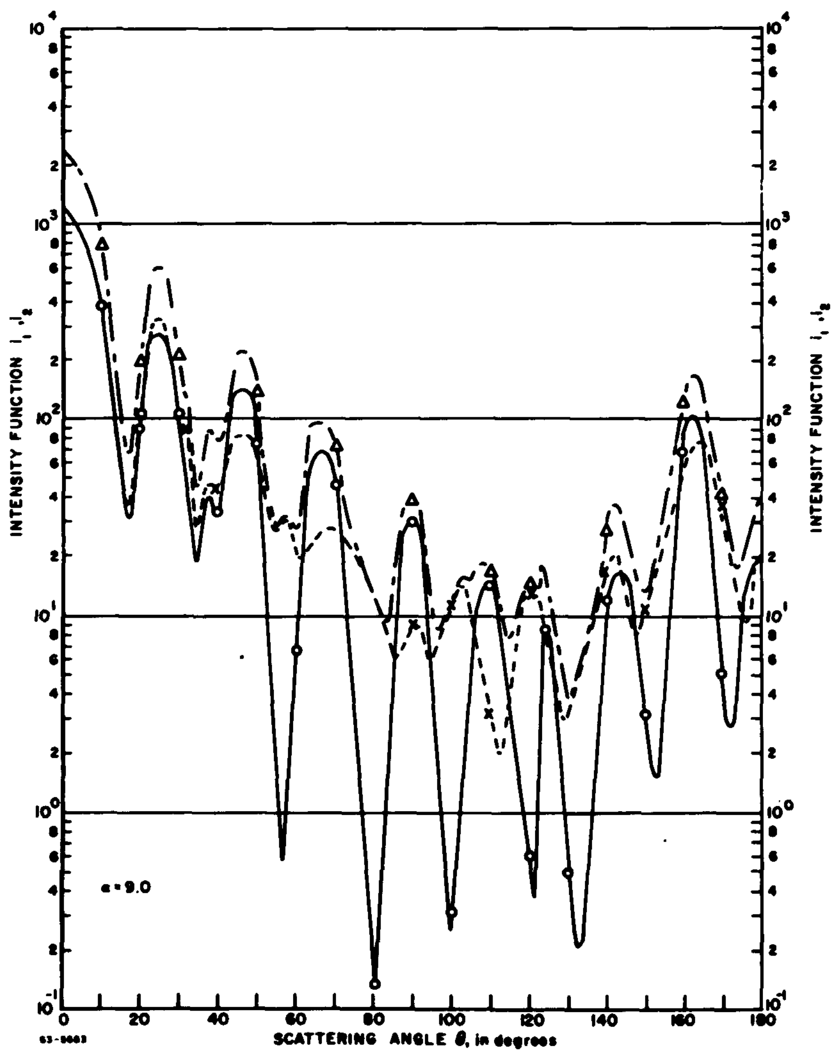


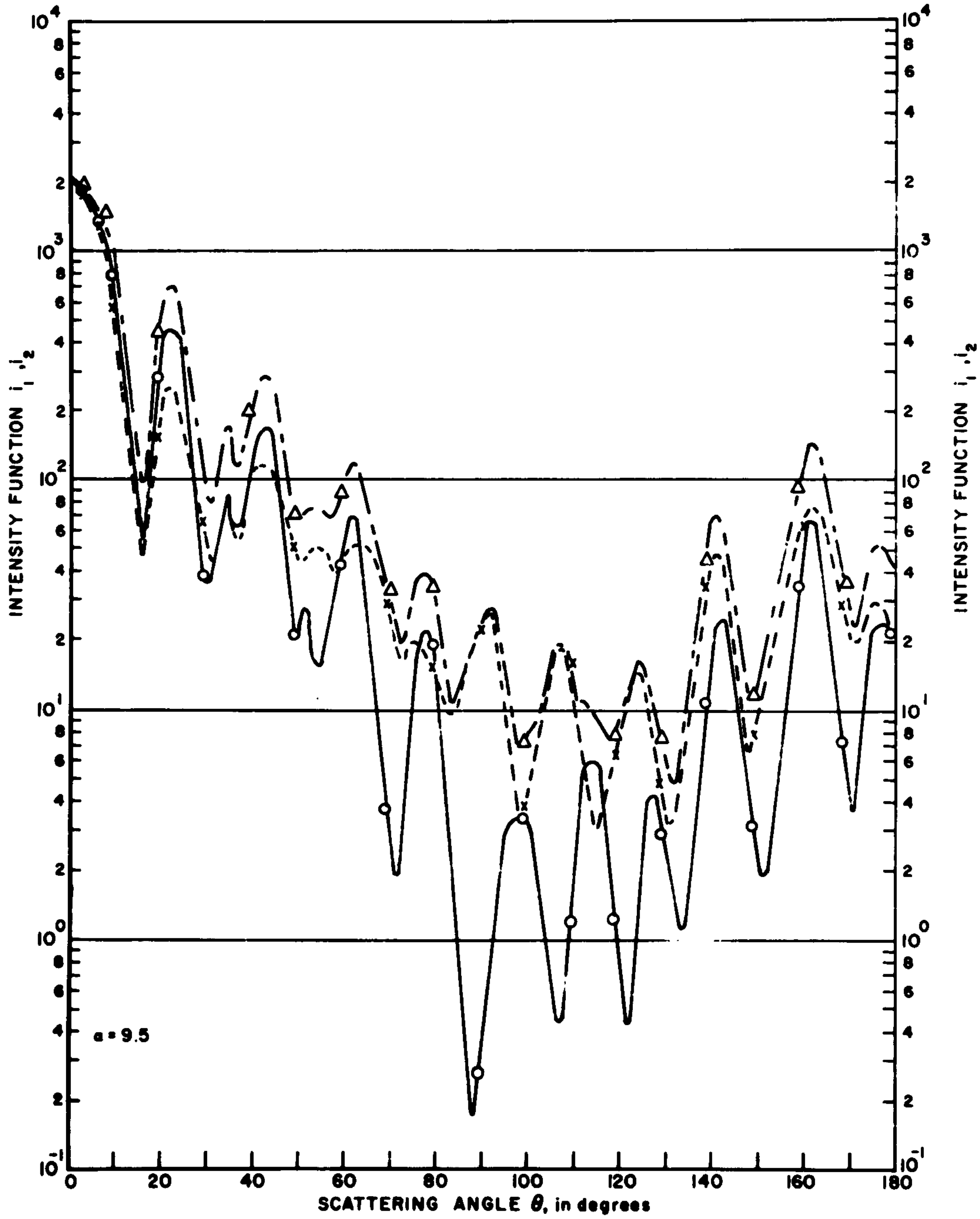
INTENSITY FUNCTION $i_1 \cdot i_2$
SCATTERING ANGLE θ, in degrees
α = 7.5
63-5680



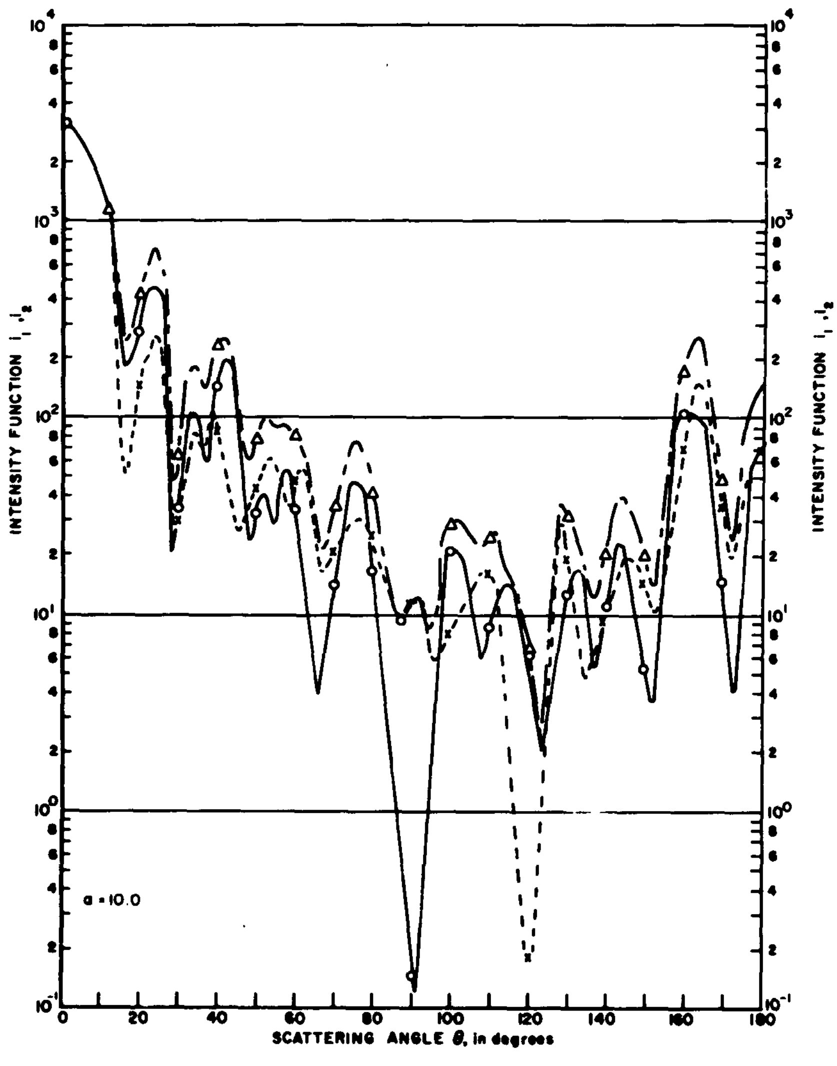
INTENSITY FUNCTION i_1, i_2
$\alpha = 8.0$
SCATTERING ANGLE θ, in degrees


INTENSITY FUNCTION i_1, i_2
α=8.5
SCATTERING ANGLE θ, in degrees


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 9.0$



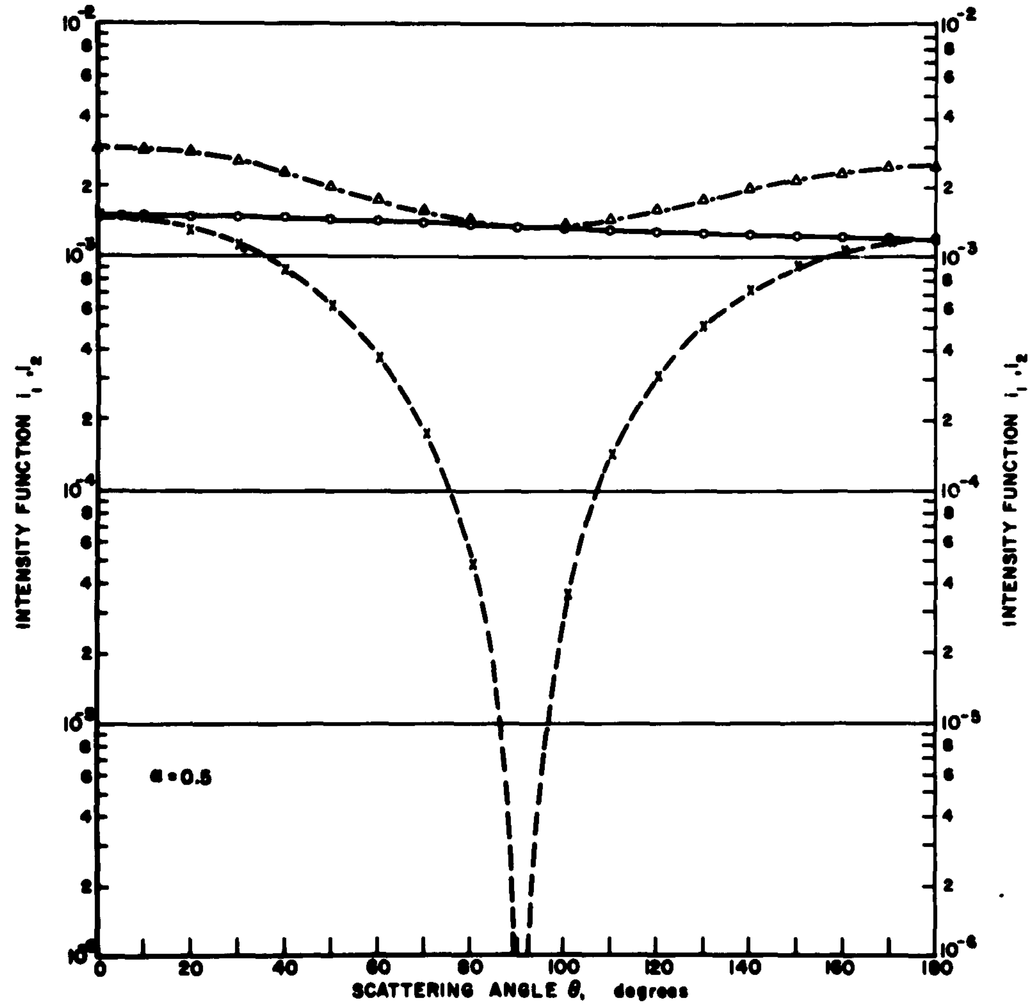
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 9.5$



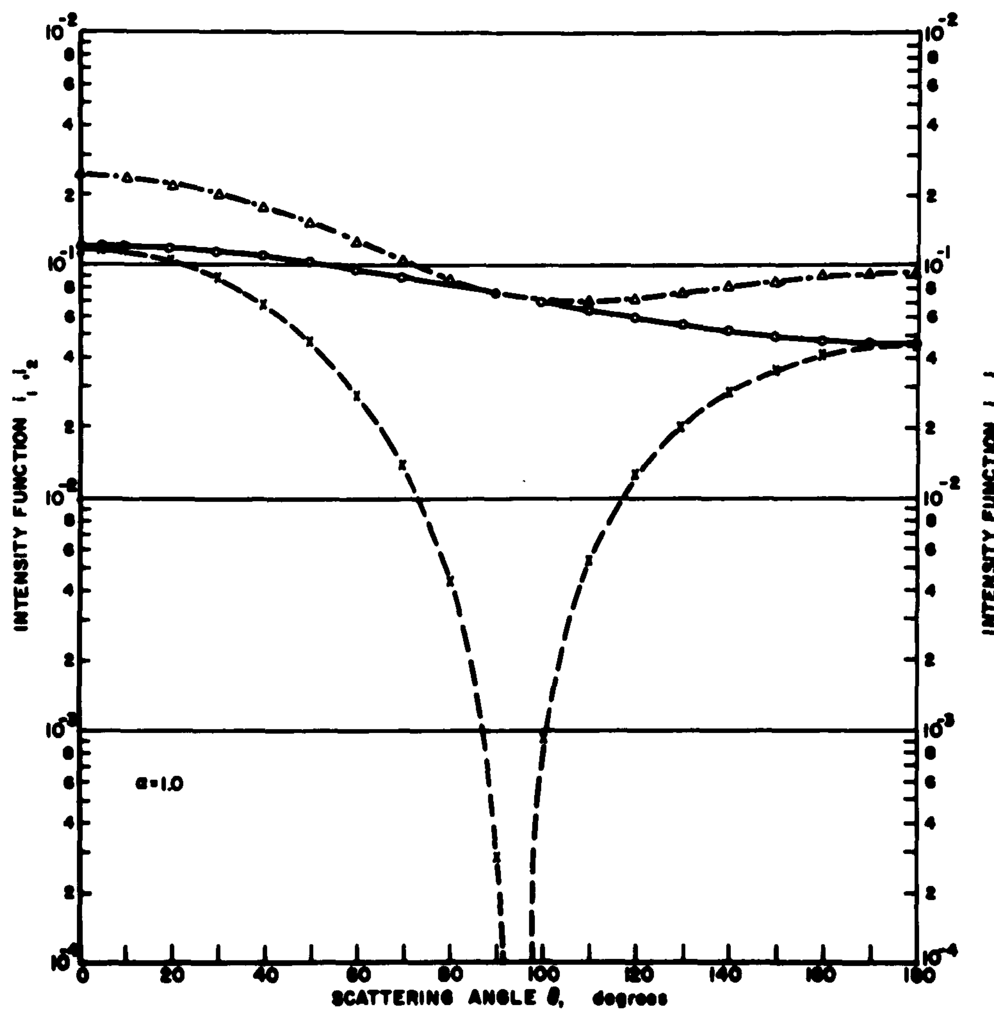
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, in degrees
$\alpha = 10.0$

6.26 Atlas of scattering diagrams

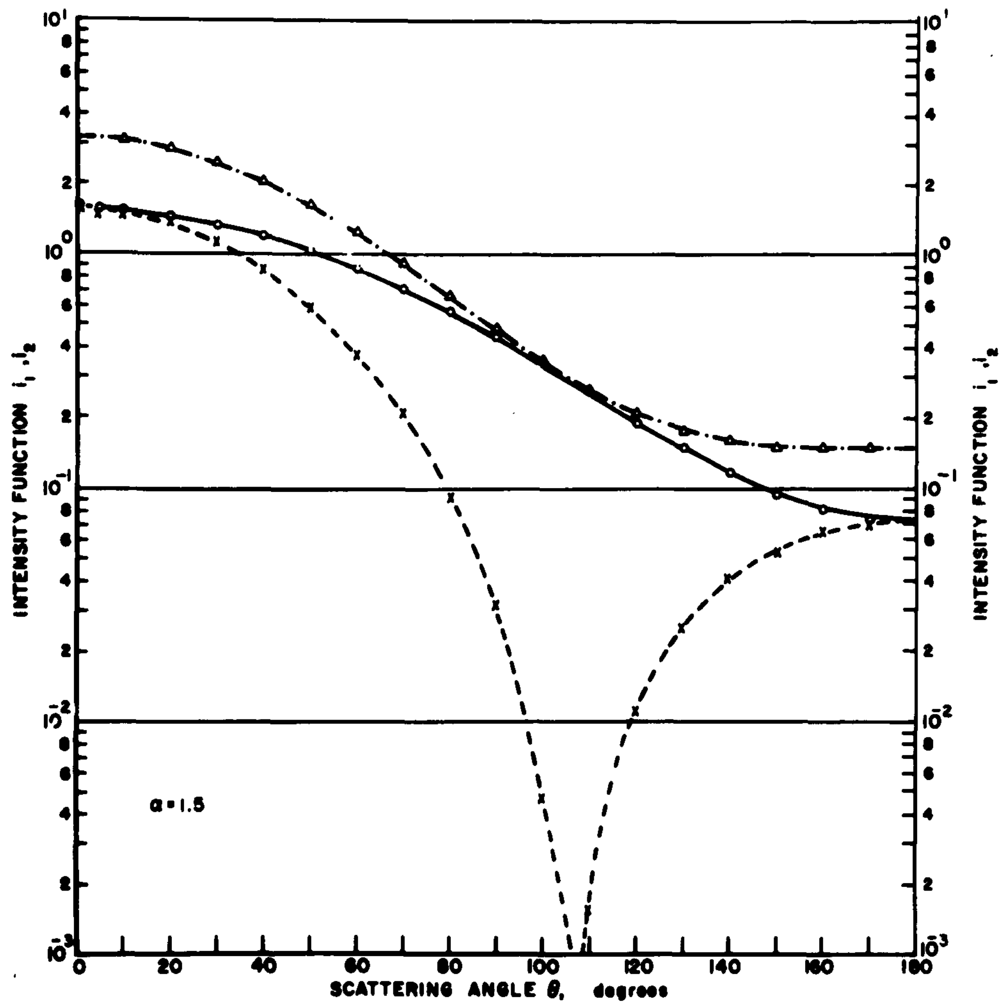
for n = 1.5



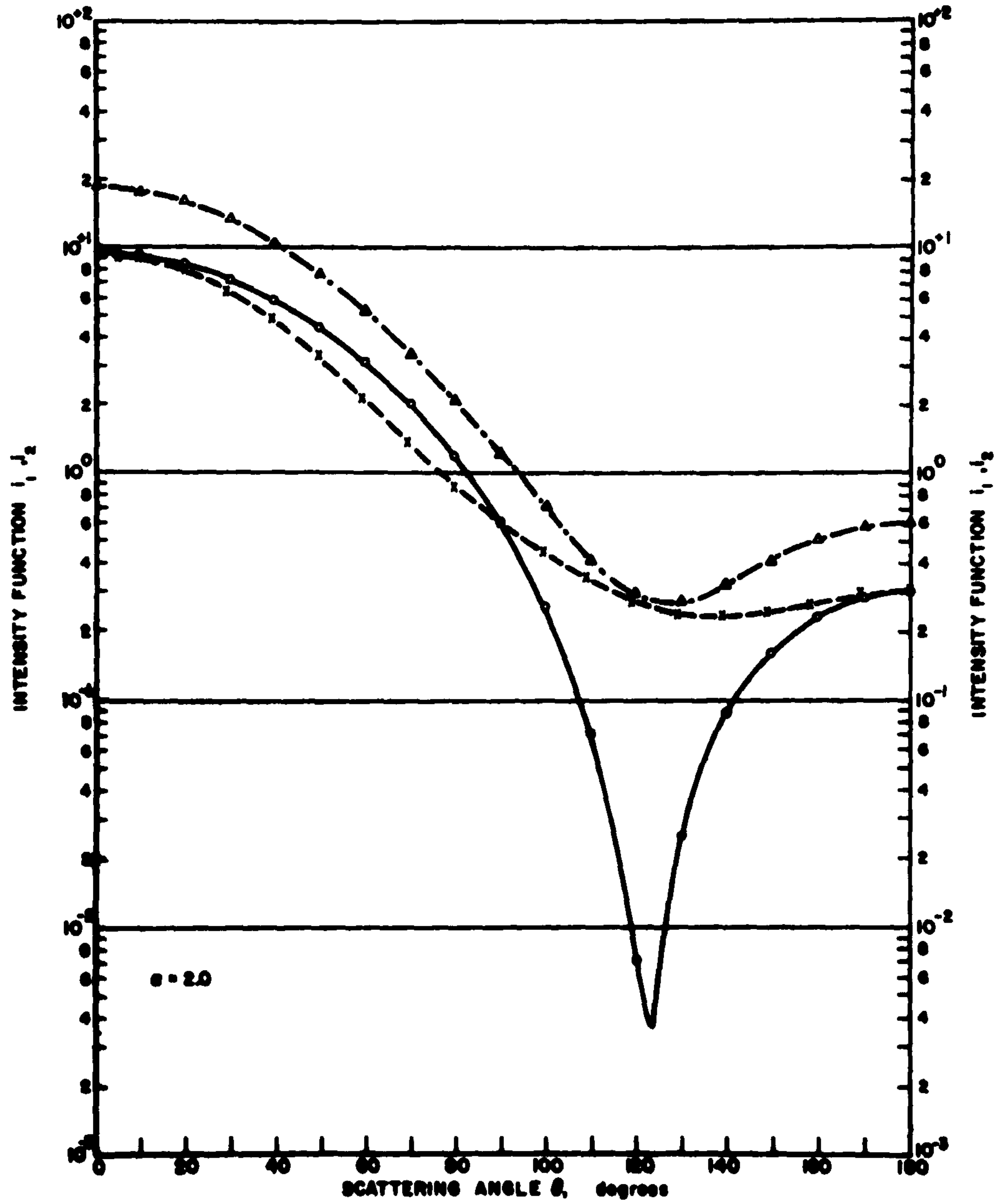
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α = 0.5

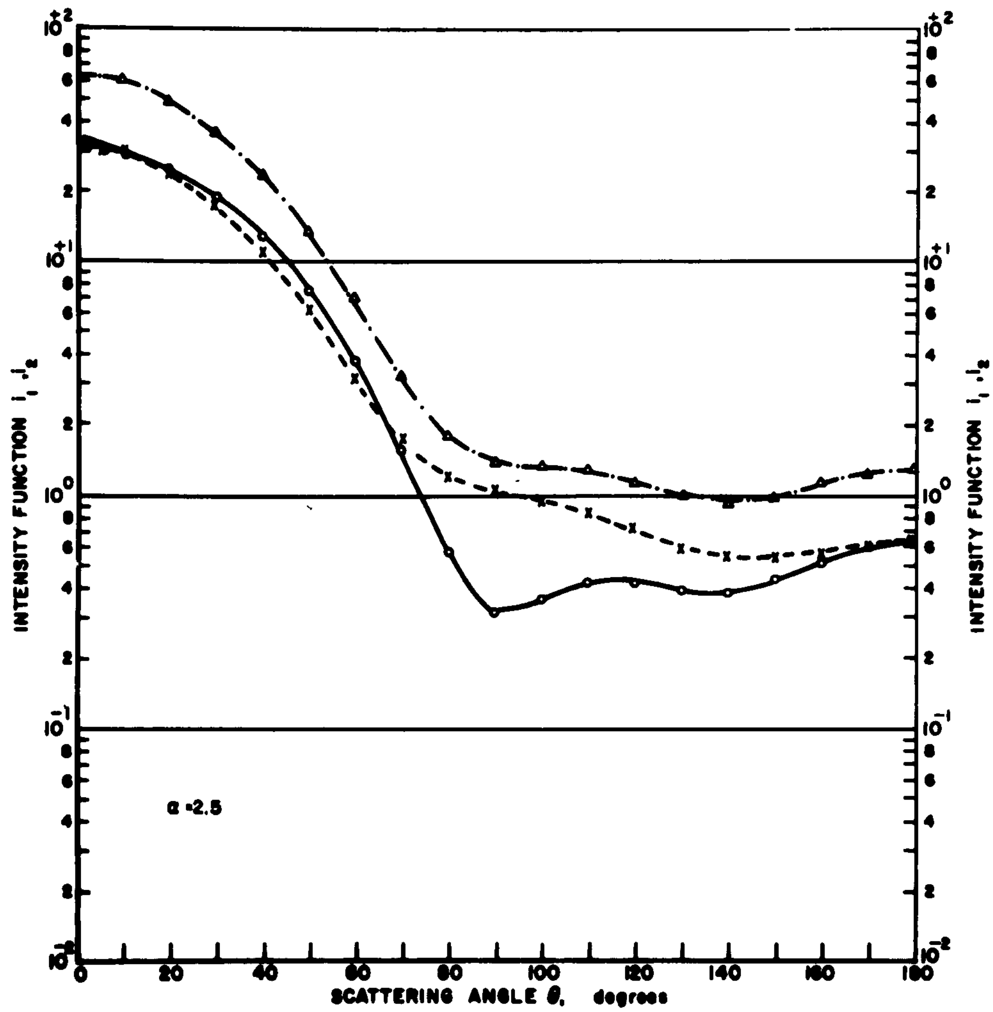


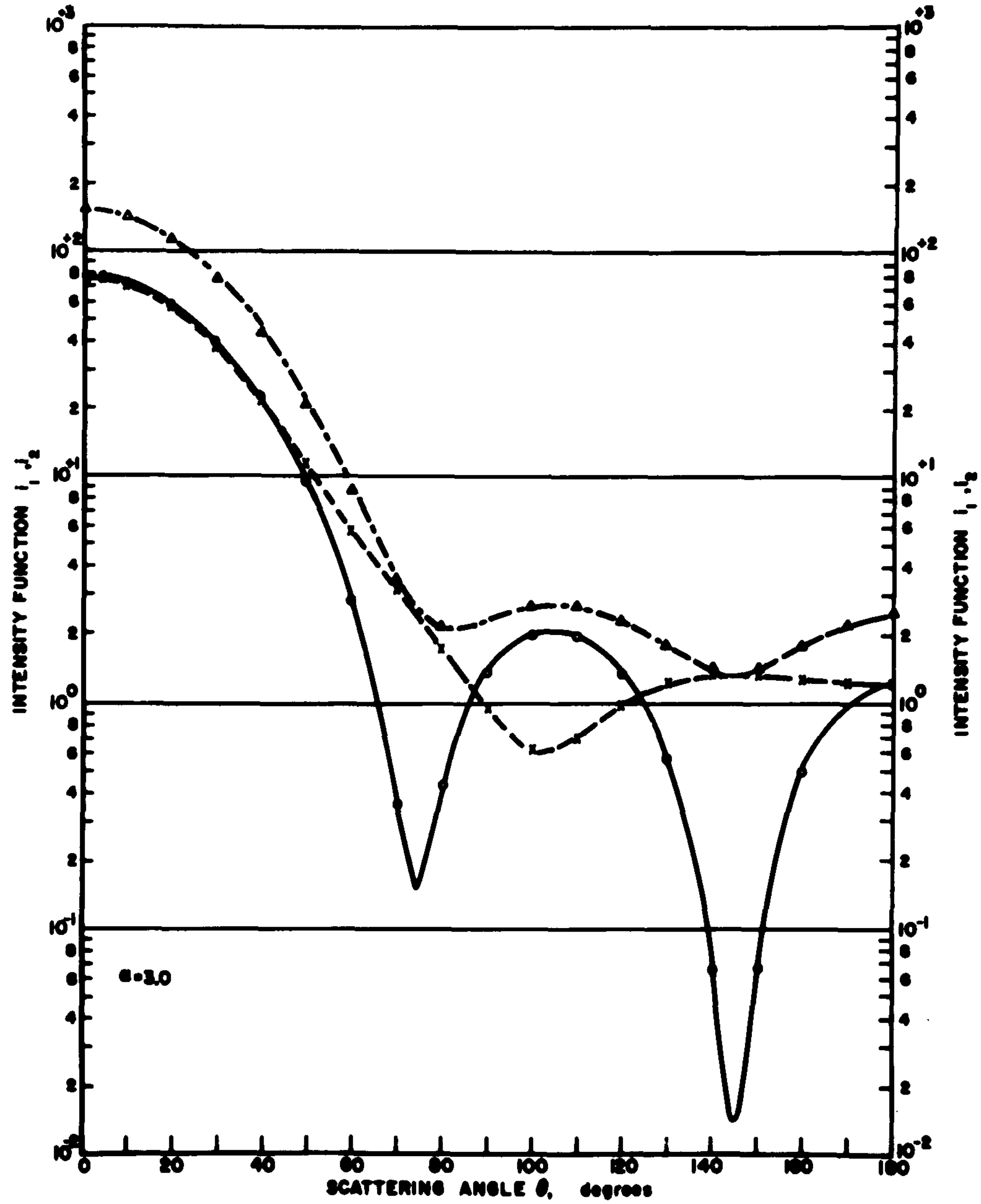
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 1.0$



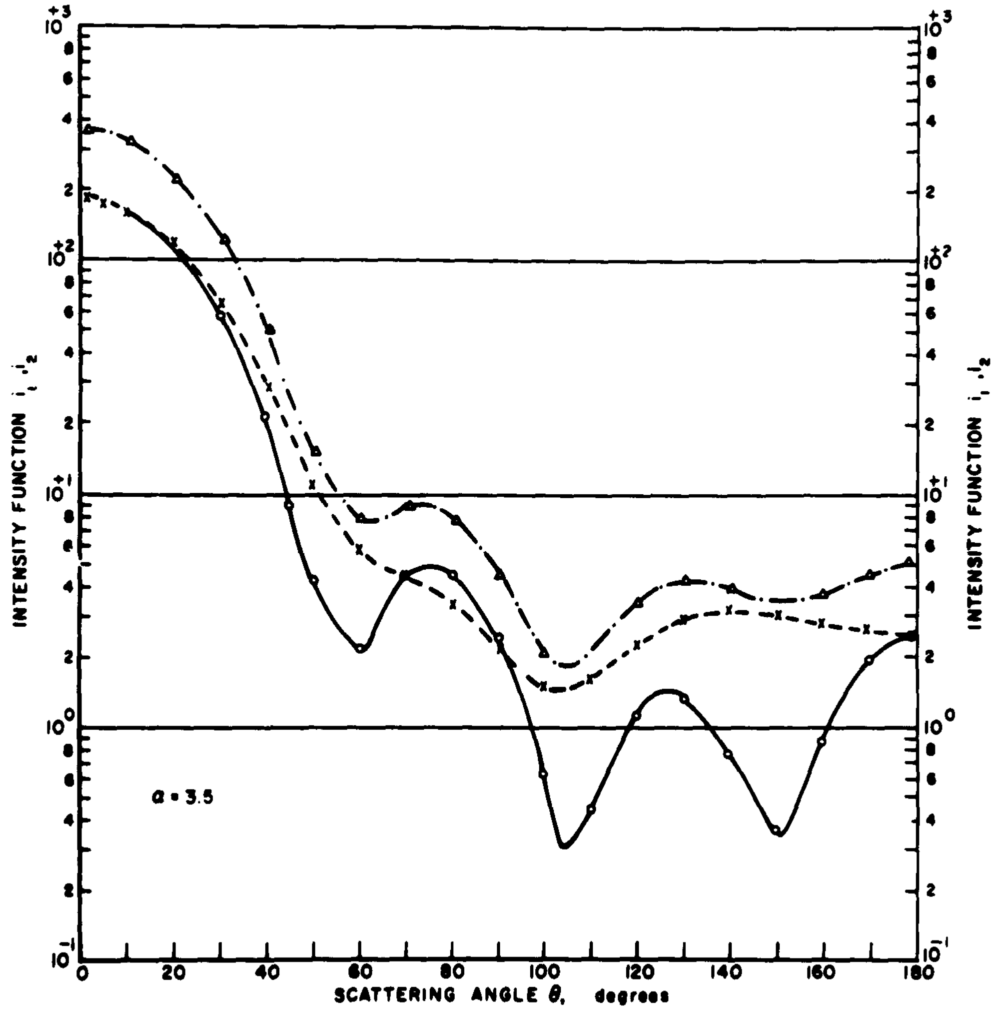
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 1.5$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α = 2.0

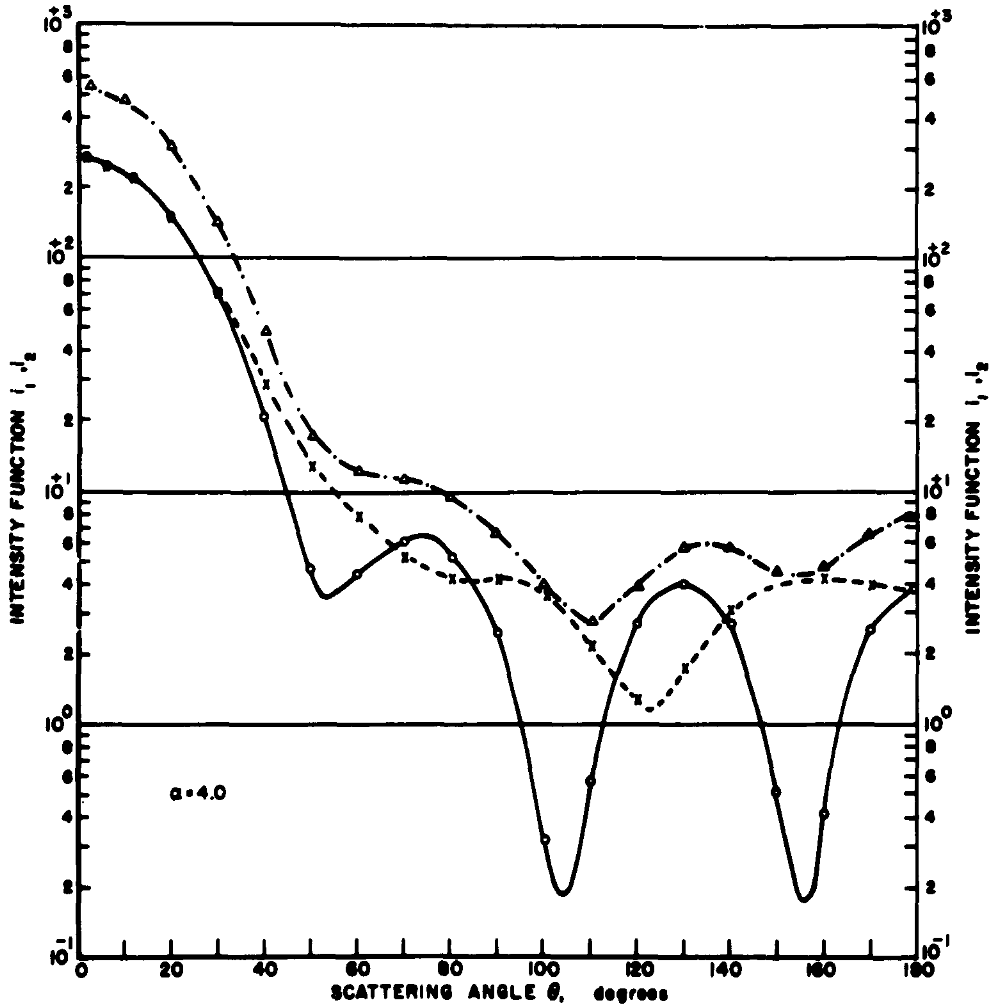

INTENSITY FUNCTION i_1+i_2
SCATTERING ANGLE θ, degrees
α =2.5
10^{+2}
10^{+1}
10^{0}
10^{-1}
10^{-2}
0
20
40
60
80
100
120
140
160
180

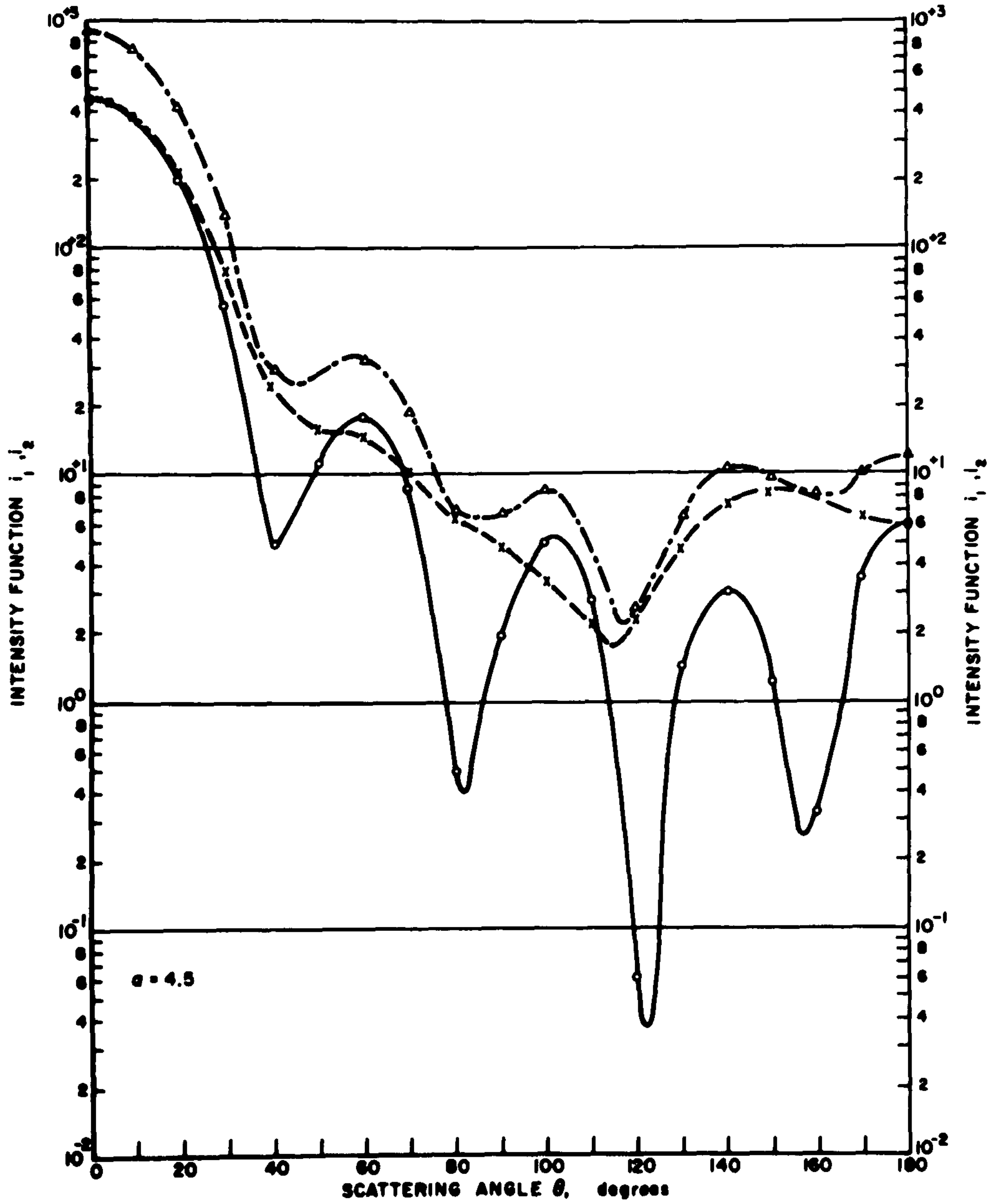


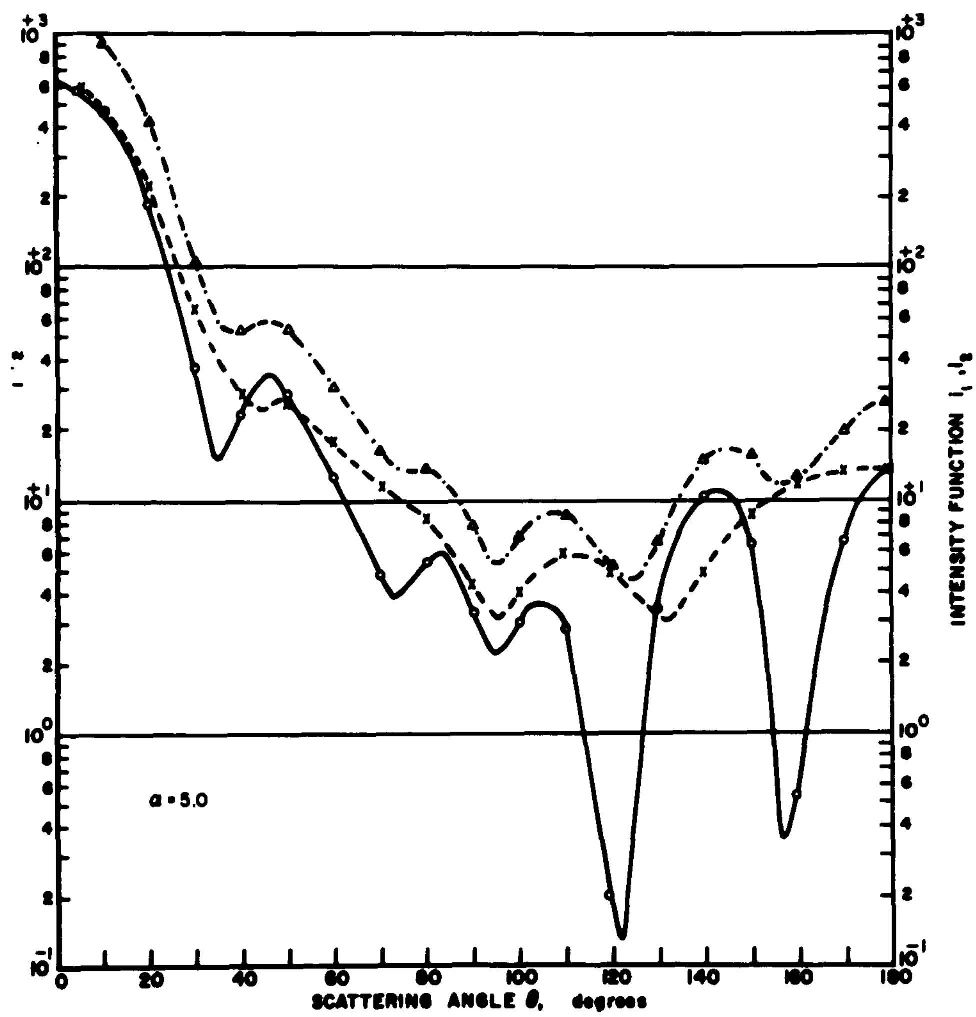
INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α=3.0



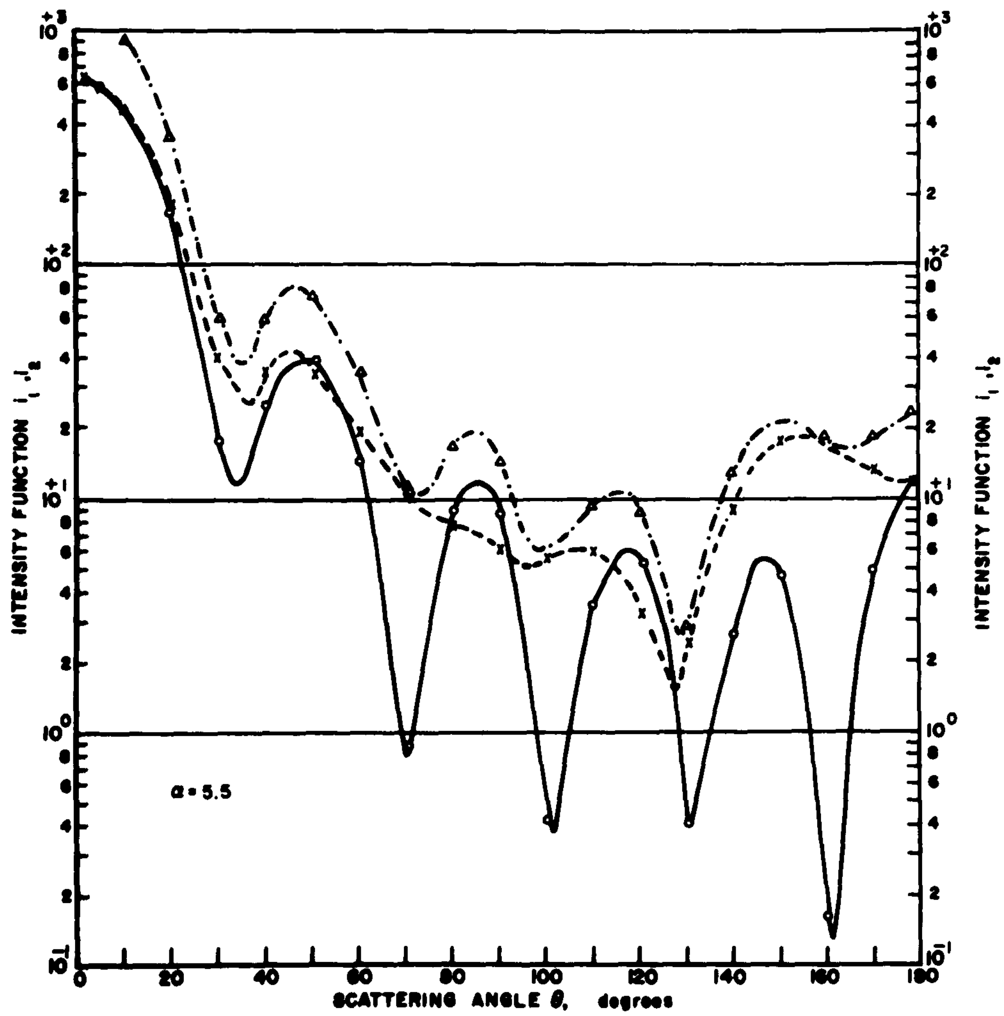
INTENSITY FUNCTION i_1, i_2
10^{+3}
10^{+2}
10^{+1}
10^{0}
10^{-1}
$\alpha = 3.5$
0 20 40 60 80 100 120 140 160 180
SCATTERING ANGLE θ, degrees

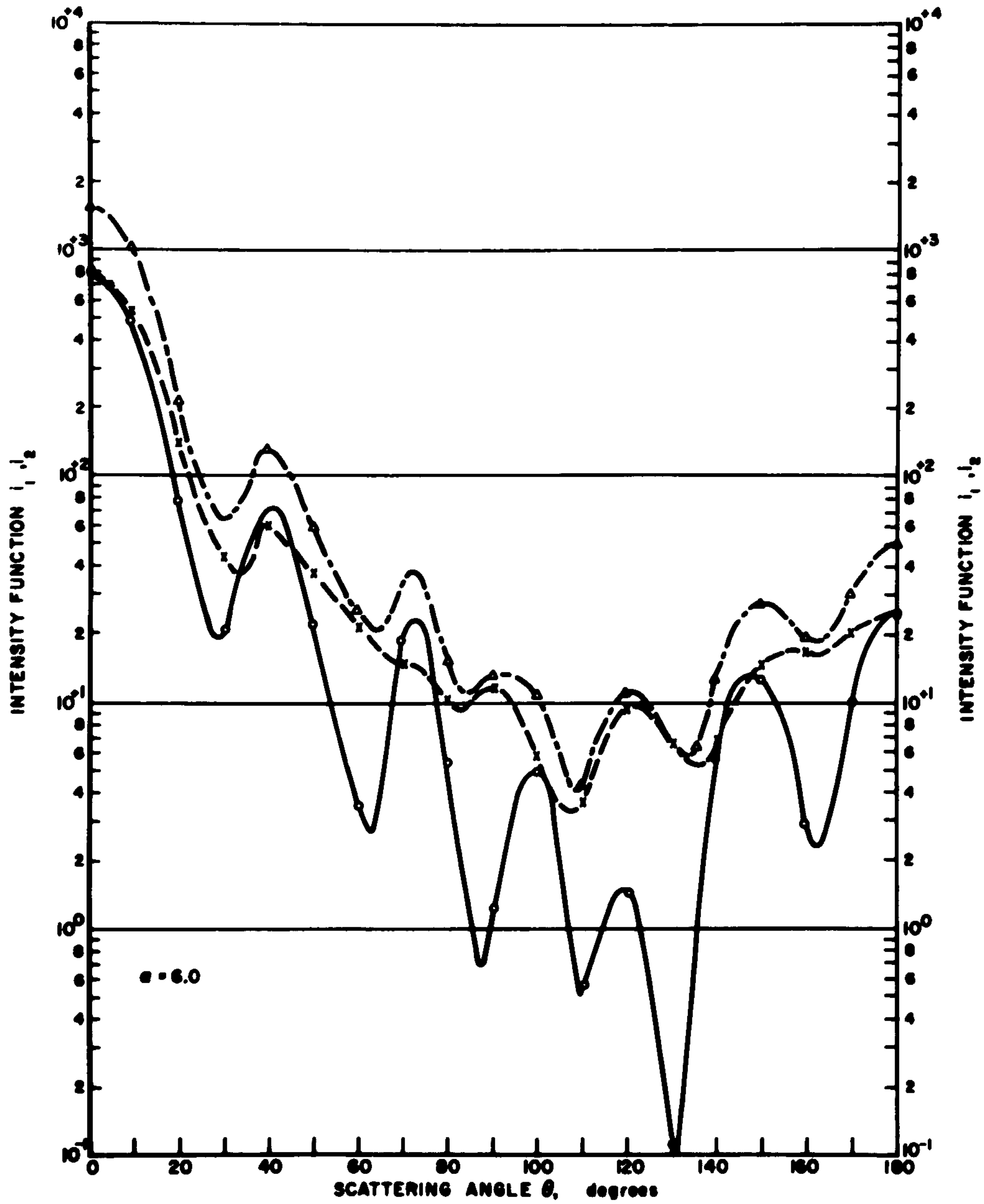

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α = 4.0

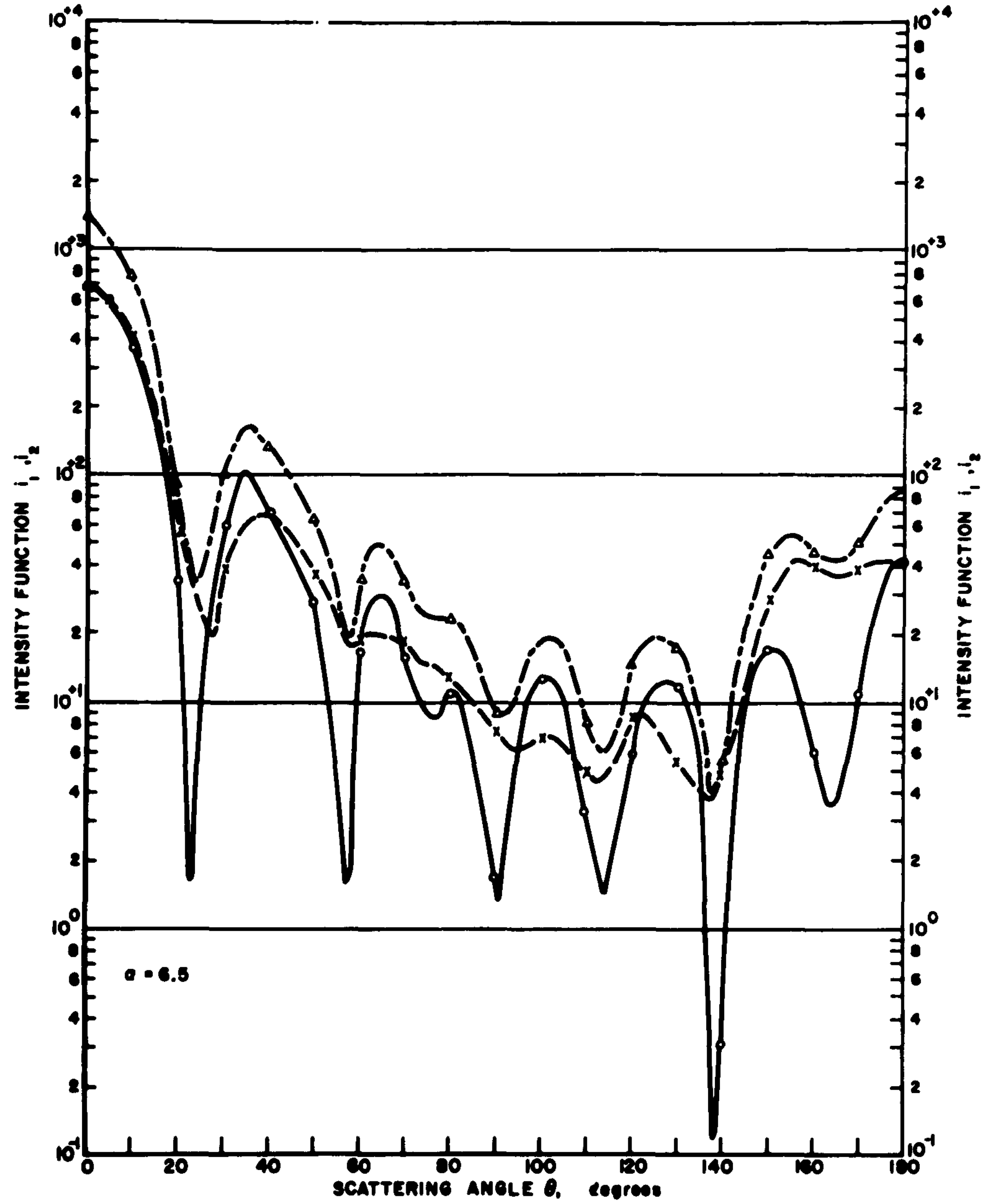

INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 4.5$



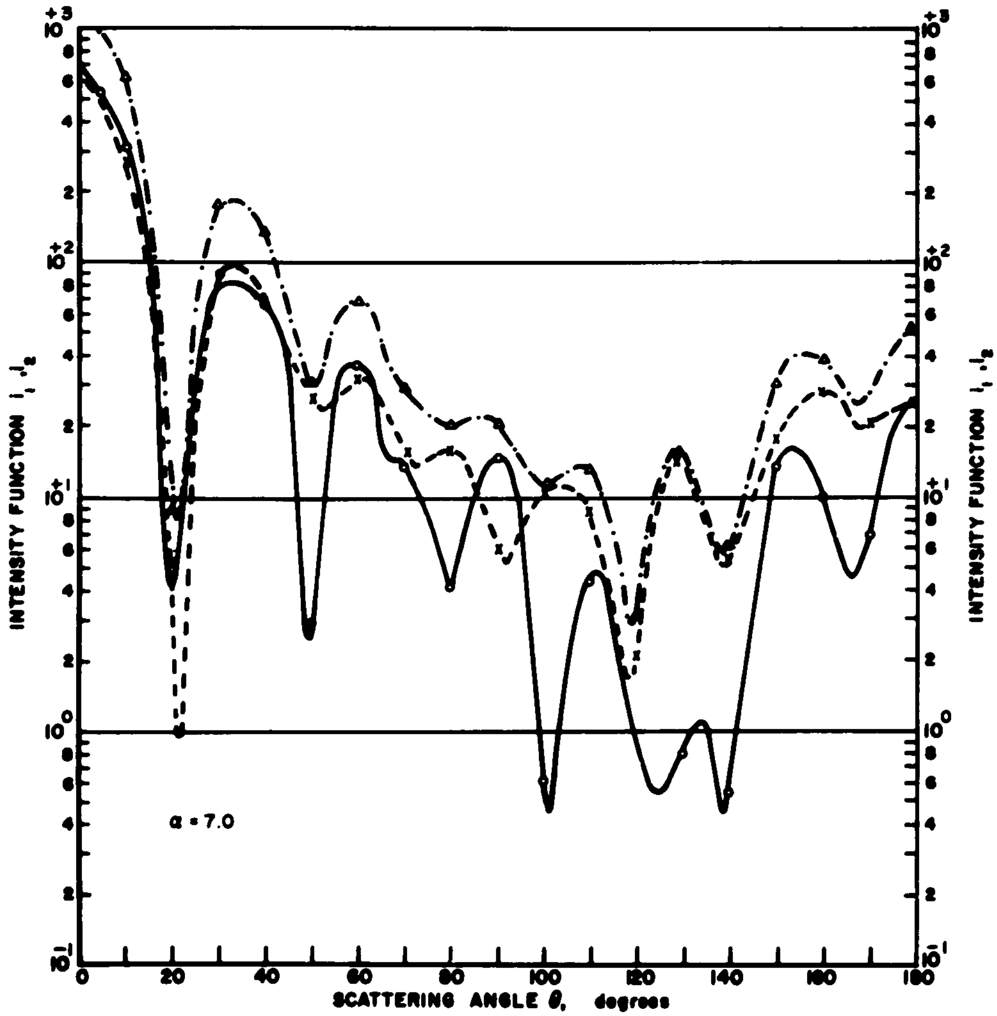
10^{+3}
8
6
4
2
10^{+2}
10^{+1}
10^{0}
10^{-1}
i_1, i_2
INTENSITY FUNCTION i_1, i_2
$\alpha = 5.0$
0
20
40
60
80
100
120
140
160
180
SCATTERING ANGLE θ, degrees


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α = 5.5

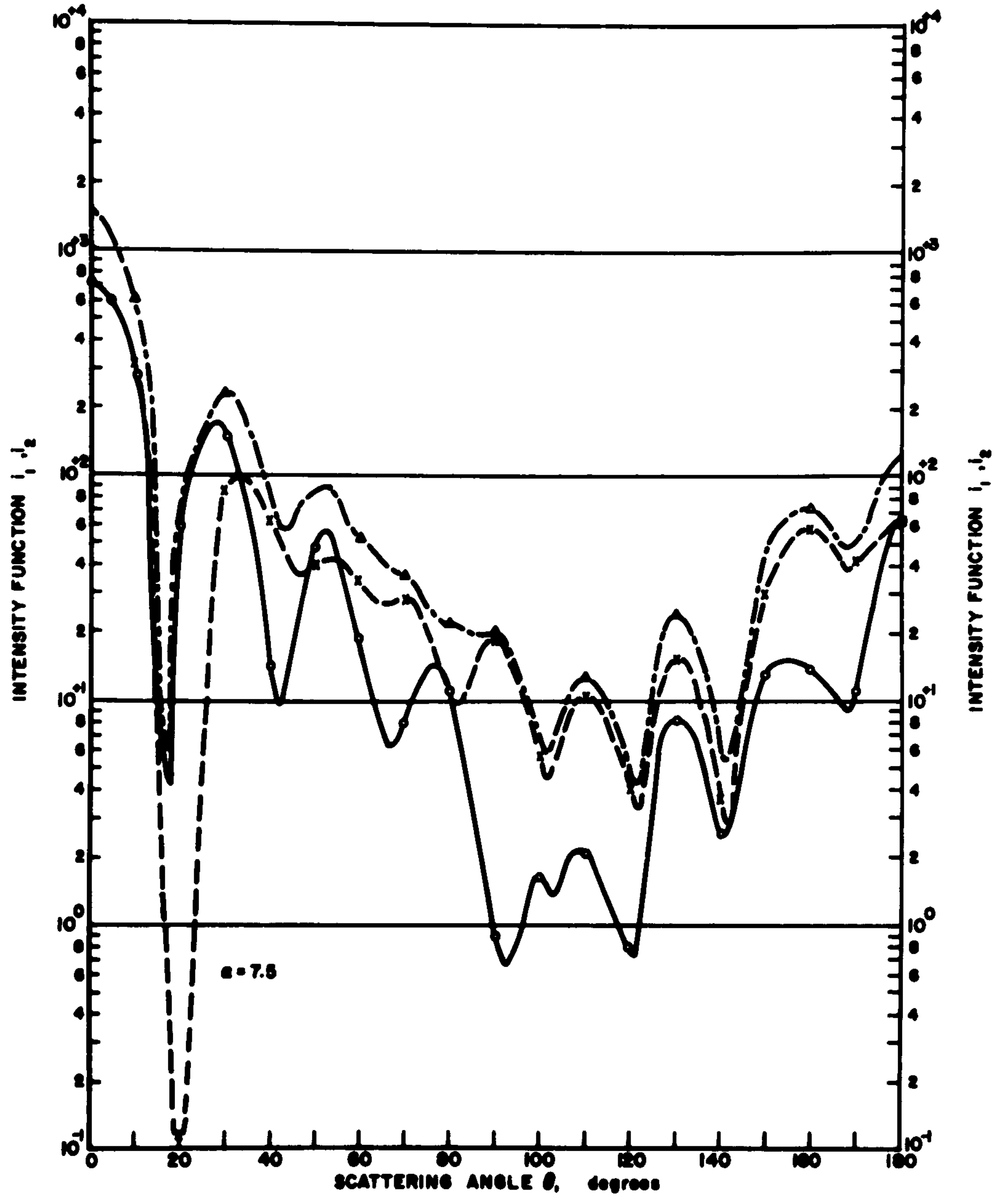

10^{+4}
10^{+3}
10^{+2}
10^{+1}
10^{0}
10^{-1}
INTENSITY FUNCTION $i_1 + i_2$
$\alpha = 6.0$
SCATTERING ANGLE θ, degrees

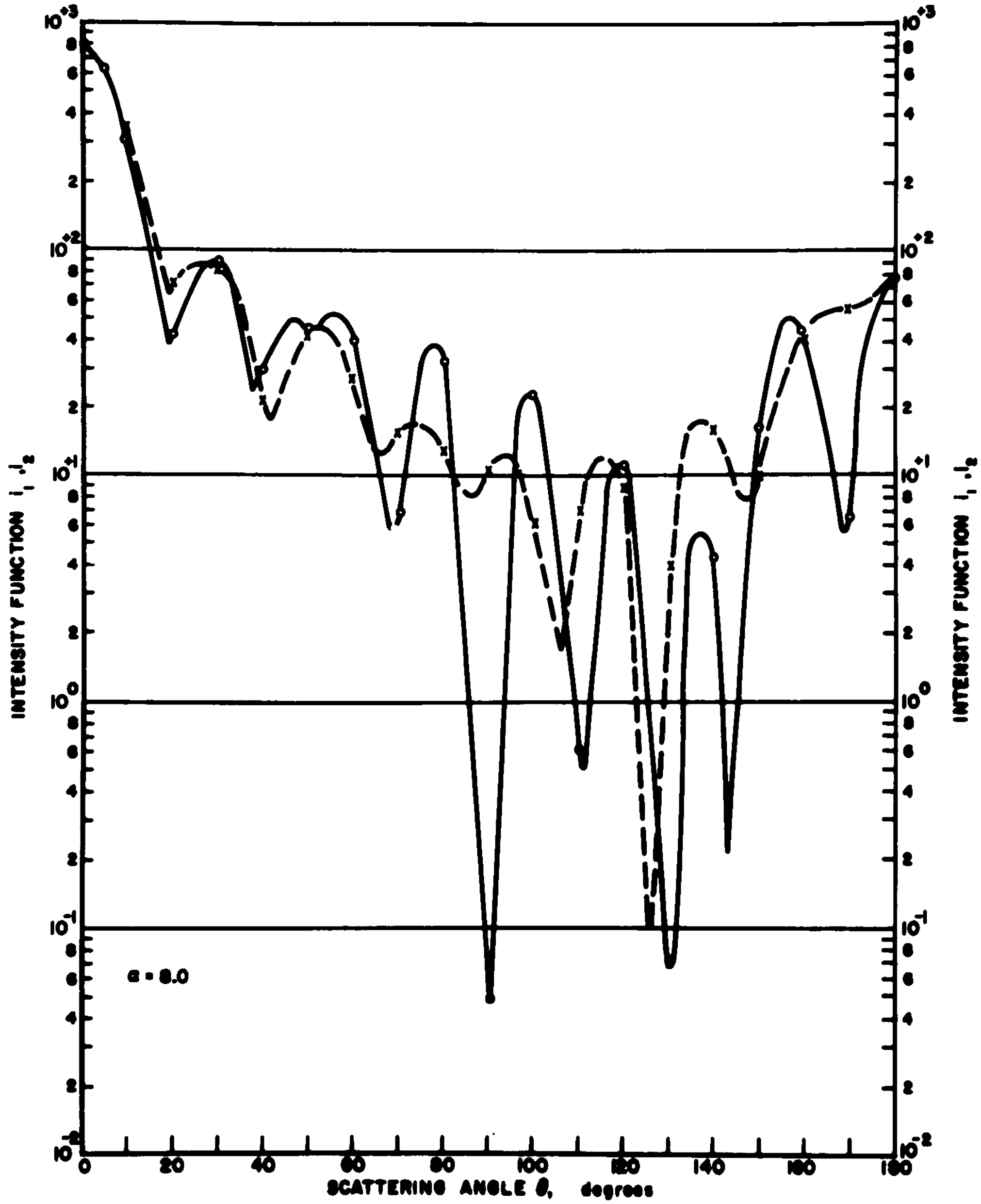


INTENSITY FUNCTION $i_1 + i_2$
SCATTERING ANGLE θ, degrees
$\alpha = 6.5$

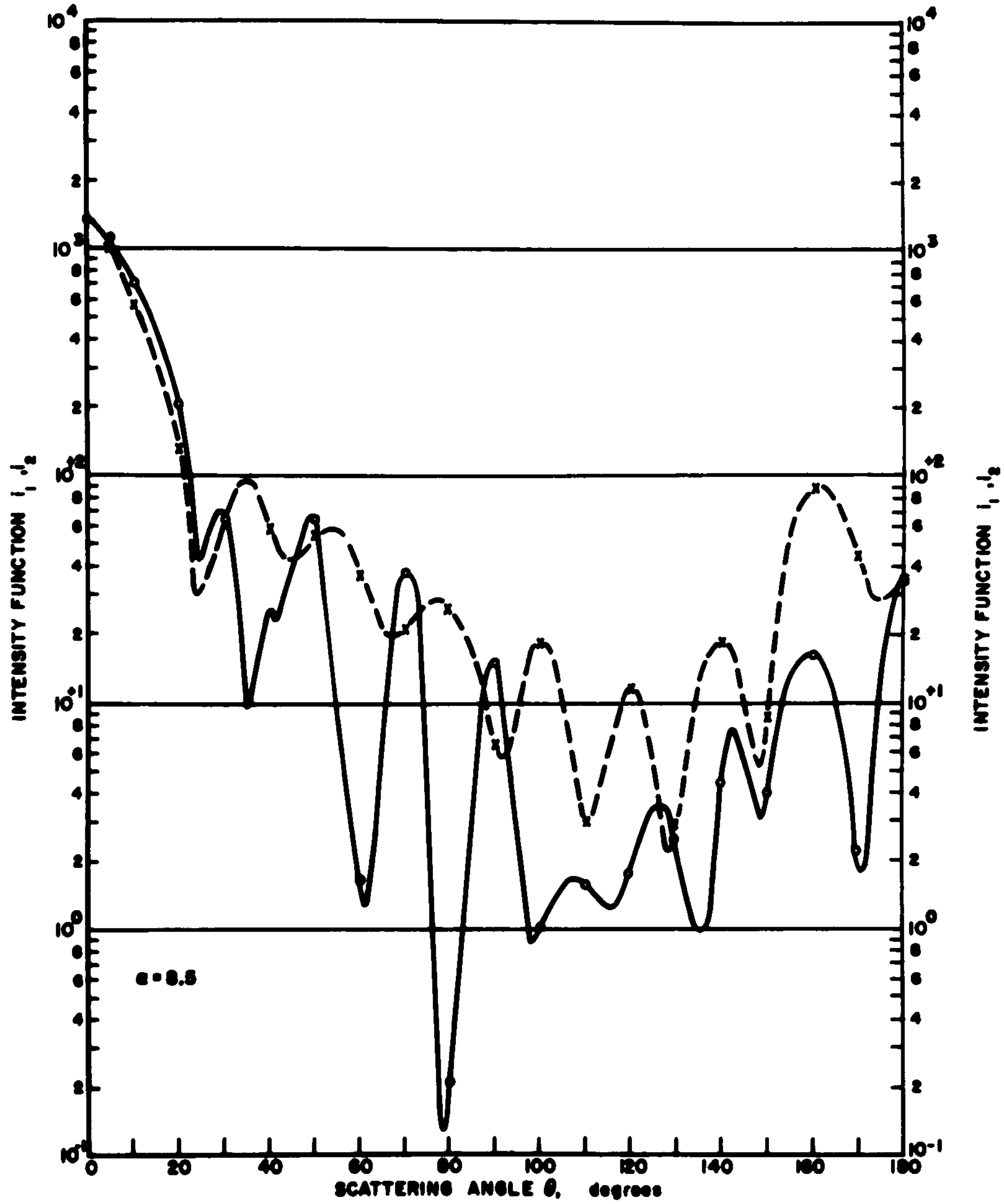


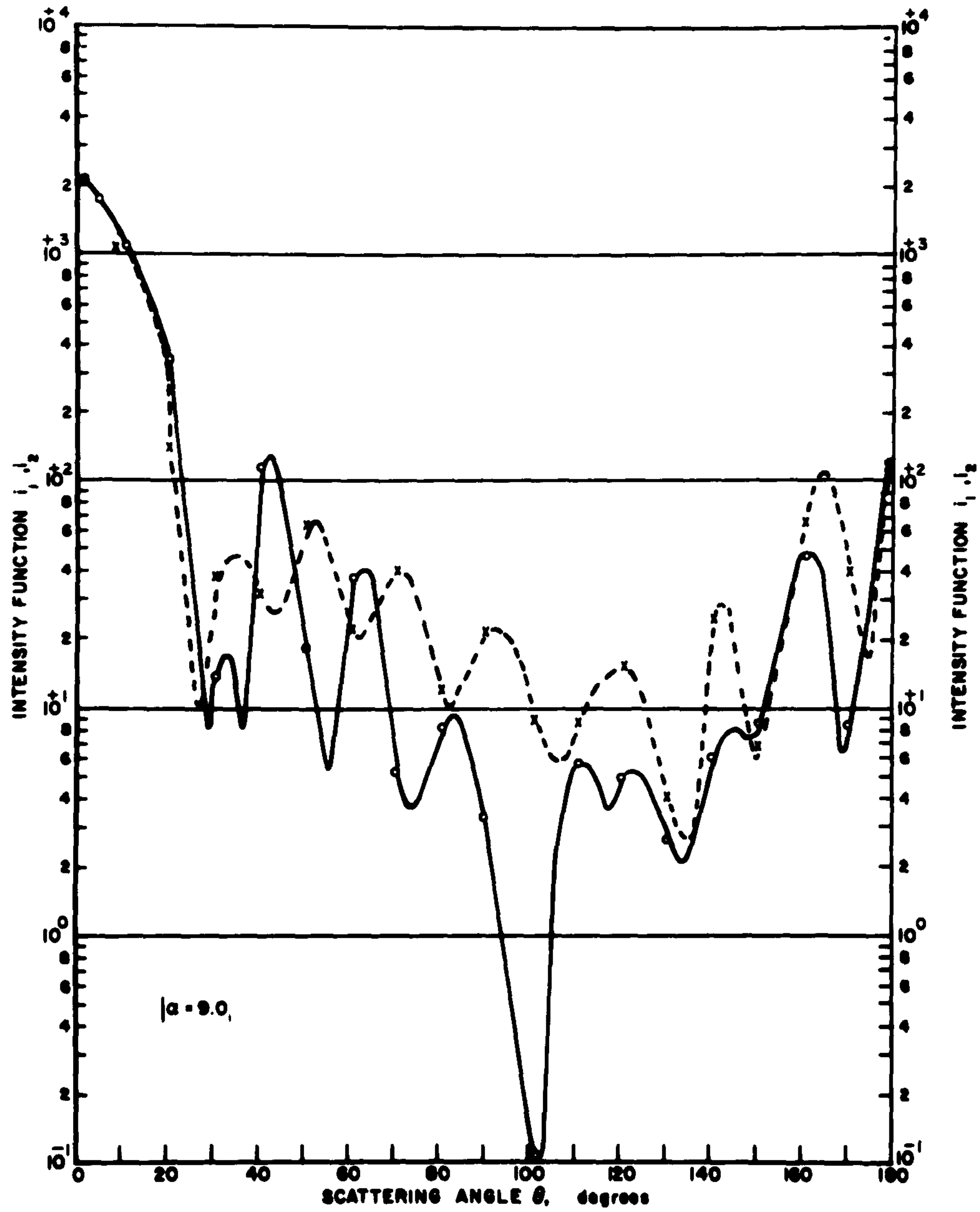
INTENSITY FUNCTION $i_1 + i_2$
SCATTERING ANGLE θ, degrees
α = 7.0

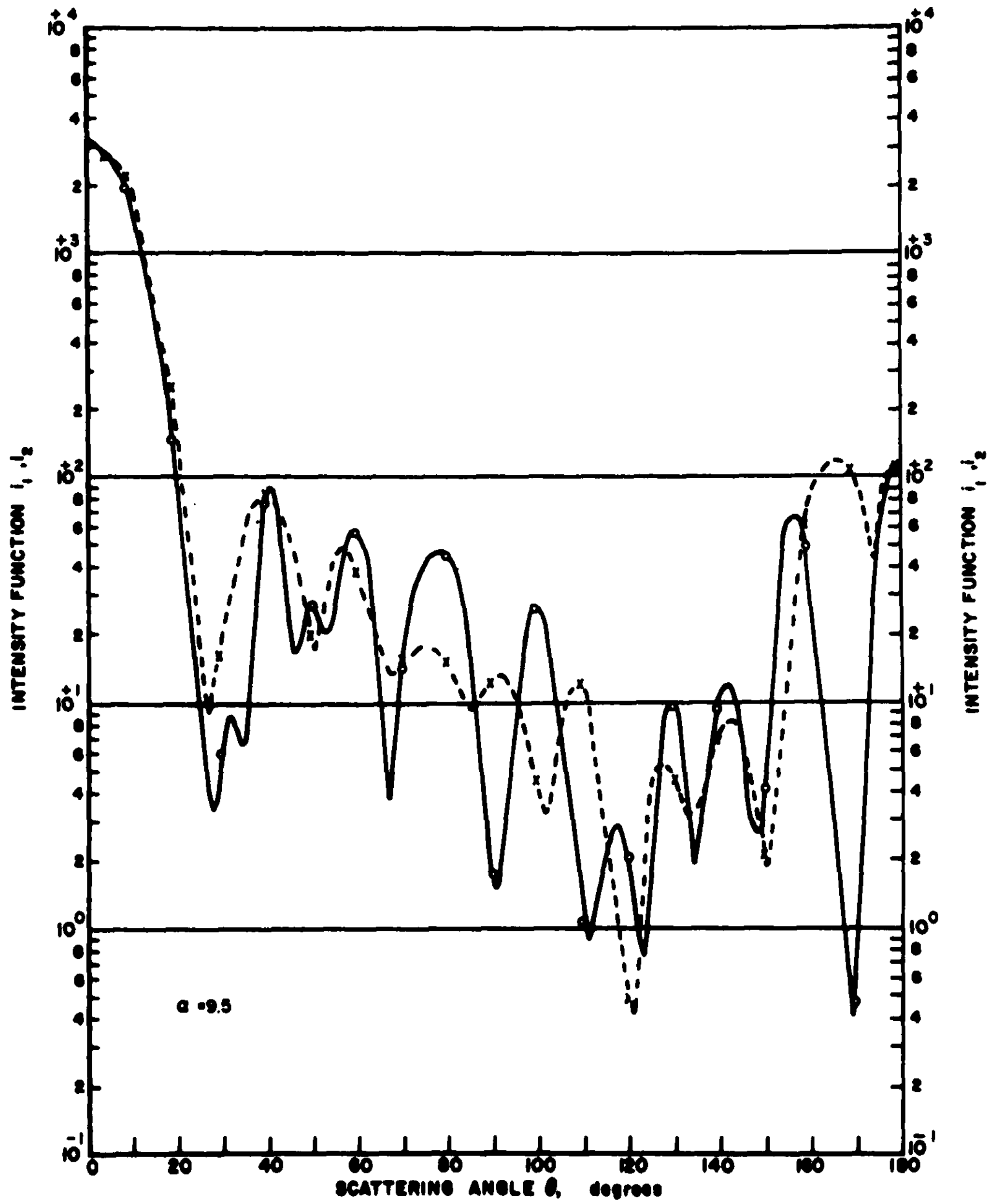

INTENSITY FUNCTION $i_1 + i_2$
SCATTERING ANGLE θ, degrees
α = 7.5

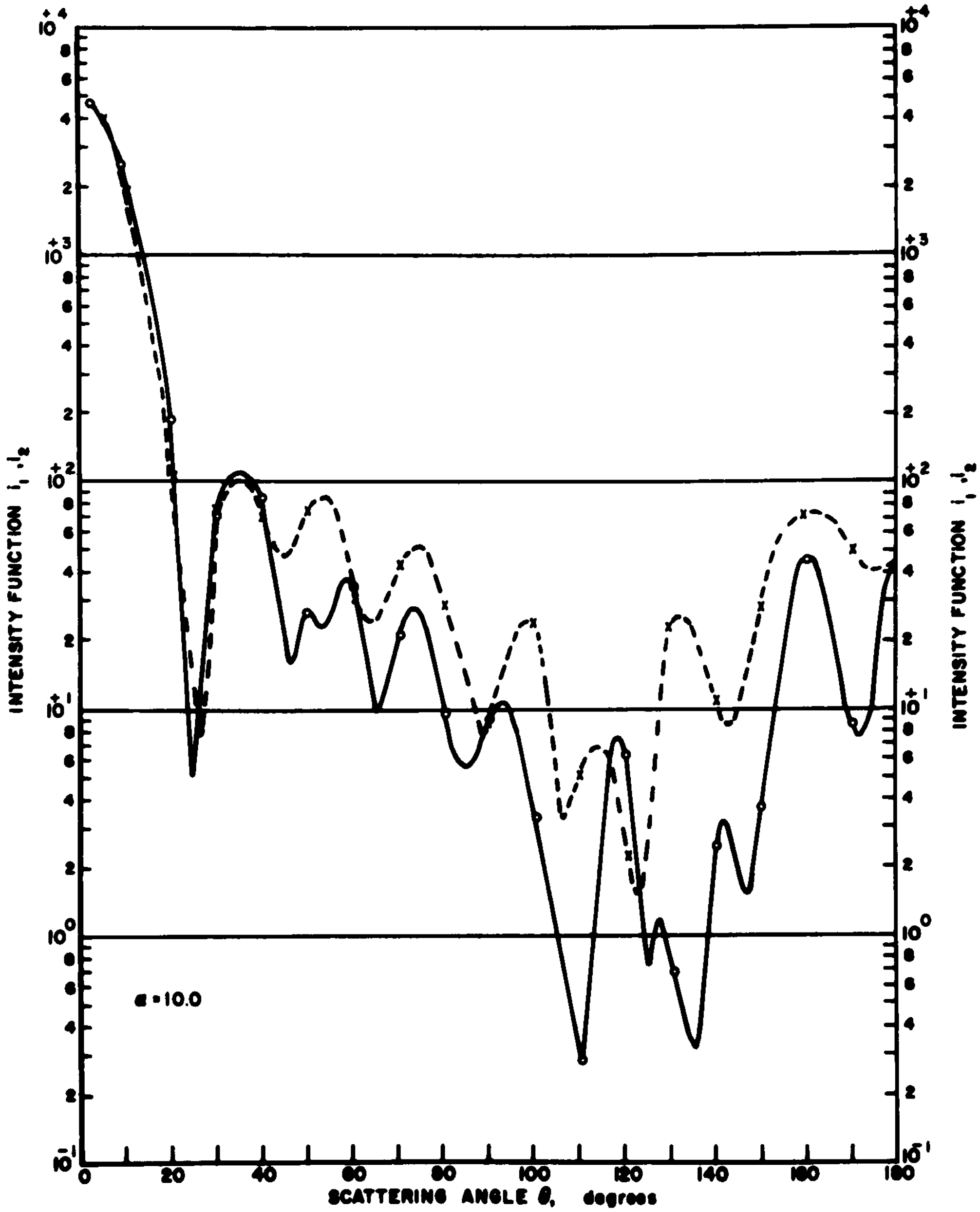


INTENSITY FUNCTION $i_1 + i_2$
10^{+3}
10^{+2}
10^{+1}
10^{0}
10^{-1}
10^{-2}
8
6
4
2
0
20
40
60
80
100
120
140
160
180
SCATTERING ANGLE θ, degrees
$\alpha = 8.0$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 8.5$


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
α = 9.0


INTENSITY FUNCTION i_1, i_2
10^{+4}
8
6
4
2
10^{+3}
10^{+2}
10^{+1}
10^{0}
10^{-1}
$\alpha = 9.5$
0
20
40
60
80
100
120
140
160
180
SCATTERING ANGLE θ, degrees


INTENSITY FUNCTION i_1, i_2
SCATTERING ANGLE θ, degrees
$\alpha = 10.0$

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Unclassified report

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2. Scattering--Mathematical analysis
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